**REG-TRIG-2425-ASM-SET 4-MATH**
**Suggested solutions**

**Multiple Choice Questions**

| | | | | |
|---|---|---|---|---|
| 1. B | 2. D | 3. B | 4. D | 5. B |
| 6. C | 7. D | 8. B | 9. B | 10. C |
| 11. D | 12. C | 13. C | 14. A | 15. A |
| 16. B | 17. B | 18. C | 19. D | 20. D |
| 21. B | 22. C | 23. A | 24. A | 25. A |
| 26. D | 27. C | 28. B | 29. D | 30. C |
| 31. A | 32. A | 33. B | 34. A | 35. B |
| 36. A | 37. A | 38. B | 39. A | 40. A |
| 41. D | 42. C | 43. B | 44. B | 45. B |

1. B

The graph passes through $(300°,\ 1)$.

A. ✗. When $x = 300°$, $y = 2\cos 330° - 1 \neq 1$.

B. ✓. When $x = 300°$, $y = 2\cos 360° - 1 = 1$.

C. ✗. When $x = 300°$, $y = 2\cos 330° + 1 \neq 1$.

D. ✗. When $x = 300°$, $y = 2\cos 360° + 1 = 3 \neq 1$.

2. D

I. ✓. Note from the graph of $y = \sin x$ that $\sin x$ is increasing on $270° < x < 360°$.

II. ✓. Note from the graph of $y = \cos x$ that $\cos x$ is increasing on $270° < x < 360°$.

III. ✓. Note that $\sin a < 0 < \cos a$.

3. B

The graph passes through $(210,\ -4)$.

$\underline{(210,\ -4)}$

A. $y = -2\cos 240° + 2 \neq -4$

B. $y = 2\cos 180° - 2 = -4$

C. $y = 2\cos 240° + 2 \neq 4$

D. $y = -2\cos 180° - 2 = 0 \neq -4$

4. D

Check the equations by the points on the graph:

| | $(0,\ 7)$ | $(90,\ 1)$ |
|---|---|---|
| A. | ✗ | |
| B. | ✗ | |
| C. | ✓ | ✗ |
| D. | ✓ | ✓ |

5. B

The graph passes through $(45°,\ 0)$.

A. ✗. When $x = 45°$, $y = -\cos 45° \neq 0$.

B. ✓. When $x = 45°$, $y = -\cos 90° = 0$.

C. ✗. When $x = 45°$, $y = -\cos 22.5° \neq 0$.

D. ✗. When $x = 45°$, $y = -\cos \frac{45°}{4} \neq 0$.

6. C

The curve passes through $(0,\ 1)$. Only option C satisfies this.

7. D

The graph passes through the following points:

| | $(75,\ 0)$ |
|---|---|
| A. | ✗ |
| B. | ✗ |
| C. | ✗ |
| D. | ✓ |

8. B

The graph passes through $(0,\ 2)$.

A. ✓. When $x = 0$, $y = 2 - 0 = 2$.

B. ✓. When $x = 0$, $y = 2 - 0 = 2$.

C. ✗. When $x = 0$, $y = 1 - 1 = 0 \neq 2$.

D. ✓. When $x = 0$, $y = 1 + 1 = 2$.

The graph passes through $(45,\ 1)$.

A. ✗. When $x = 45$, $y = 2 - \sin 22.5^\circ \neq 1$.

B. ✓. When $x = 45$, $y = 2 - \sin 90^\circ = 1$.

C.

D. ✗. When $x = 45$, $y = 1 + \cos 22.5^\circ \neq 1$.

9. B

I. ✗. Period $= \dfrac{360^\circ}{2} = 180^\circ$

II. ✗. Maximum value = 1 and minimum value = $-1$

III. ✓.

10. C

The graph passes through $(0,\ k)$, where $-1 < k < 0$.

A. ✗. $f(0) = -1$

B. ✓. $f(0) = -\dfrac{1}{2}$, which lies between $-1$ and 0.

C. ✓. $f(0) = -\dfrac{1}{2}$, which lies between $-1$ and 0.

D. ✗. $f(0) = -1$

The graph passes through $\left(90,\ -\dfrac{1}{2}\right)$.

B. ✗. $f(90) = \dfrac{\sin 45^\circ}{2} - \dfrac{1}{2} \neq -\dfrac{1}{2}$

C. ✓. $f(90) = 0 - \dfrac{1}{2} = -\dfrac{1}{2}$

11. D

Consider the graph of $y = \cos x$.

| $x$ | $90°$ | $180°$ | $270°$ |
|---|---|---|---|
| $y$ | 0 | $-1$ | 0 |

The graph of $y = \cos x$ passes through $(90°,\ 0)$, $(180°,\ -1)$ and $(270°,\ 0)$.
The answer is D.

12. C

Read from the graph of $y = \tan x$, $\tan\theta$ will increase from 0 to positive infinity.

13. C

The graph passes through $(0,\ 3)$.

A. ✗. When $x = 0$, $y = \sin 30° + 3 = 3.5 \neq 3$

B. ✗. When $x = 0$, $y = 2\sin 30° + 1 = 2 \neq 3$

C. ✓. When $x = 0$, $y = 2\sin 30° + 2 = 3$

D. ✗. When $x = 0$, $y = 3\sin 30° + 4 = 5.5 \neq 3$

14. A

If $p$ is positive, then the maximum value and the minimum value of $y = p\sin 3x° + q$ are $p + q$ and $-p + q$ respectively.

Solve $\begin{cases} p + q = 1 \\ -p + q = -3 \end{cases}$, we have $p = 2$ and $q = -1$.

15. A

The graph passes through $(0°,\ 3)$.

A. ✓. When $x = 0$, $y = 2\cos 0° + 1 = 3$.

B. ✗. When $x = 0$, $y = 2\cos 0° - 1 = 1 \neq 3$.

C. ✗. When $x = 0$, $y = 3\cos 0° + 2 = 5 \neq 3$.

D. ✗. When $x = 0$, $y = 3\cos 0° - 2 = 1 \neq 3$.

16. B

The graph passes through $(146,\ 0)$.

A. ✗. When $x = 146$, $y = -3\sin 112° \neq 0$.

B. ✓. When $x = 146$, $y = -3\sin 180° = 0$.

C. ✗. When $x = 146$, $y = -3\sin 100° \neq 0$.

D. ✗. When $x = 146$, $y = -3\sin 192° \neq 0$.

17. B

The graph passes through $(210°,\ 0)$.

| | $(210°,\ 0)$ |
|---|---|
| A. | ✗ |
| B. | ✓ |
| C. | ✗ |
| D. | ✗ |

18. C

The graph passes through $(60,\ 0)$.

A. ✗. When $x = 60$, $y = \sin 90° - 2 = -1 \neq 0$.

B. ✗. When $x = 60$, $y = \sin 30° - 2 = -\frac{3}{2} \neq 0$.

C. ✓. When $x = 60$, $y = \sin 90° - 1 = 0$.

D. ✗. When $x = 60$, $y = \sin 30° - 1 = -\frac{1}{2} \neq 0$.

19. D

The graph passes through $(180°,\ 0)$.

A. ✓. When $x = 180°$, $y = -\sin(-360°) = 0$.

B. ✗. When $x = 180°$, $y = -\sin(-90°) = -1 \neq 0$.

C. ✗. When $x = 180°$, $y = -\sin 90° = 1 \neq 0$.

D. ✓. When $x = 180°$, $y = -\sin 360° = 0$.

The graph passes through $(45°,\ -1)$.

A. ✓. When $x = 45°$, $y = -\sin(-90°) = 1 \neq -1$.

B. ✗.

C. ✗.

D. ✓. When $x = 45°$, $y = -\sin 90° = -1$.

20. D

The graph passes through $(135°,\ -5)$.

A. ✗. When $x = 135°$, $y = -5\cos 90° = 0 \neq -5$.

B. ✗. When $x = 135°$, $y = -5\cos 180° = 5 \neq -5$.

C. ✗. When $x = 135°$, $y = 5\cos 90° = 0 \neq -5$.

D. ✓. When $x = 135°$, $y = 5\cos 180° = -5$.

21. B

The graph passes through $(120,\ 3)$.

A. ✗. When $x = 120$, $y = -3\sin(120^\circ - 30^\circ) = -3 \neq 3$.

B. ✓. When $x = 120$, $y = 3\sin(120^\circ - 30^\circ) = 3$.

C. ✗. When $x = 120$, $y = -3\sin(120^\circ + 30^\circ) = -\dfrac{3}{2} \neq 3$.

D. ✗. When $x = 120$, $y = 3\sin(120^\circ + 30^\circ) = \dfrac{3}{2} \neq 3$.

22. C

The graph passes through $(0^\circ,\ 1)$ and $(90^\circ,\ 3)$.

| | $(0^\circ,\ 1)$ | $(90^\circ,\ 3)$ |
|---|---|---|
| A. | ✓ | ✗ |
| B. | ✗ | |
| C. | ✓ | ✓ |
| D. | ✗ | |

23. A

The graph passes through the following points:

| | $(b,\ 2)$ | $(0,\ \ominus)$ |
|---|---|---|
| A. | ✓ | ✓ |
| B. | ✗ | |
| C. | ✗ | |
| D. | ✓ | ✗ |

$\ominus$ represents a negative number (the $y$-intercept).

24. A

I. ✓. For $180^\circ < \theta < 225^\circ$, $0 < \tan\theta < 1$
Since $\cos\theta < 0$, $\dfrac{\sin\theta}{\cos\theta} < 1 \quad \Rightarrow \quad \sin\theta > \cos\theta$

II. ✓. $\sin\theta < 0 < \tan\theta$

III. ✗. $\tan\theta > 0$ but $\cos\theta < 0$

25. A

$\cos 0^\circ + \cos 1^\circ + \cos 2^\circ + \ldots + \cos 180^\circ$

$= (\cos 0^\circ + \cos 180^\circ) + (\cos 1^\circ + \cos 179^\circ) + \ldots + (\cos 89^\circ + \cos 91^\circ) + \cos 90^\circ$

$= (1 - 1) + (\cos 1^\circ - \cos 1^\circ) + \ldots + (\cos 89^\circ - \cos 89^\circ) + 0$

$= 0$

26. D

$$\frac{\tan 45^\circ}{1-\cos\theta} + \frac{2\sin 30^\circ}{1+\sin(90^\circ+\theta)} = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$$
$$= \frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$

27. C

$$\frac{\cos 180^\circ}{1+\sin(90^\circ+\theta)} + \frac{\cos 360^\circ}{1+\sin(270^\circ+\theta)} = \frac{-1}{1+\cos\theta} + \frac{1}{1-\cos\theta}$$
$$= \frac{-(1-\cos\theta)+(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{2\cos\theta}{\sin^2\theta}$$

28. B

Let $x = 5$ and $r = 13$. Then $y = \sqrt{13^2 - 5^2} = 12$.

$\sin\theta = \frac{12}{13}$

$$\cos(180^\circ+\theta) - \cos(90^\circ+\theta) = -\cos\theta + \sin\theta$$
$$= -\frac{5}{13} + \frac{12}{13}$$
$$= \frac{7}{13}$$

29. D

I. ✗. Take $A = 90^\circ$. Then $B = 90^\circ$ and $\sin A + \sin B = \sin 90^\circ + \sin 90^\circ = 2 \neq 0$.

II. ✓. $\cos A + \cos B = \cos A + \cos(180^\circ - A) = \cos A - \cos A = 0$

III. ✓. $\tan A + \tan B = \tan A + \tan(180^\circ - A) = \tan A - \tan A = 0$

30. C

$$\frac{\cos 60^\circ}{1-\cos(90^\circ-\theta)} + \frac{\cos 240^\circ}{1-\cos(270^\circ-\theta)} = \frac{0.5}{1-\sin\theta} + \frac{-0.5}{1+\sin\theta}$$
$$= \frac{0.5(1+\sin\theta)-(0.5)(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{\sin\theta}{\cos^2\theta} = \frac{\tan\theta}{\cos\theta}$$

31. A

$$\sin^2(\theta-90^\circ) + \sin^2(-\theta) = \cos^2\theta + \sin^2\theta$$
$$= 1$$

32. A

$$\begin{aligned}(\cos(90^\circ + \theta) + 1)(\sin(360^\circ - \theta) - 1) &= (-\sin\theta + 1)(-\sin\theta - 1)\\ &= \sin^2\theta - 1\\ &= -\cos^2\theta\end{aligned}$$

33. B

$$\begin{aligned}\cos(90^\circ - \theta)\tan(270^\circ - \theta) &= \sin\theta \times \frac{1}{\tan\theta}\\ &= \sin\theta \times \frac{\cos\theta}{\sin\theta}\\ &= \cos\theta\end{aligned}$$

34. A

$$\begin{aligned}\tan(\theta - 180^\circ)\sin(270^\circ + \theta) &= \tan\theta(-\cos\theta)\\ &= -\frac{\sin\theta}{\cos\theta} \times \cos\theta\\ &= -\sin\theta\end{aligned}$$

35. B

$$\begin{aligned}\frac{24}{1 + 3\cos^2\theta + 5\cos^2(90^\circ - \theta)} &= \frac{24}{1 + 3\cos^2\theta + 5\sin^2\theta}\\ &= \frac{24}{1 + 3(\sin^2\theta + \cos^2\theta) + 2\sin^2\theta}\\ &= \frac{12}{2 + \sin^2\theta}\end{aligned}$$

Since $0 \le \sin^2\theta \le 1$, the required greatest value is $\frac{12}{2+0} = 6$.

36. A

$$\frac{\sin(90^\circ - \theta)\tan(180^\circ - \theta)}{\cos(360^\circ + \theta)} = \frac{\cos\theta(-\tan\theta)}{\cos\theta} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$$

37. A

$$\begin{aligned}\tan(90^\circ + \theta) &= \frac{3}{2}\\ -\frac{1}{\tan\theta} &= \frac{3}{2}\\ \tan\theta &= -\frac{2}{3}\\ \frac{6\sin\theta}{2\cos\theta - 3\sin\theta} &= \frac{\frac{6\sin\theta}{\cos\theta}}{2 - \frac{3\sin\theta}{\cos\theta}}\\ &= \frac{6\tan\theta}{2 - 3\tan\theta}\\ &= -1\end{aligned}$$

38. B

$\cos^2(180^\circ - \theta) - [\sin^2(270^\circ - \theta)\sin^4\theta + \cos^2(90^\circ + \theta)\cos^4\theta]$

$= \cos^2\theta - [\cos^2\theta\sin^4\theta + \sin^2\theta\cos^4\theta]$

$= \cos^2\theta - \sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= \cos^2\theta(1 - \sin^2\theta) = \cos^4\theta$

39. A

$$\frac{\cos(180^\circ - \theta)}{\tan(90^\circ - \theta)} - \sin(-\theta) = \frac{-\cos\theta}{\frac{1}{\tan\theta}} + \sin\theta = -\sin\theta + \sin\theta = 0$$

40. A

I. ✓.

II. ✗. Range of the function is $-1 \le y \le 1$.

III. ✗. Note that $\sin[3((x + 120)^\circ)] = \sin(3x^\circ + 360^\circ) = \sin 3x^\circ$.
The least positive period of the function is not 1080.

41. D

The domain of the function $y = \sin x$ is the set of all real numbers.

I. ✓.

II. ✗. Note that the domain does not include 30°.

III. ✓.

42. C

$$\frac{120}{3\cos^2(90^\circ - \theta) + 5\cos^2\theta} = \frac{120}{3\sin^2\theta + 5\cos^2\theta}$$

$$= \frac{120}{3(\sin^2\theta + \cos^2\theta) + 2\cos^2\theta}$$

$$= \frac{120}{3 + 2\cos^2\theta}$$

When $\cos^2\theta = 0$, $\frac{120}{3 + 2\cos^2\theta} = 40$.

When $\cos^2\theta = 1$, $\frac{120}{3 + 2\cos^2\theta} = 30$.

Required greatest value is 40.

43. B

$$\frac{1}{3\sin^2 x + 2\cos^2 x} = \frac{1}{3(1-\cos^2 x) + 2\cos^2 x}$$
$$= \frac{1}{3 - \cos^2 x}$$

When $\cos^2 x = 0$, $\frac{1}{3 - \cos^2 x} = \frac{1}{3}$.

When $\cos^2 x = 1$, $\frac{1}{3 - \cos^2 x} = \frac{1}{2}$.

Required maximum value $= \frac{1}{2}$.

44. B

For $0° \le \theta \le 90°$, $0 \le \sin\theta \le 1$.

When $\sin\theta = 1$, $\frac{6}{5 + 3\sin\theta} = \frac{6}{5+3} = \frac{3}{4}$.

When $\sin\theta = 0$, $\frac{6}{5 + 3\sin\theta} = \frac{6}{5}$.

Required value is $\frac{6}{5}$.

45. B

When $\cos\theta = -1$, $\frac{4 - \cos\theta}{2 - \cos\theta} = \frac{4+1}{2+1} = \frac{5}{3}$.

When $\cos\theta = 1$, $\frac{4 - \cos\theta}{2 - \cos\theta} = \frac{4-1}{2-1} = 3$.

Required least value is $\frac{5}{3}$.