REG-VAR-2425-ASM-SET 3-MATH
Suggested solutions

Conventional Questions

1. (a) Let the total expenditure, the rent, the number of participants and the cost of food per participant be $\$t$, $\$r$, n and $\$c$ respectively.

$$\begin{cases} 15\,750 = r + 85c \\ 21\,000 = r + 120c \end{cases}$$ 1M 1M

Solving, we have $r = 3000$ and $c = 150$. 1A
The rent and the cost of food per participant are \$3000 and \$150 respectively.

(b) Total expenditure $= 3000(1 + 50\%) + 150(100)$
$= \$19\,500$ 1A
$< \$(200 \times 100)$
Therefore, the organization does not need to bear any cost. 1A

2. (a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 72 = 2a + (2)^2 b \\ -60 = -a + (-1)^2 b \end{cases}$$ 1M

Solving, we have $a = 52$ and $b = -8$. 1A
So, $f(x) = 52x - 8x^2$.

(b) $52x - 8x^2 = 260$
$-8x^2 + 52x - 260 = 0$
$\Delta = 52^2 - 4(-8)(-260) = -5616 < 0$ 1M
So, the equation has zero real roots. 1A

3. (a) Let $C = a + \dfrac{b}{n^2}$, where a and b are non-zero constants. 1A

$$\begin{cases} 82\,200 = a + b \\ 22\,200 = a + \dfrac{b}{4} \end{cases}$$ 1M

Solving, we have $a = 2200$ and $b = 80\,000$. 1A
Required cost $= 2200 + \dfrac{80\,000}{100^2} = \$2208$ 1A

(b) $\dfrac{80\,000}{n^2} > 0$ for all positive integers n.
$C = 2200 + \dfrac{80\,000}{n^2} > 2200$ for all positive integers n. 1M
The claim is disagreed. 1A

4. (a) Let $\log_2 N = a + bt$, where a and b are non-zero constants. 1A

$$\begin{cases} 4 = a + 5b \\ 7 = a + 20b \end{cases}$$ 1M

Solving, we have $a = 3$ and $b = \frac{1}{5}$. 1A

Thus, $\log_2 N = 3 + \frac{t}{5}$.

(b) When $t = 0$, $\log_2 N = 3$ 1A

Thus, initial number of bacteria $= 2^3 = 8$ thousand 1A

(c) When $t = 60$, $\log_2 N = 3 + \frac{60}{5} = 15$.

So, number of bacteria $= 2^{15} \approx 32\,768$ thousand > 30 million 1A

The claim is agreed. 1A

5. (a) Let $C = a + bn$, where a and b are non-zero constants. 1A

$$\begin{cases} 2400 = a + 100b \\ 4200 = a + 300b \end{cases}$$ 1M

Solving, we have $a = 1500$ and $b = 9$. 1A

Thus, $C = 1500 + 9n$.

(b) Average cost per watch after crisis $= \frac{1500(1 + 20\%)}{n} + 9 = 1.2\left(\frac{1500}{n}\right) + 9$ 1A

$$\text{Percentage} = \frac{1.2\left(\frac{1500}{n}\right) + 9 - \left(\frac{1500}{n}\right) - 9}{\left(\frac{1500}{n}\right) + 9} \times 100\%$$ 1M+1A

$$= \frac{0.2 \times \frac{1500}{n}}{\frac{1500}{n} + 9}$$

$$< \frac{0.2 \times \frac{1500}{n}}{\frac{1500}{n}} \times 100 = 20\%$$

Thus, the percentage increase is less than 20%. 1A

6. (a) Let $C = a + bV$, where a and b are non-zero constants. 1A

$$\begin{cases} 25 = a + 40b \\ 29 = a + 48b \end{cases}$$ 1M

Solving, we have $a = 5$ and $b = \frac{1}{2}$ 1A

Required cost $= 5 + \frac{28}{2} = \$19$ 1A

(b) Cost of the bigger car $= 5 + \frac{1}{2}\left(28 \times 4^{\frac{3}{2}}\right)$ 1M+1M

$= \$117 \neq \$19 \times 8$

The claim is disagreed. 1A

7. (a) Let $y = a + b\log_2(x+1)$, where a and b are non-zero constants. 1A

$$\begin{cases} 29 = a + b\log_2(3+1) \\ 45 = a + b\log_2(15+1) \end{cases}$$ 1M

Solving, we have $a = 13$ and $b = 8$. 1A

Therefore, $y = 13 + 8\log_2(x+1)$.

(b) $2[\log_2(x+1)]^2 + 3 = 13 + 8\log_2(x+1)$ 1M

$2[\log_2(x+1)]^2 - 8\log_2(x+1) - 10 = 0$

$\log_2(x+1) = -1 \quad \text{or} \quad 5$ 1A

$x = 2^{-1} - 1 \text{ (rejected)} \quad \text{or} \quad 2^5 - 1$ 1M

$= 31$

Required time is 31 min after the starting of experiment. 1A

8. (a) Let $C = ar^2 + br^3$, where a and b are non-zero constants. 1A

$$\begin{cases} 64a + 512b = 80 \\ 100a + 1000b = 150 \end{cases}$$ 1M

Solving, we have $a = \frac{1}{4}$ and $b = \frac{1}{8}$. 1A

Required cost $= \frac{12^2}{4} + \frac{12^3}{8} = \$252$ 1A

(b) Let the radius of the smaller balls be R cm.

$\frac{4}{3}\pi(12)^3 = 10 \times \frac{4}{3}\pi R^3$ 1A

$R \approx 5.57$

Percentage change $= \dfrac{10\left(\frac{R^2}{4} + \frac{R^3}{8}\right) - 252}{252} \times 100\%$ 1M

$\approx 16.5\%$ 1A

9. (a) Let $P = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} 30\,000 = a(100) + b(100)^2 \\ 37\,500 = a(150) + b(150)^2 \end{cases}$$ 1M

Solving, we have $a = 400$ and $b = -1$. 1A

Required profit $= 400(350) - (350)^2$

$= \$17\,500$ 1A

(b) $400x - x^2 = 45\,000$

$x^2 - 400x + 45\,000 = 0$

$\Delta = (-400)^2 - 4(1)(45\,000)$ 1M

$= -20\,000$

< 0

The equation $x^2 - 400x + 45\,000 = 0$ has no real roots.
The owner cannot obtain a profit of \$45 000 by selling all these trousers. 1A

10. (a) Let $f(x) = ax^2 + bx$, where a and b are non-zero constants. 1A

$$\begin{cases} -75 = 25a - 5b \\ 25 = 25a + 5b \end{cases}$$ 1M

Solving, we have $a = -1$ and $b = 10$. 1A
Thus, $f(x) = -x^2 + 10x$.

(b) (i) Coordinates of the common vertex are $(h,\ 25)$. 1M
Since $f(5) = 25$, we have $h = 5$. 1A

(ii) $-\frac{1}{4}(x-5)^2 + 25 = 0$

$$(x-5)^2 = 100$$

$$x = -5 \quad \text{or} \quad 15$$

Coordinates of A are $(-5,\ 0)$ and $OA = 5$. 1A
Coordinates of B are $(-5,\ f(-5)) = (-5,\ -75)$.

$$-75 = -x^2 + 10x$$

$$x^2 - 10x - 75 = 0$$

$$x = 15 \quad \text{or} \quad -5$$

Coordinates of C are $(15,\ -75)$ and $BC = 15 + 5 = 20$. 1A
Area of trapezium $ABCO = \frac{(5+20)(75)}{2} = \frac{1875}{2}$ 1A

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