REG-TRIG-2425-ASM-SET 3-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. A	3. D	4. B	5. D
6. B	7. D	8. A	9. C	10. C
11. A	12. A	13. C	14. A	15. A
16. C	17. B	18. A	19. D	20. B
21. B	22. C	23. C	24. C	

1. $\boxed{\text{C}}$

I. ✓. $x = 90°$ only.

II. ✗. $x = 0°$ or $180°$.

III. ✓. $x = 90°$ only.

2. $\boxed{\text{A}}$

$$\sin\theta = 5\tan\theta$$
$$\sin\theta = \frac{5\sin\theta}{\cos\theta}$$
$$\sin\theta\left(1 - \frac{5}{\cos\theta}\right) = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \cos\theta = 5 \text{ (rejected)}$$
$$\theta = 0° \quad \text{or} \quad 180°$$

3. $\boxed{\text{D}}$

$$3\tan^2 x = \tan x$$
$$\tan x(3\tan x - 1) = 0$$
$$\tan x = 0 \quad \text{or} \quad \frac{1}{3}$$

When $\tan x = 0$, $x = 0°$ or $180°$ or $360°$
When $\tan x = \frac{1}{3}$, there are 2 roots (in quadrants I and III)
Altogether 5 roots.

4. $\boxed{\text{B}}$

$$2\tan\theta = \sin\theta$$
$$2\sin\theta = \sin\theta\cos\theta$$
$$\sin\theta(2 - \cos\theta) = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \cos\theta = 2 \text{ (rejected)}$$

There are 3 roots.

5. $\boxed{\text{D}}$

We have $\sin\theta = 0$ or $\tan\theta = -3$.

When $\sin\theta = 0$, $\theta = 0°$ or $180°$ or $360°$.

When $\tan\theta = -3$, there are two roots.

Thus, there are 5 roots.

6. $\boxed{\text{B}}$

We have $2\sin^2 x = 1$ or $\sin x = 0$.

When $\sin x = 0$, $x = 180°$

When $2\sin^2 x = 1$, $\sin x = \pm\dfrac{1}{\sqrt{2}}$ and there are four roots within the required range.

Thus, there are 5 roots.

7. $\boxed{\text{D}}$

$$\sin\theta = \sin^3\theta$$

$$0 = \sin\theta(\sin^2\theta - 1)$$

$$\sin\theta = 0 \quad \text{or} \quad \pm 1$$

With reference to the graph of sine, there are 5 roots.

8. $\boxed{\text{A}}$

$$3\cos^2\theta + \cos\theta - 4 = 0$$

$$\cos\theta = 1 \quad \text{or} \quad -\frac{4}{3} \text{ (rejected)}$$

$$\theta = 0°$$

There is 1 root.

9. $\boxed{\text{C}}$

$$\frac{2}{1-\sin\theta} - \frac{2}{1+\sin\theta} = 4$$

$$2(1+\sin\theta) - 2(1-\sin\theta) = 4(1-\sin\theta)(1+\sin\theta)$$

$$4\sin^2\theta + 4\sin\theta - 4 = 0$$

$$\sin\theta \approx 0.618 \quad \text{or} \quad -1.62$$

$$\theta \approx 38.2° \quad \text{or} \quad 142°$$

10. $\boxed{\text{C}}$

$$7\cos^2 x = \cos x + 6$$

$$7\cos^2 x - \cos x - 6 = 0$$

$$\cos x = 1 \quad \text{or} \quad -\frac{6}{7}$$

When $\cos x = 1$, $x = 0°$ (exclude $360°$)

When $\cos x = -\dfrac{6}{7}$, $x = 180° - \cos^{-1}\dfrac{6}{7}$ or $180° + \cos^{-1}\dfrac{6}{7}$

There are 3 roots.

11. $\boxed{\text{A}}$

$9\cos^4\theta - 13\cos^2\theta + 4 = 0$

$$\cos^2\theta = 1 \quad \text{or} \quad \frac{4}{9}$$

$$\cos\theta = \pm 1 \quad \text{or} \quad \pm\frac{2}{3}$$

When $\cos\theta = \pm 1$, $\theta = 180°$

When $\cos\theta = \pm\frac{2}{3}$, there are 3 roots (one root per quadrant).

There are total 4 roots.

12. $\boxed{\text{A}}$

$3\tan^2\theta + \tan\theta - 2 = 0$

$$\tan\theta = \frac{2}{3} \quad \text{or} \quad -1$$

There are two roots for $\tan\theta = \frac{2}{3}$ and two roots for $\tan\theta = -1$.

Thus, there are 4 roots.

13. $\boxed{\text{C}}$

$$4\cos^2\theta - 7\sin\theta - 7 = 0$$

$$4(1 - \sin^2\theta) - 7\sin\theta - 7 = 0$$

$$-4\sin^2\theta - 7\sin\theta - 3 = 0$$

$$\sin\theta = -1 \quad \text{or} \quad -\frac{3}{4}$$

$\sin\theta = -1$ has one root and $\sin\theta = -\frac{3}{4}$ has two roots.

14. $\boxed{\text{A}}$

$$\cos x = \tan x$$

$$\cos^2 x = \sin x$$

$$-\sin^2 x - \sin x + 1 = 0$$

$$\sin x \approx -1.62 \text{ (rejected)} \quad \text{or} \quad 0.618$$

There are 2 roots and hence 2 intersections.

15. $\boxed{\text{A}}$

$$\frac{3}{\tan^2(90° - \alpha)} - \frac{5}{\sin(270° + \alpha)} = \frac{1}{\cos^2\alpha}$$

$$3\tan^2\alpha + \frac{5}{\cos\alpha} = \frac{1}{\cos^2\alpha}$$

$$3\sin^2\alpha + 5\cos\alpha = 1$$

$$-3\cos^2\alpha + 5\cos\alpha + 2 = 0$$

$$\cos\alpha = -\frac{1}{3} \quad \text{or} \quad 2 \text{ (rejected)}$$

There are two roots for $\cos\alpha = -\frac{1}{3}$. Required number of roots is 2.

16. $\boxed{\text{C}}$

$$\cos^2 x - \sin x = 1$$
$$1 - \sin^2 x - \sin x = 1$$
$$\sin x(\sin x + 1) = 0$$
$$\sin x = 0 \quad \text{or} \quad -1$$

There are three roots for $\sin x = 0$ and one root for $\sin x = -1$.

Thus, there are 4 roots.

17. $\boxed{\text{B}}$

$$5(1 - \sin^2 x) = \sin x + 1$$
$$0 = 5\sin^2 x + \sin x - 4$$
$$\sin x = 0.8 \quad \text{or} \quad -1$$

When $\sin x = 0.8$, there are two roots; when $\sin x = -1$, there is one root.

There are total 3 roots.

18. $\boxed{\text{A}}$

$$3\tan x = 2\sin(90^\circ - x)$$
$$\frac{3\sin x}{\cos x} = 2\cos x$$
$$3\sin x = 2\cos^2 x$$
$$3\sin x = 2(1 - \sin^2 x)$$
$$2\sin^2 x + 3\sin x - 2 = 0$$
$$\sin x = 0.5 \quad \text{or} \quad -2 \text{ (rejected)}$$
$$x = 30^\circ \quad \text{or} \quad 150^\circ$$

There are 2 roots.

19. $\boxed{\text{D}}$

$$2\cos^2\theta = 2 - \sin\theta$$
$$2(1 - \sin^2\theta) = 2 - \sin\theta$$
$$-2\sin^2\theta + \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{2}$$

When $\sin\theta = 0$, $\theta = 0^\circ$ or 180° or 360°.

When $\sin\theta = \frac{1}{2}$, $\theta = 30^\circ$ or 150°.

There are 5 roots.

20. $\boxed{\text{B}}$

$$4 + \cos(90^\circ + \theta) = 4\cos^2\theta$$
$$4 - \sin\theta = 4(1 - \sin^2\theta)$$
$$4\sin^2\theta - \sin\theta = 0$$
$$\sin\theta = 0 \quad \text{or} \quad \frac{1}{4}$$

The equation $\sin\theta = 0$ has one root.

The equation $\sin\theta = \frac{1}{4}$ has two roots.

Thus, the equation $4 + \cos(90^\circ + \theta) = 4\cos^2\theta$ has 3 roots.

21. $\boxed{\text{B}}$

$$\frac{1 + \cos\theta}{\sin\theta} = 3\tan\theta$$
$$\cos\theta(1 + \cos\theta) = 3\sin^2\theta$$
$$\cos\theta + \cos^2\theta = 3(1 - \cos^2\theta)$$
$$4\cos^2\theta + \cos\theta - 3 = 0$$
$$\cos\theta = \frac{3}{4} \quad \text{or} \quad -1$$

Note that when $\cos\theta = -1$, we have $\theta = 180^\circ$ and $\sin\theta = 0$, which should be rejected.

There are two roots for $\cos\theta = \frac{3}{4}$.

Thus, there are 2 roots.

22. $\boxed{\text{C}}$

A. 3 roots

B. 3 roots

C. 4 roots

D. 2 roots

23. $\boxed{\text{C}}$

$$3 - 6\cos^2\theta = 8\sin\theta\cos\theta$$
$$3(\sin^2\theta + \cos^2\theta) - 6\cos^2\theta = 8\sin\theta\cos\theta$$
$$3\sin^2\theta - 8\sin\theta\cos\theta - 3\cos^2\theta = 0$$
$$\frac{3\sin^2\theta}{\cos^2\theta} - \frac{8\sin\theta\cos\theta}{\cos^2\theta} - 3 = 0$$
$$3\tan^2\theta - 8\tan\theta - 3 = 0$$
$$\tan\theta = -\frac{1}{3} \quad \text{or} \quad 3$$

24. $\boxed{\text{C}}$

$$\begin{aligned}
\sin\theta\tan\theta &= 1 \\
\frac{\sin^2\theta}{\cos\theta} &= 1 \\
1-\cos^2\theta &= \cos\theta \\
-\cos^2\theta-\cos\theta+1 &= 0 \\
\cos\theta &= \frac{1\pm\sqrt{1^2-4(-1)(1)}}{-2} \\
&= \frac{-1\mp\sqrt{5}}{2}
\end{aligned}$$

Since $\sin\theta\tan\theta = 1$, $\sin\theta$ and $\tan\theta$ are both positive or both negative.

In any of the case, $\cos\theta > 0$.

Thus, $\cos\theta = \dfrac{\sqrt{5}-1}{2}$.

Conventional Questions

25\. (a) $\sin(x - 25°) = \dfrac{1}{2}$

$x - 25° = 30° \quad \text{or} \quad 180° - 30°$

$x = 55° \quad \text{or} \quad 175°$ 1A

(b) $\sqrt{3}\tan(x + 26°) = -1$

$\tan(x + 26°) = -\dfrac{1}{\sqrt{3}}$ 1M

$x + 26° = 180° - 30° \quad \text{or} \quad 360° - 30°$

$x = 124° \quad \text{or} \quad 304°$ 1A

(c) $\cos(70° + x) = \cos 80°$

$70° + x = 80° \quad \text{or} \quad 360° - 80°$

$x = 10° \quad \text{or} \quad 210°$ 1A

(d) $\dfrac{\cos 147°}{\sin(100° - x)} = 1$

$\cos 147° = \sin(100° - x)$

$\sin(x - 100°) = -\cos 147°$ 1M

$x - 100° = 57° \quad \text{or} \quad 180° - 57°$

$x = 157° \quad \text{or} \quad 223°$ 1A

26\. (a) $3\cos^2 x - 10\cos x + 3 = 0$

$\cos x = \dfrac{1}{3} \quad \text{or} \quad 3 \text{ (rejected)}$ 1A

$x \approx 70.5° \quad \text{or} \quad 289°$ 1A

(b) $12\cos^2 x = \sin x + 6$

$12(1 - \sin^2 x) = \sin x + 6$ 1M

$-12\sin^2 x - \sin x + 6 = 0$

$\sin x = \dfrac{2}{3} \quad \text{or} \quad -\dfrac{3}{4}$ 1A

$\sin x = \dfrac{2}{3} \qquad \text{or} \qquad \sin x = -\dfrac{3}{4}$

$x \approx 41.8° \quad \text{or} \quad 138° \qquad x \approx 229° \quad \text{or} \quad 311°$ 1A

(c) $\tan x + 1 = \tan(270° - x)$

$\tan x + 1 = \dfrac{1}{\tan x}$

$\tan^2 x + \tan x = 1$

$\tan^2 x + \tan x - 1 = 0$ 1M

$\tan x = \dfrac{-1 \pm \sqrt{5}}{2}$ 1A

$$\tan x = \frac{-1+\sqrt{5}}{2} \quad \text{or} \quad \tan x = \frac{-1-\sqrt{5}}{2}$$

$$x \approx 31.7^\circ \quad \text{or} \quad 212^\circ \qquad x \approx 122^\circ \quad \text{or} \quad 302$$ 1A

(d) $$9\sin x - \frac{10}{\tan x} = 0$$

$$9\sin x - \frac{10\cos x}{\sin x} = 0$$

$$9\sin^2 x - 10\cos x = 0$$

$$9(1-\cos^2 x) - 10\cos x = 0$$ 1M

$$-9\cos^2 x - 10\cos x + 9 = 0$$

$$\cos x = \frac{-5+\sqrt{106}}{9} \quad \text{or} \quad \frac{-5-\sqrt{106}}{9} \text{ (rejected)}$$ 1A

$$x \approx 54.0^\circ \quad \text{or} \quad 306^\circ$$ 1A

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