1 Conventional Questions

1. (a) Median = 3 1A

Inter-quartile range = 4 − 2 1M

= 2 1A

(b) Required probability $= \frac{30}{9 + 12 + 18 + 30}$ 1M

$= \frac{10}{23}$ 1A

2. (a) The inter-quartile range is 10 kg.

$55 - (40 + h) = 10$ 1M

$h = 5$

Mode = 52 kg 1A

Mean $= \frac{37 + 39 + \ldots + 65}{26}$

= 51 kg 1A

(b) Required probability $= \frac{14}{26}$ 1M

$= \frac{7}{13}$ 1A

3. (a) The range of the distribution is 25.

$78 - (50 + a) = 25$ 1M

$a = 3$

Mean $= \frac{53 + 53 + \ldots + 78}{30} = 65$ g 1A

Mode = 72 g 1A

(b) Required probability $= \frac{26}{30}$ 1M

$= \frac{13}{15}$ 1A

4. (a) $47 = (50 + c) - (10 + a)$ 1M

$c - a = 7$

$$33 = \frac{(10 + a) + 14 + 18 + \ldots + (50 + c)}{18}$$ 1M

$a + b + c = 10$

Since $0 \le a \le 4$, $0 \le b \le 3$ and $7 \le c \le 9$, we have

$(a,\ b,\ c) = (0,\ 3,\ 7)$ or $(1,\ 1,\ 8)$. 1A

(b) Required probability $= \frac{10}{18}$ 1M

$= \frac{5}{9}$ 1A

5. (a) Mean = 6.4 1A

Median = 6.5 1A

Mode = 5 and 8 1A

(b) (i) $\frac{50 \times 6.4 + 15n}{50 + 15} = 6.4 + 0.6$ 1M

$n = 9$ 1A

(ii) Least possible mode is 4. 1A

Greatest possible mode is 9. 1A

6. (a) $11 = 28 - \frac{(20 + c) + 13}{2}$ 1M

$c = 1$ 1A

(b) $(30 + b) - (10 + a) \ge 25$

$b - a \ge 5$

$$23 = \frac{(10 + a) + 12 + 13 + \ldots + (30 + b)}{16}$$ 1M

$a + b = 6$

We have $a = 0$ and $b = 6$. 1A+1A

7. (a) $71 - (50 + h) = 16$ 1M

$h = 5$ 1A

$\dfrac{(60 + k) + (60 + k)}{2} = 68$ 1M

$k = 8$ 1A

(b) Inter-quartile range is 11 kg. 1A

Variance is $32.85\ \text{kg}^2$. 1A

(c) Original inter-quartile range is 11 kg.

The weights of the two players withdrawn from the team are 55 kg and 71 kg.

New inter-quartile range $= 68 - 57 = 11$ kg 1M

The inter-quartile range remains unchanged. 1A

8. (a) $96 - (60 + a) = 36$ 1M

$a = 0$ 1A

$\dfrac{(80 + b) + 89}{2} - 71 = 17$

$b = 7$ 1A

(b) (i) The original median and mode of the distribution are 81.5 and 84 respectively.

The scores of the two students are lower than 81.5. 1M

The mode of the distribution will be 84, which remains unchanged.

The claim is agreed. 1A

(ii) Since the range is reduced, one of the removed datum is 60.

The mean is the least when the score of the other student is 80. 1M

Least possible mean

$= \dfrac{62 + 64 + \ldots + 96}{18}$

$= 80$ 1A

9. (a) $\dfrac{(10 + a) + (10 + a) + \ldots + 45}{20} = 29$ 1M

$2a + 3b = 21$

Note that a and b are integers with $0 \le a \le 9$ and $4 \le b \le 8$.

We have $(a,\ b) = (3,\ 5)$ or $(0,\ 7)$. 1A+1A

(b) 35 1A

(c) The inter-quartile range is the least when $b = 5$.

Least inter-quartile range

$= 35 - 24.5$ 1M

$= 10.5$ 1A

10. (a) $159 - (120 + a) = 35$

$$a = 4$$ 1A

$$\frac{(140 + b) + 141}{2} - 128 = 13$$ 1M

$$b = 1$$ 1A

(b) Let x cm and y cm be the lengths of the two trial pipes, where $x \leq y$.

$$\frac{x + y + 137(20)}{22} = 137 - 1$$ 1M

$$x + y = 252$$

The range is increased by 1 cm.

Suppose $x = 123$. Then $y = 252 - 123 = 129$. 1M

Suppose $y = 160$. Then $x = 252 - 160 = 92$, the new range is 68 cm.

This case is rejected.

Thus, the lengths of the two trial pipes are 123 cm and 129 cm.

11. (a) The range of the distribution is 42 kg.

$$(80 + n) - 41 = 42$$ 1M

$$n = 3$$ 1A

The mean of the distribution is 60 kg.

$$\frac{41 + 43 + \ldots + 83}{21} = 60$$

$$m = 9$$ 1A

(b) Let x kg and y kg be the weights of the two members, where $x \leq y$.

$$\frac{x + y + 61 \times 21}{23} = 60 + 1$$ 1M

$$x + y = 143$$

The new range is 43 kg.

Case 1: $x = 40$

$$40 + y = 143$$ 1M

$$y = 103$$

The new range is 63 kg and this case is rejected.

Case 2: $y = 84$

$$x + y = 143$$

$$x = 59$$

The new range is 43 kg.

Thus, the weights of the two members are 59 kg and 84 kg respectively. 1A+1A

12. (a) Range = 74 − 40 1M

$= 34$ kg

Inter-quartile range $= \dfrac{(60 + b) + 69}{2} - \dfrac{48 + 50}{2}$ 1M

$= (0.5b + 15.5)$ kg

$34 - (0.5b + 15.5) = 14$

$b = 9$ 1A

(b) Let $(40 + a)$ kg be the weight of the player, where $a < 8$.

New upper quartile $= \dfrac{69 + 69}{2} = 69$ kg

New lower quartile $= 69 - (20 + 1.5) = 47.5$ kg 1M

$\dfrac{(40 + a) + 48}{2} = 47.5$ 1M

$a = 7$ 1A

The weight of the player is 47 kg.

13. (a) (i) The mean of the data is 20 cm.

$\dfrac{18 + 23 + \ldots + x}{8} = 20$ 1M

$x = 19$ 1A

(ii) Range = 26 − 13 = 13 cm 1A

Mode = 26 cm 1A

(b) (i) Least possible value $= \dfrac{14 + 18}{2}$ 1M

$= 16$ cm 1A

Greatest possible value $= \dfrac{23 + 26}{2}$

$= 24.5$ cm 1A

(ii) 26 cm, 26 cm, 26 cm and 26 cm. 1A

14. (a) Range = 44 − 14 = 30 1M

Inter-quartile range = $(30 + b) - 19 = 11 + b$ 1M

$$30 = 2(11 + b)$$

$$b = 4$$ 1A

(b) (i) The mean of the distribution is at least 27.

$$\frac{14 + 15 + \ldots + 44}{19} \geq 27$$ 1M

$$a \geq 2$$

The least possible value of a is 2. 1A

(ii) Mean age of two new trees $= \frac{18 + 32}{2}$ 1M

$= 25$

Since 25 < 27, the new mean of the ages of the trees in the garden must be decreased.

The claim is agreed. 1A

15. (a) Mean $= \frac{2.7 + 3.1 + 2.9 + 2.8 + 3.4 + 2.8 + 3.3}{7}$

$= 3$ kg 1A

Range = 3.4 − 2.7

$= 0.7$ kg 1A

(b) (i) Let x kg be the required total weight.

$$\frac{7(3) + 3.3 + 3.4 + x}{11} = 3$$ 1M

$$x = 5.3$$ 1A

(ii) Let a kg and b kg be the weights of the two newly-added baby girls.

Suppose the range remains unchanged, we have $2.7 \leq a \leq 3.4$ and $2.7 \leq b \leq 3.4$. 1M

We have $a + b \geq 2.7 + 2.7 = 5.4$, which is a contradiction.

Thus, it is not possible. 1A

16. (a) $2a - (a - 32) = 4(118 - a)$ 1M

$a = 88$

Lower quartile is \$88. 1A

Range $= 2a - (a - 32)$

$= \$120$ 1A

(b) $\dfrac{219\,468 + 102 \times 92 + 54h + 54k}{2017 + 210} \geq 108$ 1M

$k \geq 216 - h$

Note that $h \leq 110$.

$k \geq 216 - 110$

$k \geq 106$

The values of h and k are greater than 105. 1M

There are 108 (54 + 54) new books with selling prices greater than \$105.

There are 102 new books with selling prices less than \$105.

The new median of the selling prices is not less than the median before adding 210 new books. 1M

The new median is not less than \$105.

The claim is disagreed. 1A

17. (a) Interquartile range $= 2.8 - 1.9$ 1A

$= 0.9\text{ h}$ 1A

(b) (i) $m = 2.4$ 1A

$n = 1.1 + 2.0 = 3.1$ 1A

(ii) The interquartile range of John's running time (0.9 hours) is smaller than that of Peter's running time (1.1 hours). 1M

John should be chosen. 1A

(iii) According to the past performance,

probability of John to break the record $= \dfrac{5}{19}$; 1A

probability of Peter to break the record $\leq 0.25 < \dfrac{5}{19}$.

John should be chosen. 1A

18. (a) Median = 64 1A

Range = 84 − 40 = 44 1A

Inter-quartile range = 75 − 54 = 21 1A

(b) (i) The new inter-quartile range = 80 − 55 = 25 > 21. 1M

No. The distribution is not less dispersed than that in the first term. 1A

(ii) Number of students who get Grade A in the first term is 2.

In the second term, it is possible that the highest six scores are

80 80 80 80 80 88

such that the upper quartile and the maximum are 80 and 88 respectively.

In this case, number of students who get Grade A is 1, which is not 3 more than that in the first term. 1M

The claim is incorrect. 1A

19. (a) Median = 26.5 min 1A

Range = 42.3 − 16.8 = 25.5 min 1A

(b) Let t minutes be the time taken by the new student.

$$28 \times 35 + t = (28 - 0.3) \times 36$$

$$t = 17.2$$ 1A

Range = 42.3 − 16.8 = 25.5 1M

The claim is not correct. 1A

20. (a) $28 = 159 - (130 + a)$

$a = 1$ 1A

$$145 = \frac{131 + 132 + \ldots + 159}{16}$$ 1M

$b = 1$ 1A

Interquartile range = 151 − 141 = 10 cm 1A

(b) (i) For group B, interquartile range = 168 − 157 = 11 cm > 10 cm. 1M

The distribution of heights of students in group B is more dispersed than that in group A. 1A

(ii) Median of the distribution in group B (162 cm) is higher than the maximum of the distribution in group A (159 cm). 1M

The claim is agreed. 1A

21. (a) $55 - 49 > 64 - k$ 1M

$k > 58$

Required value is 59. 1A

(b) Median score of Test B (63) is greater than the highest score of Test A (k). 1A

The claim is agreed. 1A

22. (a) Range $= 13.1 - 1.8$ 1M

$= 11.3 \text{ g/100mL}$ 1A

Interquartile range $= 9.2 - 5.4$

$= 3.8 \text{ g/100mL}$ 1A

(b) New mean $= \dfrac{7.2 \times 20 + 2.4 + 4.6 + 7.5 + 10.4 + 13.4}{20 + 5}$ 1M

$= 7.292 \text{ g/100mL}$ 1A

New median is the 13th datum in ascending order.

New median is 7.5 g/100mL. 1M+1A

23. (a) Mean $= \dfrac{28 + 29 + \ldots + 69}{18}$

$= 42$ 1A

Median $= \dfrac{40 + 42}{2} = 41$ 1A

(b) (i) 39 1A

(ii) Let x, y and z be the new data, where $x \le y \le z$.

The median remains unchanged, we have $y = 41$. 1A

The mean of x, y and z is also 42.

Case 1: $x = 27$

Mean $= \dfrac{27 + 41 + z}{3} = 42$

$z = 58$ 1A

Standard deviation ≈ 10.4

Case 2: $z = 70$

Mean $= \dfrac{x + 41 + 70}{3} = 42$

$x = 15$ (rejected)

Thus, $x = 27$, $y = 41$ and $z = 58$.

The standard deviation is 10.4. 1A

24. (a) $87 - 55 = 2[(70 + n) - (60 + m)]$ 1M+1M

$n = m + 6$

Since the mode is 67, we have $0 < m < 3$.

Thus, $(m, n) = (1, 7)$ or $(2, 8)$. 1A+1A

(b) The weight of the left player is 87 kg. 1A

Case 1: $m = 1$ and $n = 7$

Standard deviation ≈ 8.34 kg 1M

Case 2: $m = 2$ and $n = 8$

Standard deviation ≈ 8.36 kg

The greatest possible standard deviation is 8.36 kg. 1A

25. (a) (i) Mode = 39

Thus, $a = b = 9$. 1A

(ii) $\dfrac{(50 + c) + 51}{2} - \dfrac{(30 + d) + 30}{2} = 21$ 1M

$c - d = 1$

Range $= (60 + d) - (20 + c)$

$= 40 - (c - d)$

$= 39$ 1A

(b) Mean $= \dfrac{(20 + c) + 25 + 26 + \ldots + (60 + d)}{20}$ 1M

$= \dfrac{830 + 2(c + d)}{20}$

Since $c - d = 1$, $1 \le c \le 5$ and $2 \le d \le 5$, we have $3 \le c + d \le 9$. 1M

$\dfrac{830 + 2(3)}{20} = 41.8 \le \text{mean} \le \dfrac{830 + 2(9)}{20} = 42.4$

Thus, mean = 42 and $c + d = 5$. 1A

Solving, we have $c = 3$ and $d = 2$.

Standard deviation ≈ 11.9 1A

26. (a) $57 = \dfrac{41 + 47 + \ldots + (70 + a)}{12}$ 1M

$a = 5$ 1A

(b) Range $= 75 - 41 = 34$ kg 1A

Interquartile range $= 66 - 49.5 = 16.5$ kg 1A

Standard deviation ≈ 10.7 kg 1A

27. Mean $= \dfrac{20 + 23 + \ldots + 63}{25} = 45$ 1A

Lower quartile = 35 1A

Standard deviation ≈ 12.2 1A

28. (a) Mode of the distribution is 38.

$$38 = \frac{21 + 22 + \ldots + 69}{18} \quad \text{1M}$$

$$2h + 2k = 20$$

The inter-quartile range is one third of the range.

$$(40 + k) - (20 + h) = \frac{69 - 21}{3} \quad \text{1M}$$

$$k - h = -4$$

Solving, we have $h = 7$ and $k = 3$. 1A+1A

(b) (i) Original median is 38.

New median is the 10th datum, which is also 38. 1M

There is no change in the median of the distribution. 1A

(ii) Let x be the numbers of Chinese characters typed by the newly added student in one minute.

Case 1: $x = 20$

Standard deviation ≈ 12.9

Case 2: $x = 70$

Standard deviation ≈ 14.1

The least possible standard deviation is 12.9. 1M

Thus, it is impossible that the standard deviation of the distribution is less than 12.7. 1A

29. (a) $(50 + c) - 24 = 33$ 1M

$c = 7$ 1A

(b) $\frac{43 + (40 + b)}{2} - \frac{(20 + a) + 26}{2} \geq 20$ 1M

$b - a \geq 3$

There are only two cases.

Case 1: mode is 24.

We have $a = 4$ and $b \neq 3$. 1M

$b - 4 \geq 3$

$b \geq 7$

When $a = 4$ and $b = 7$, standard deviation ≈ 10.3 1M

When $a = 4$ and $b = 8$, standard deviation ≈ 10.4

Case 2: mode is 43.

We have $a \neq 4$ and $b = 3$.

$3 - a \geq 3$

$a \leq 0$ (rejected)

Thus, the greatest possible standard deviation is 10.4. 1A

30. (a) $\frac{13 + x + 2}{32} = \frac{9}{16}$ 1M

$x = 3$ 1A

$y + 13 + x + 2 = 32$

$y = 14$ 1A

(b) 0.856 1A

31. (a) $\frac{k + 14}{k + 14 + 8 + 7 + 3} = \frac{5}{8}$ 1M

$8k + 112 = 5k + 160$

$k = 16$ 1A

(b) 1.24 1A

(c) Median = 2 1M

Mode = 1

The median and the mode are not equal. 1A

32. (a) The mode is 9. We have $x > 10$.

The median is 9.5.

$$7 + 9 + x = 10 + y + 8$$ 1M

$$y = x - 2$$

We have $(x,\ y) = (11,\ 9)$ or $(12,\ 10)$. 1A+1A

(b) When $x = 11$ and $y = 9$, standard deviation ≈ 1.61. 1M

When $x = 12$ and $y = 10$, standard deviation ≈ 1.59.

The least possible standard deviation is 1.59. 1A

(c) The ages of the four leaving members are 7, 7, 7 and 9. 1M

When $x = 11$ and $y = 9$, mean $= \dfrac{7(4) + 8(9) + \ldots + 12(8)}{4 + 9 + \ldots + 8} = 9.7$.

When $x = 12$ and $y = 10$, mean $= \dfrac{7(4) + 8(9) + \ldots + 12(8)}{4 + 9 + \ldots + 8} \approx 9.71$.

The greatest possible mean is 9.71. 1A

33. (a) Median = 4 1A

Inter-quartile range $= 6 - 2$

$= 4$ 1A

Standard deviation ≈ 1.99 1A

(b) (i) 8 1A

(ii) 31 1A

34. (a) $(50 + b) - (20 + a) = 36$ 1M

$$b - a = 6$$

Mean $= \dfrac{(20 + a) + 23 + \ldots + (50 + b)}{18} > 38$ 1M

$$a + b > 8$$

We have $(a,\ b) = (3,\ 9)$ or $(2,\ 8)$. 1A+1A

(b) Suppose $a = 3$ and $b = 9$.

Standard deviation is 10.30. 1M

Suppose $a = 2$ and $b = 8$.

Standard deviation is 10.27.

The greatest possible standard deviation is 10.30. 1A

35. (a) Let $\overline{x}$ be the mean of the scores of the examination.

$$\frac{71-\overline{x}}{6} = 1.5 \quad \text{1M}$$

$$\overline{x} = 62 \quad \text{1A}$$

(b) Score of David = $62 - 2.5(6) = 47$ 1M

Range of scores $\geq 71 - 47 = 24 > 23$

The claim is disagreed. 1A

36. (a) Let x marks be the score of Amy in the English examination.

$$\frac{x-60}{10} = -0.6 \quad \text{1M}$$

$$x = 54 \quad \text{1A}$$

(b) Standard score of Amy in the Chinese examination

$$\frac{54-59}{8}$$

$= -0.625$ 1A

< -0.6

Amy performs better in the English examination.

The claim is not correct. 1A

37. (a) $$\frac{3.1 + (0.1-k) + (0.1+k) + k}{4} = 0 \quad \text{1M}$$

$$k = -3.3 \quad \text{1A}$$

(b) Let σ marks be the standard deviation of the scores of the students.

$$0.1 - (-3.3) = \frac{74-40}{\sigma}$$

$$\sigma = 10 \quad \text{1A}$$

Standard score of Peter = $\frac{72-40}{10}$

$= 3.2$

> 3.1

The claim is incorrect. 1A

38. Let σ be the standard deviation of the scores.

$$-3.5 = \frac{28 - 70}{\sigma} \quad \text{1M}$$

$$\sigma = 12$$

Highest possible score of a student

$= 72 + 28$

$= 100$ 1A

Greatest possible standard score of a student

$= \frac{100 - 70}{12}$

$= 2.5$

The standard score of a student cannot exceed 2.5. 1A

39. (a) Let μ minutes be the mean of the distribution.

$$\frac{190 - \mu}{20} + \frac{240 - \mu}{20} = 0.5 \quad \text{1M}$$

$$\mu = 210 \quad \text{1A}$$

The mean is 210 minutes.

(b) The median is 220 minutes, which is greater than the mean (210 minutes). 1M

The claim is agreed. 1A

40. (a) Let x marks be the score of Peter in the English test.

$$\frac{x - 52}{16} = 0.5 \quad \text{1M}$$

$$x = 60 \quad \text{1A}$$

(b) Standard score of Peter in the Chinese test

$= \frac{68 - 66}{8}$

$= 0.25$

< 0.5 1A

Peter performs better in the English test than in the Chinese test.

The claim is not correct. 1A

41. Let $\$\mu$ and $\$\sigma$ be the mean and the standard deviation of the salaries respectively.
Let $\$C$ and $\$J$ be the salaries of Chris and John respectively.
We have $C - J = 3000$. 1M

$$\frac{C-\mu}{\sigma} - \frac{J-\mu}{\sigma} = 1 - (-2) \quad \text{1M}$$
$$\frac{C-J}{\sigma} = 3$$
$$\frac{3000}{\sigma} = 3$$
$$\sigma = 1000 \quad \text{1A}$$

$$\text{Variance} = 1000^2$$
$$= \$1\,000\,000 \quad \text{1A}$$

42. (a) Let $\overline{x}$ be the mean of the scores.

$$\frac{74 - \overline{x}}{12.5} = 0.96 \quad \text{1M}$$
$$\overline{x} = 62 \quad \text{1A}$$

(b) Standard score of Cora

$$= \frac{60 - 62}{12.5}$$
$$= -0.16$$

Note that $-0.16 + 1 = 0.84 < 0.96$. 1M
The claim is not correct. 1A

43. (a) $y = 7$ 1A

$$67 - \frac{1}{2}[(40 + x) + 51] = 18$$
$$x = 7 \quad \text{1A}$$

(b) $\text{Mean} = \dfrac{34 + 35 + \ldots + 83}{20}$

$= 58$

Standard deviation $= \sigma \approx 13.5$

Required standard score $= \dfrac{62 - 58}{\sigma}$ 1M

≈ 0.296 1A

(c) Sum of the two deleted data $= 58 \times 2 = 116$
The only possible set of deleted data is $\{42, 74\}$. 1M
In this case, new standard deviation is 13.2, which is lower than the original standard deviation.
The new mean is also 58.
Thus, the new standard score increases. 1M
The claim is disagreed. 1A

44. (a) $\dfrac{13 - x}{2} = 1.5$ 1M

$x = 10$ 1A

(b) The mean score of the 3 new people is also 10.

$$\frac{p + (p + 1) + (p + 5)}{3} = 10$$ 1M

$$p = 8$$

The scores of these 3 people are 8, 9 and 13.

There are 2 people with scores lower than the mean.

Thus, 2 people have negative standard score. 1A

45. (a) Let the mean and standard deviation of the distribution be μ and σ respectively.

$$\begin{cases} \dfrac{60 - \mu}{\sigma} = 1.25 \\ \dfrac{44 - \mu}{\sigma} = 0.25 \end{cases}$$ 1M

Solving, we have $\mu = 40$ and $\sigma = 16$. 1A+1A

(b) New standard score of Carol $= \dfrac{44(1 + 10\%) - 40(1 + 10\%)}{16(1 + 10\%)}$ 1M

$= 0.25$

The claim is not correct. 1A

46. (a) Let $\$\sigma$ be the standard deviation of the salaries of salesmen of the company.

$$\frac{3000}{\sigma} = 1 - (-2)$$ 1M

$$\sigma = 1000$$

Salary of Chris = $20\,000 + 1(1000) = \$21\,000$ 1A

Salary of John = $20\,000 - 2(1000) = \$18\,000$ 1A

(b) Difference of standard scores of them

$$= \frac{21\,000(1 + 10\%) - 20\,000(1 + 10\%)}{1000(1 + 10\%)} - \frac{18\,000(1 + 10\%) - 20\,000(1 + 10\%)}{1000(1 + 10\%)}$$ 1M

$= 3$

The difference of standard scores remains unchanged.

The claim is disagreed. 1A

47. (a) Standard score $= \dfrac{74 - 64}{4}$ 1M

$= 2.5$ 1A

(b) Standard score of Samuel after the adjustment

$= \dfrac{74(1 + 10\%) - 64(1 + 10\%)}{4(1 + 10\%)}$ 1M

$= 2.5$

< 2.75

Sophia performs better in the test. 1A

2 Multiple Choice Questions

1. C

Number of data = 8 + 6 + 6 + 4 + 6 = 30

Inter-quartile range = 9 − 3

= 6

2. D

I. ✓.

II. ✗.

$$q = (50 + p) - 42$$

$$q - p = 8$$

III. ✓.

Since $2 \leq p \leq 5$ and the inter-quartile range is $q = p + 8$.

We have $10 \leq q \leq 13$.

3. B

The upper quartile of the distribution is 210 g.

Required probability $= \dfrac{7}{24}$

4. C

Range = $(40 + b) - (10 + a) \le 36$

$$b - a \le 6$$

I. ✓. Note that $a \le 2$ and $b \ge 5$.

Range $= (40 + b) - (10 + a)$

$= 30 + (b - a)$

$\ge 30 + (5 - 2)$

$= 33$

II. ✗. Take $b = 8$ and $a = 2$, the range of the above distribution is 36.

III. ✓. Median $= \dfrac{30 + 32}{2} = 31$

5. A

Median $= \dfrac{(20 + n) + 25}{2} \le 24$ and Interquartile range $= (30 + n) - (10 + m) \ge 18$

$n \le 3$

$n - m \ge -2$

$m - n \le 2$

I. ✓. $m \le n + 2 \le 3 + 2 = 5$ and $m \ge 0$.

II. ✓. From the stem-and-leaf diagram, $n \ge 1$. Combine with $n \le 3$, we have $1 \le n \le 3$.

III. ✗. It is possible that $m = n = 1$ such that all conditions are satisfied.

6. A

I. ✗.

Mean $= \dfrac{4 + 5 + \ldots + v}{11} = \dfrac{71 + u + v}{11}$

Take $u = v = 4$, we have mean $= \dfrac{79}{11} \ne 7$.

II. ✓.

The median is the 6th datum, which must be 7.

III. ✗.

Take $u = v = 5$, we have mode $= 5 \ne 7$.

7. B

Consider the inter-quartile range.

$$(30 + n) - (10 + m) > 24$$
$$n - m > 4$$

I. ✓.

Note that $0 \le n \le 8$ and $n - m > 4$.

We have $0 \le m < n - 4 \le 8 - 4 = 4$.

II. ✗.

Take $n = 8$ and $m = 0$.

The inter-quartile range is 28, which is greater than 24.

Thus, it is possible that $n = 8$.

III. ✓.

8. A

Range of the integers is 10. There is one datum equal to 14.

Take $m = 14$ for simplicity.

I. ✓.

The mean is the greatest when $m = n = 14$.

Greatest possible mean $= \dfrac{4 + 5 + \ldots + 14}{8} = 8.5$

II. ✓.

The median is the average of the 4th and 5th data.

No matter the value of n, the median is still 7.

III. ✗.

Take $m = 14$ and $n = 7$.

Inter-quartile range $= 8.5 - 6 = 2.5 < 4$

The least possible inter-quartile range is not greater than 2.5.

9. A

I. ✓.

The mean is maximum when $m = n = 20$.

Maximum mean $= \dfrac{5 + 7 + 8 + \ldots + 20}{12} \approx 12.7 < 13$

II. ✗.

When $m = n = 20$, median is 14.

III. ✗.

When $m = 9$ and $n = 20$, the mode has two values 9 and 15.

10. B

We have $4 \le x \le 11$ and $4 \le y \le 11$.

I. ✗.

Take $x = y = 5$. The range of the group of numbers is 7, but $y - x = 0$.

II. ✓.

Since $0 < 4 \le x \le 11$ and $0 < 4 \le y \le 11$, we have

$$4 \times 4 \le xy \le 11 \times 11$$
$$16 \le xy \le 121$$

III. ✗.

Take $x = y = 4$. The range of the group of numbers is 7.

$$\text{Mean} = \frac{x + y + 4 + 6 + 7 + 11}{6}$$
$$= 6$$

The mean may not be greater than 6.

11. B

$$2 + 10 + 14 + m + 2 = 40$$
$$m = 12$$

$$n - 1 = 5$$
$$n = 6$$

We have $x = 3$ and $z = 3$.

$$y = \frac{1(2) + 2(10) + 3(14) + 4(12) + 6(2)}{40}$$
$$= 3.1$$

12. D

A. ✗. Mode = 30

B. ✗. Median = 30

C. ✗. Lower quartile = 25

D. ✓.

13. D

We have $m = n = 5$.

I. ✓.

II. ✓. $\text{Mean} = \dfrac{1 + 2 + 5 + 12 + \ldots + 5}{10} = 5.2$

III. ✓. Range = $12 - 1 = 11$

14. B

Required number $= \frac{1}{4} \times 40$

$= 10$

15. A

Easy.

16. C

The upper quartile is 9.

The lower quartile is 1.

Interquartile range $= 9 - 1 = 8$

Remarks: The minimum is equal to the lower quartile in this case.

17. A

I. ✓.

II. ✓. Interquartile range $= 26 - 20 = 6\,^\circ\text{C}$

III. ✗. Range $= 30 - 16 = 14\,^\circ\text{C}$

18. B

The data are more concentrated on the higher values.

The maximum, upper quartile and median should be closed to each other.

Options A and D should be wrong.

The minimum, lower quartile and the median are 22, 40 and 47 respectively.

Lower quartile should appear closer to the median than the minimum.

The answer is B.

19. B

50% of the data lies between lower quartile and upper quartile.

20. B

$70 - 40 = 3(a - 48)$

$a = 58$

21. C

Inter-quartile range = 85 − 70

= 15 marks

22. A

I. ✓.

Inter-quartile range = 20 − 17 = 3

II. ✗.

Mean cannot be obtained from the diagram.

III. ✓.

Median is 19, which is greater than 18.

23. A

Since median is 6, one of the unknown is 6.

Since the mode is 7, one of the unknown is 7.

Take $x = 6$ and $y = 7$ for simplicity.

$$\text{Mean} = \frac{2+3+3+6+7}{5}$$

$$= 4.2$$

24. B

Inter-quartile range = 9 − 4 = 5 °C

25. A

I. ✓.

II. The minimum is 3 and the lower quartile is 4, which are different.

III. The median is 3 and the minimum is 3, which are equal.

IV. The lower quartile is 5 and the median is 5, which are equal.

26. D

In the cumulative frequency curve, steeper curve represents more data in the corresponding class.

The data is more concentrated in the lower part.

Minimum, lower quartile, median and upper quartile will be closed to each other.

27. D

A. ✗.

The median is 1.

B. ✗.

The lower quartile is 0.

C. ✗.

The mode is 0.

D. ✓.

28. D

$$\text{Mean} = 6 = \frac{2 + 4 + 6 + 6 + x + x + x + y}{8}$$

$3x + y = 30$

The only possible cases are $(x,\ y) = (6,\ 12)$ or $(8,\ 6)$ or $(9,\ 3)$.

I. ✗.

Take $x = 9$ and $y = 3$.

The mean and the median is 6, while the mode is 9.

II. ✓.

$(x,\ y)$	$(6,\ 12)$	$(8,\ 6)$	$(9,\ 3)$
Range	10	6	7

The greatest possible range is 10.

III. ✓.

$(x,\ y)$	$(6,\ 12)$	$(8,\ 6)$	$(9,\ 3)$
Variance	7	4	7

The least possible variance is 4.

29. D

I. ✗.

Take $x = y = 3$. The range is 3, which is less than 5.

The mode is 3, instead of 5.

II. ✓.

The median is the greatest when x and y take the maximum value.

Since the range does not exceed 5, the maximum value of x and y are 7.

In this case, the median is 5.

Thus, the greatest possible median is 5.

III. ✓.

The variance is greater when the numbers are more dispersed.

Take $x = 7$ and $y = 2$, the variance is 3.01.

The greatest possible variance is not less than 3.01, which means it exceeds 3.

30. A

Steeper curve represents more data in the corresponding class.

In distribution X, the data is more concentrated in both upper and lower parts.

The standard deviation is greater.

In distribution Z, the data is more concentrated in the middle part.

The standard deviation is smaller.

31. B

Calculator work.

32. D

Let μ and σ be the mean and the standard deviation of the test scores respectively.

$$\frac{m}{n} = -\frac{3}{2}$$

$$\frac{67-\mu}{\sigma} \div \frac{82-\mu}{\sigma} = -\frac{3}{2}$$

$$\frac{67-\mu}{82-\mu} = -\frac{3}{2}$$

$$134 - 2\mu = -246 + 3\mu$$

$$\mu = 76$$

33. B

Let μ and σ be the mean and the standard deviation of the scores in the test respectively.

$$\frac{96.5-\mu}{\sigma}-\frac{58-\mu}{\sigma}=4-(-1.5)$$

$$\frac{96.5-58}{\sigma}=5.5$$

$$\sigma=7$$

34. B

Let $\bar{x}$ and σ be the mean and the standard deviation respectively.

$$y+(-y)=\frac{54-\bar{x}}{\sigma}+\frac{74-\bar{x}}{\sigma}$$

$$0=(54-\bar{x})+(74-\bar{x})$$

$$\bar{x}=64$$

Consider the student with score 70.

$$1.5=\frac{70-64}{\sigma}$$

$$\sigma=4$$

Thus, $y=\dfrac{54-64}{4}=-2.5$.

35. B

Let μ marks and σ marks be the mean and the standard deviation of the scores.

$$\begin{cases}\dfrac{82-\mu}{\sigma}=1.5\\[2mm]\dfrac{58-\mu}{\sigma}=-2.5\end{cases}$$

Solving, we have $\mu=73$ and $\sigma=6$.

Required standard score $=\dfrac{70-73}{6}$

$=-0.5$

36. B

Let z be the standard score of Alice.

$$\frac{196-136}{62}<z<\frac{200-119}{62}$$

$$0.968<z<1.31$$

The answer is B.

37. A

I. ✓.

The standard score of Mary is higher.

II. ✓.

Difference between Mary's score and the mean is 0.8σ, where σ is the standard deviation.

Difference between John's score and the mean is 1.05σ, which is greater than 0.8σ.

III. ✗.

There is no information about the distribution.

This may not be true.

38. B

Let μ and σ be the mean and the standard deviation of the test scores respectively.

$$\begin{cases} \dfrac{54-\mu}{\sigma} = -1.5 \\ \dfrac{65-\mu}{\sigma} = 1.25 \end{cases}$$

Solving, we have $\mu = 60$ and $\sigma = 4$.

39. D

Let μ and σ be the mean and the standard deviation of the test scores respectively.

$$\begin{cases} \dfrac{90-\mu}{\sigma} = 6 \\ \dfrac{36-\mu}{\sigma} = -3 \end{cases}$$

Solving, we have $\mu = 54$ and $\sigma = 6$.

$$\begin{aligned} s &= \frac{57-54}{6} \\ &= 0.5 \end{aligned}$$

40. A

Let $\bar{x}$ marks be the mean score.

Difference in standard score

$$\begin{aligned} &= \frac{72-\bar{x}}{8} - \frac{56-\bar{x}}{8} \\ &= \frac{72-56}{8} \\ &= 2 \end{aligned}$$

41. D

I. ✗. New median = $(m + 7) \times 4 = 4m + 28$

II. ✗. New variance = $4^2 v = 16v$

42. A

I. ✓.
Let m be the mean of $\{a, b, c, d, e\}$.
The means of P and Q are $x + m$ and $y + m$ respectively.
The mean of P is greater than the mean of Q.

II. ✓.
The ranges of P and Q are both $e - a$.

III. ✗.
The variances of P and Q are equal to the variance of $\{a, b, c, d, e\}$.

43. B

$$\begin{aligned}\text{Standard deviation} &= \sqrt{9} \times 4 \\ &= 12\end{aligned}$$

44. D

New mean = $2(a - 2)$
New interquartile range = $2b$ (won't be affected by addition or subtraction)
New variance = $4c$ (remember to square the ratio)

45. B

The new set of numbers are formed by the following steps:

(1) Multiply each number by 5.

(2) Add 2 to each number.

I. ✓.

II. ✗.
We have $q_2 = 5q_1 \neq 5q_1 + 2$.

III. ✓.
We have $v_2 = 5^2 v_1 = 25v_1$.

46. A

I. ✓.

New mean = $(m + 10)(-0.8)$

$= -0.8m - 8$

II. ✓.

New variance = $0.8^2 v$

$= 0.64v$

III. ✗.

Standard deviation cannot be negative.

New standard deviation is $0.8\sqrt{v}$.

47. D

From new to old,

multiply by 4 and then subtract 3 from the resulting number.

Thus, mean of original set = $4m - 3$ and variance = $16v$.

48. A

Required variance is equal to the variance of the numbers −7, −1, 0, 4, 4, 6, 9 and 17.

Required variance is 45.

END OF PAPER