REG-LOG-2425-ASM-SET 6-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. D	3. B	4. A	5. A
6. B	7. D	8. A	9. B	10. B
11. A	12. D	13. B	14. D	15. C
16. A	17. D	18. D	19. A	20. B
21. D	22. C	23. D	24. C	

1. C

A. ✗. $y = 2^x \longrightarrow y = -2^x \neq \left(\frac{1}{2}\right)^x$

B. ✗. $y = 3(2^x) \longrightarrow y = -3(2^x) \neq 3\left(\frac{1}{2}\right)^x$

C. ✓. $y = \log_2 x \longrightarrow y = -\log_2 x = \frac{\log x}{-\log 2} = \frac{\log x}{\log \frac{1}{2}} = \log_{\frac{1}{2}} x$

D. ✗. $y = 10^x \longrightarrow y = -10^x \neq \log x$

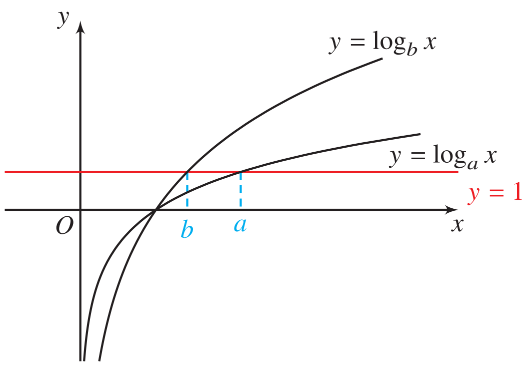
2. D

For $a > 1$, the rate of decrease of $\log_a x$ increases as x decreases.
The answer is D.

3. B

Draw the line $y = 1$.

The line intersects the graphs at $(a,\ 1)$ and $(b,\ 1)$.
The x-intercepts of the graphs are both 1.
From the graph, we have $1 < b < a$.



4. A

A. ✓.

B. ✗. When $x = 2$, $y = \frac{1}{2}\log 2 \neq 1$.

C. ✗. When $x = 1$, $y = \log(1 - 2) = \log(-1)$ is undefined.

D. ✗. When $x = 1$, $y = \log(1 + 2) = \log 3 \neq 0$.

5. $\boxed{A}$

A. ✓.

B. ✗. The graph passes through $(1,\ 0)$ but $y = 5^1 = 5 \neq 0$.

C. ✗. The graph passes through $(1,\ 0)$ but $y = 0.2^1 = 0.2 \neq 0$.

D. ✗. When $y = 1$, $x = 0.2^1 = 0.2$ but the graph shown does not pass through $(0.2,\ 1)$ obviously.

6. $\boxed{B}$

$$\begin{aligned}\frac{AC}{AB} &= \frac{\log_c k}{\log_b k} \\ &= \frac{\log k}{\log c} \div \frac{\log k}{\log b} \\ &= \frac{\log b}{\log c} \\ &= \log_c b\end{aligned}$$

7. $\boxed{D}$

Suppose the equation of L is $x = k$.

I. ✗. $y = \log_b x$ and the line $y = 1$ intersect at $(b,\ 1)$.
The x-intercept of the graph of $y = \log_b x$ is 1.
Thus, we have $0 < b < 1$ from the graph.

II. ✓. The coordinates of A and B are $\left(k,\ \log_a k\right)$ and $\left(k,\ \log_b k\right)$. Since $AC = BC$,

$$\begin{aligned}\log_a k &= -\log_b k \\ \frac{\log k}{\log a} &= -\frac{\log k}{\log b} \\ \log b &= -\log a \\ \log ab &= 0 \\ ab &= 1\end{aligned}$$

III. ✓. The x-intercept of the curves is 1.
Thus, $OC > 1$.

8. **A**

When $y = 0$,

$$0 = \log_3 x \quad \text{and} \quad 0 = \log_4 x$$
$$x = 1 \qquad\qquad x = 1$$

When $y = 1$,

$$1 = \log_3 x \quad \text{and} \quad 1 = \log_4 x$$
$$x = 3 \qquad\qquad x = 4$$

The answer is A.

9. **B**

A. ✗. When $x = 1$, $y = \log 2 \neq 0$.

B. ✓.

C. ✗. When $x = 1$, $y = \log \frac{1}{2} \neq 0$.

D. ✗. When $y = 1$, $x = 10^1 = 10$ but the graph does not pass through $(10,\ 1)$ obviously.

10. **B**

The coordinates of A and B are $(0,\ k)$ and $(0,\ q)$ respectively.

$$q = ka^x$$
$$a^x = \frac{q}{k}$$
$$x = \log_a \frac{q}{k}$$

The coordinates of C are $\left(\log_a \frac{q}{k},\ q\right)$.

I. ✓.
The y-coordinate of A is negative.

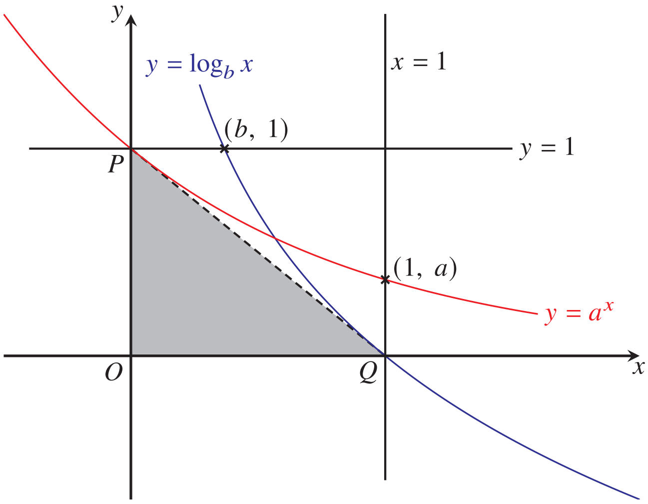
II. ✗.
The value of ka^x approaches zero when x increases.
Thus, $0 < a < 1$.

III. ✓.

$$BC < OA$$
$$0 - \log_a \frac{q}{k} < 0 - k$$
$$\log_a \frac{q}{k} > k$$

11. $\boxed{\text{A}}$

Note that the coordinates of P and Q are $(0,\ 1)$ and $(1,\ 0)$ respectively.



I. ✓. Observe the y-coordinates. We have $0 < a < 1$.

II. ✓. The graph of reflection image of $y = a^x$ is $y = \log_a x$.
We have $a = b$ and $\dfrac{a}{b} = 1$.

III. ✗. Note that $0 < a < 1$ and $0 < b < 1$.

$$\begin{aligned}\text{Area of } \triangle OPQ &= \frac{(1)(1)}{2} \\ &= \frac{1}{2} \\ &\neq \frac{1}{2}ab\end{aligned}$$

12. $\boxed{\text{D}}$

The equation of C_2 is

$$\begin{aligned} y &= -\log_4 x \\ &= \frac{\log x}{-\log 4} \\ &= \frac{\log x}{\log \frac{1}{4}} \\ &= \log_{\frac{1}{4}} x \end{aligned}$$

13. $\boxed{\text{B}}$

I. ✓.

II. ✓.

III. ✗. The graph of $y = \log_a x$ should pass through $(1,\ 0)$.

14. D

$$0 = \log_a(ax)$$
$$ax = 1$$
$$x = \frac{1}{a}$$

The coordinates of P are $\left(\frac{1}{a},\ 0\right)$.

$$0 = \log_b(x + b) \quad \text{and} \quad y = \log_b(0 + b)$$
$$x + b = 1 \qquad\qquad\qquad\qquad = 1$$
$$x = 1 - b$$

The coordinates of Q and R are $(1 - b,\ 0)$ and $(0,\ 1)$ respectively.

Required area

$$= \frac{1}{2} \times \left(\frac{1}{a} - (1 - b)\right) \times 1$$
$$= \frac{1}{2a} + \frac{b}{2} - \frac{1}{2}$$

15. C

The x-intercept of two graphs are 1.

Let the coordinates of C be $(c,\ 0)$.

I. ✓.

The graph of $y = \log_{\frac{1}{b}} x$ cuts the line $y = 1$ at $\left(\frac{1}{b},\ 1\right)$, which lies between the lines $x = 0$ and $x = 1$.

$$0 < \frac{1}{b} < 1$$
$$b > 1$$

II. ✗.

Suppose that $AC = CB$.

$$\log_a c = -\log_{\frac{1}{b}} c$$
$$\frac{\log c}{\log a} = \frac{\log c}{\log b}$$
$$a = b$$

The statement is not true in this case.

III. ✓.

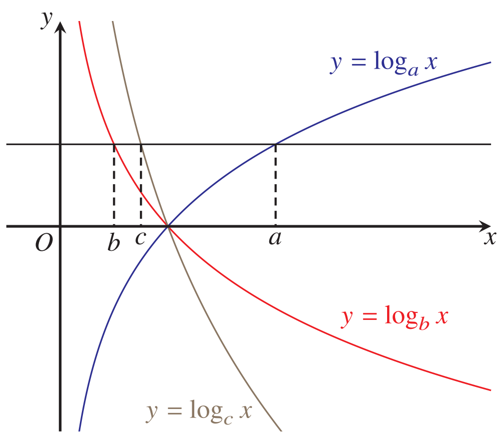
$$\begin{aligned}\frac{AB}{BC} &= (\log_a c - \log_{\frac{1}{b}} c) \div (-\log_{\frac{1}{b}} c) \\ &= \left(\frac{\log c}{\log a} + \frac{\log c}{\log b}\right) \div \frac{\log c}{\log b} \\ &= \frac{\log b}{\log a} + 1 \\ &= \frac{\log b + \log a}{\log a} \\ &= \log_a ab\end{aligned}$$

16. A

Draw the line $y = 1$.

Note that the x-intercepts of the graphs are 1.

We have $0 < b < c < 1 < a$.



17. D

A. ✗. The graph passes through $(1,\ 0)$ but $\log_2 0$ is undefined.

B. ✗. The graph passes through $(1,\ 0)$ but $\log_{0.5} 0$ is undefined.

C. ✗. When $y = 1$, $x = 2^1 = 2$. The graph shown does not pass through $(2,\ 1)$ obviously.

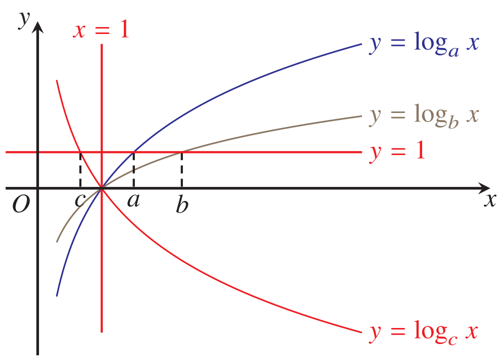
D. ✓.

18. D

Draw the line $y = 1$.

The line intersects the graphs at $(a,\ 1)$, $(b,\ 1)$ and $(c,\ 1)$.

From the graph, we have $c < a < b$.



19. A

For the graph of $y = \log_{\frac{1}{3}} x$.

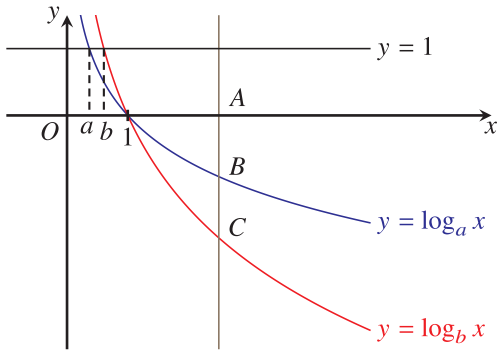
x	3	1	$\frac{1}{3}$
y	-1	0	1

The graph passes through the points $(3,\ -1)$, $(1,\ 0)$ and $\left(\frac{1}{3},\ 1\right)$.

The answer is A.

20. B

Draw the line $y = 1$. We have $0 < a < b < 1$.



I. ✗.

II. ✗.

III. ✓. Let the equation of ABC be $x = k$.
The coordinates of B and C are $(k,\ \log_a k)$ and $(k,\ \log_b k)$ respectively.

$$\begin{aligned}\frac{BC}{AB} &= \frac{\log_a k - \log_b k}{-\log_a k} \\ &= \frac{\frac{\log k}{\log a} - \frac{\log k}{\log b}}{-\frac{\log k}{\log a}} \\ &= \frac{\log a}{\log b} - 1 \\ &= \log_b a - \log_b b \\ &= \log_b \frac{a}{b}\end{aligned}$$

21. D

The coordinates of A are $(a,\ 1)$.
The coordinates of B are $(a,\ \log_b a)$.
The coordinates of C are $(a,\ 0)$.
Since $0 < b < a < 1$, we have $\log_b a = \frac{\log b}{\log a} < \frac{\log a}{\log a} = 1$.
Thus, the point B lies below A.

$$\begin{aligned}\frac{AB}{BC} &= \frac{1 - \log_b a}{\log_b a - 0} \\ &= \frac{1}{\log_b a} - 1 \\ &= 1 \div \frac{\log a}{\log b} - 1 \\ &= \frac{\log b}{\log a} - 1 \\ &= \log_a b - 1\end{aligned}$$

22. C

We have the following table of values.

x	$\frac{1}{3}$	1	3
y	-2	0	2

The graph passes through the points $\left(\frac{1}{3},\ -2\right)$, $(1,\ 0)$ and $(3,\ 2)$.
The answer is D.

23. D

I. ✗.

The graph of $y = \log_b x$ intersects the straight line $y = 1$ at $(b,\ 1)$.

The x-intercept of the graph of $y = \log_b x$ is 1.

Compare the x-coordinates of two points, we have $b < 1$.

II. ✓.

Let the equation of L be $x = k$, where $k > 1$.

The coordinates of A and B are $(k,\ \log_a k)$ and $(k,\ \log_b k)$ respectively.

$$\begin{aligned} AC &= BC \\ \log_a k &= -\log_b k \\ \frac{\log k}{\log a} &= -\frac{\log k}{\log b} \\ \log b &= -\log a \\ \log ab &= 0 \\ ab &= 1 \end{aligned}$$

III. ✓.

The x-intercepts of two curves are 1.

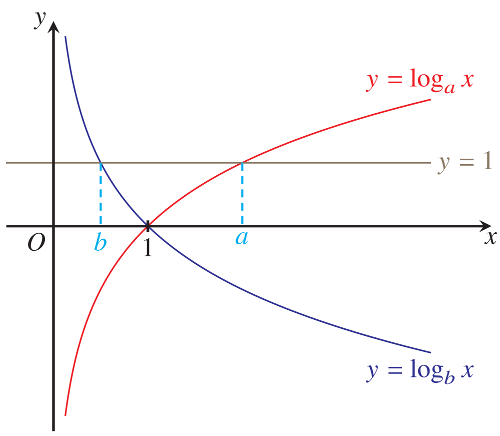
24. C

Draw the line $y = 1$.

The line intersects the graph at $(b,\ 1)$ and $(a,\ 1)$.

From the graph, we have $0 < b < 1 < a$.

The result follows.



Conventional Questions

25. G passes through $(-12,\ 0)$ and $(0,\ 1)$.

$$\begin{cases} 0 = a + \log_b(-12+16) \\ 1 = a + \log_b 16 \end{cases}$$ 1M

$$1 - 0 = \log_b 16 - \log_b 4$$ 1M

$$1 = \log_b 4$$

$$b = 4$$ 1A

When $b = 4$, $a = 1 - \log_4 16 = -1$.

$$y = -1 + \log_4(x+16)$$

$$4^{y+1} = x + 16$$

$$x = 4^{y+1} - 16$$ 1A

26. We have

$$\begin{cases} 0 = m^2 - n \\ -12 = m - n \end{cases}$$ 1M

$$0 + 12 = m^2 - m$$ 1M

$$0 = m^2 - m - 12$$

$$m = 4 \quad \text{or} \quad -3 \text{ (rejected)}$$

When $m = 4$, $n = m + 12 = 16$. 1A

$$4^x - 16 > 2021$$

$$4^x > 2037$$

$$x \log 4 > \log 2037$$ 1M

$$x > 5.50$$ 1A

 END OF PAPER