REG-MAE-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. B	3. B	4. C	5. C
6. A	7. C	8. A	9. B	10. B
11. C	12. C	13. B	14. B	15. A
16. D	17. A	18. B	19. C	20. B
21. B	22. C	23. A	24. C	25. C
26. A	27. C	28. C	29. B	30. C

1. $\boxed{\text{C}}$

$bpx^2 + (bq + 1)x + (br + c) = 0$

$b(px^2 + qx + r) + x + c = 0$

The roots are the x-coordinates of the intersection of $x + by + c = 0$ and $y = px^2 + qx + r$.

The roots are x_2 and x_5.

2. $\boxed{\text{B}}$

$P\ (a,\ 0)$, $Q\ (b,\ 0)$ and $R\ (0,\ c)$.

$$c = (0 - a)(0 - b)$$
$$ab = c$$

Required solution are $(a,\ 0)$ and $(0,\ ab)$.

3. $\boxed{\text{B}}$

Put $x = -1$ and $x = 2$ into $y - 2x - 3 = 0$, we have $y = 1$ and $y = 7$ respectively.

$$\begin{cases} 1 = 5(-1)^2 + a(-1) + b \\ 7 = 5(2)^2 + a(2) + b \end{cases}$$

Solving, we have $a = -3$ and $b = -7$.

4. $\boxed{\text{C}}$

For the point $(0,\ 5)$,

$$0 - 2(5) + r = 0 \quad \text{and} \quad 5 = -2(0)^2 + p(0) + q$$
$$r = 10 \qquad\qquad q = 5$$

At point P, when $x = -2$,

$$(-2) - 2y + 10 = 0 \quad \text{and} \quad 4 = -2(-2)^2 + p(-2) + 5$$
$$y = 4 \qquad\qquad p = -3.5$$

5. $\boxed{\text{C}}$

$$3x + 8 = (x + 1)^2 + 5$$
$$0 = x^2 - x - 2$$
$$x = 2 \quad \text{or} \quad -1$$

We have $(x,\ y) = (2,\ 14)$ or $(-1,\ 5)$.

Thus, $y = 5$ or 14.

6. $\boxed{\text{A}}$

$$-9x - 7 = x^2 + 6x + 7$$
$$0 = x^2 + 15x + 14$$
$$x = -1 \quad \text{or} \quad -14$$

When $x = -1$, $y = -9(-1) - 7 = 2$; when $x = -14$, $y = -9(-14) - 7 = 119$.

7. $\boxed{\text{C}}$

$$-4x - 1 = 2x^2 - x - 3$$
$$0 = 2x^2 + 3x - 2$$
$$x = -2 \quad \text{or} \quad \frac{1}{2}$$

When $x = -2$, $y = -4(-2) - 1 = 7$; when $x = \frac{1}{2}$, $y = -4\left(\frac{1}{2}\right) - 1 = -3$.

$(x,\ y) = \left(\frac{1}{2},\ -3\right)$ or $(-2,\ 7)$

8. $\boxed{\text{A}}$

$$x^2 + 4x = -6x - 25$$
$$x^2 + 10x + 25 = 0$$
$$x = -5$$

9. $\boxed{\text{B}}$

$$3x^2 - x(7x - 16) = 12$$
$$-4x^2 + 16x - 12 = 0$$
$$x = 1 \quad \text{or} \quad 3$$

When $x = 1$, $y = 7(1) - 16 = -9$; when $x = 3$, $y = 7(3) - 16 = 5$.

10. $\boxed{\text{B}}$

Rewrite the equations for easier checking: $\begin{cases}(y-x)(y+x)=20\\ x-3y=-14\end{cases}$.

Check if the integral coordinates can satisfy the equation:

		$(y-x)(y+x)=20$	$x-3y=-14$
A.	(7, 7)	✗	
B.	(4, 6)	✓	✓
C.	(−4, 6)	✓	✗
D.	(6, −4)	✓	✗

11. $\boxed{\text{C}}$

$$5(-2)+4(-1)+k=0$$
$$k=14$$
$$5x+4y+14=0 \longrightarrow y=\frac{-7}{2}-\frac{5x}{4}.$$
$$2x^2-2\left(-\frac{7}{2}-\frac{5x}{4}\right)^2+4=5x\left(-\frac{7}{2}-\frac{5x}{4}\right)$$
$$\frac{41}{8}x^2-\frac{41}{2}=0$$
$$x=\pm 2$$

Only option C has the root with $x=2$.

12. $\boxed{\text{C}}$

$$x-\frac{y}{2}=2 \quad\longrightarrow\quad y=2x-4$$
$$5x^2+9x-(2x-4)=2$$
$$5x^2+7x+2=0$$
$$x=-\frac{2}{5}\quad\text{or}\quad -1$$

13. $\boxed{\text{B}}$

$$3\left(\frac{4}{3}x+1\right)=6x^2+x-15$$
$$0=6x^2-3x-18$$
$$x=2\quad\text{or}\quad -\frac{3}{2}$$

When $x=-\frac{3}{2}$, $y=\frac{4}{3}\left(-\frac{3}{2}\right)+1=-1$; when $x=2$, $y=\frac{4}{3}(2)+1=\frac{11}{3}$.

14. $\boxed{\text{B}}$

$5 = (-1)^2 - b(-1) + 1$

$b = 3$

$5 = a(-1) + 3$

$a = -2$

15. $\boxed{\text{A}}$

$p = 7^2 - 3(7) - 5$

$p = 23$

$23 = 3(7) + q$

$q = 2$

We have $\begin{cases} y = 3x + 2 \\ y = x^2 - 3x - 5 \end{cases}$.

$$\begin{aligned} 3x + 2 &= x^2 - 3x - 5 \\ 0 &= x^2 + 6x - 7 \\ x &= 7 \quad \text{or} \quad -1 \end{aligned}$$

When $x = -1$, $y = 3(-1) + 2 = -1$.
The other solution is $(-1,\ -1)$.

16. $\boxed{\text{D}}$

Put $x = 3y - 1$ into the first equation,

$$\begin{aligned} (3y - 1)^2 - y(3y - 1) - 3y^2 &= 3 \\ (9 - 3 - 3)y^2 + (-6 + 1)y + (1 - 3) &= 0 \\ y = 2 \quad \text{or} \quad -\frac{1}{3} \end{aligned}$$

Only option D satisfies this.

17. $\boxed{\text{A}}$

We have

$$\begin{cases} 7(2) = a(6) + b \\ \quad 6 = (2)^2 + a(2) + b \end{cases}$$

Solving, we have $a = 3$ and $b = -4$.

18. $\boxed{\text{B}}$

$-3x^2 + 8x - 1 = 5x - k$

$-3x^2 + 3x + k - 1 = 0$

$\Delta = 3^2 - 4(-3)(k - 1) \geq 0$

$k \geq \frac{1}{4}$

19. $\boxed{\text{C}}$

$$2x - 3 = x^2 - 4x + (k + 2)$$
$$0 = x^2 - 6x + (k + 5)$$
$$\Delta = 6^2 - 4(1)(k + 5) < 0$$
$$-4k + 16 < 0$$
$$k > 4$$

20. $\boxed{\text{B}}$

$$x - k = x^2 - 7x + 2$$
$$0 = x^2 - 8x + 2 + k$$
$$\Delta = 8^2 - 4(1)(2 + k) = 0$$
$$-4k + 56 = 0$$
$$k = 14$$

21. $\boxed{\text{B}}$

$$x^2 + (2x + 5)^2 = k$$
$$5x^2 + 20x + 25 - k = 0$$
$$\Delta = 20^2 - 4(5)(25 - k) \geq 0$$
$$20k - 100 \geq 0$$
$$k \geq 5$$

22. $\boxed{\text{C}}$

$$kx^2 - (x + 1)^2 + x(x + 1) = 1$$
$$kx^2 - x - 2 = 0$$
$$\Delta = 1^2 - 4(k)(-2) < 0$$
$$8k + 1 < 0$$
$$k < -\frac{1}{8}$$

23. $\boxed{\text{A}}$

$$5 - x = 4x^2 - 5x + 2$$
$$0 = 4x^2 - 4x - 3$$
$$x = \frac{3}{2} \quad \text{or} \quad -\frac{1}{2}$$

Since x-coordinate of A is negative, required x-coordinate is $-\frac{1}{2}$.

24. $\boxed{\text{C}}$

$3x - 14 = 2x^2 - 9x + c$

$0 = 2x^2 - 12x + (c + 14)$

$\Delta = 12^2 - 4(2)(c + 14) = 0$

$c = 4$

25. $\boxed{\text{C}}$

$\dfrac{10x - k}{2} = 6x^2 - 7x - 5$

$0 = 6x^2 - 12x + \dfrac{k}{2} - 5$

$\Delta = 12^2 - 4(6)\left(\dfrac{k}{2} - 5\right) < 0$

$k > 22$

26. $\boxed{\text{A}}$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$x = 2 \quad \text{or} \quad -1$

We have $(x,\ y) = (2,\ 4)$ or $(-1,\ 1)$.

Denote the intersection of AB and the y-axis by C.

The coordinates of C are $(0,\ 2)$.

Required area $= \dfrac{(OC)(1)}{2} + \dfrac{(OC)(2)}{2}$

$= 3$

27. $\boxed{\text{C}}$

$x^2 - 5k + k = 3x - 2$

$x^2 - 8x + (k + 2) = 0$

$\Delta = 8^2 - 4(1)(k + 2) = 0$

$-4k + 56 = 0$

$k = 14$

28. $\boxed{\text{C}}$

The coordinates of F are $(2,\ 0)$.

$x^2 - 2x - 3 = \dfrac{3}{2}x - 3$

$x^2 - 3.5x = 0$

$x = 0 \quad \text{or} \quad 3.5$

The coordinates of D and E are $(0,\ -3)$ and $(3.5,\ 2.25)$.

Required area $= \dfrac{(BF)(3)}{2} + \dfrac{(BF)(2.25)}{2}$

$= \dfrac{21}{8}$

29. $\boxed{\text{B}}$

$$2x + 1 = x^2 - 6x + k$$
$$0 = x^2 - 8x + (k - 1)$$
$$\alpha + \beta = -\frac{-8}{1}$$
$$= 8$$

30. $\boxed{\text{C}}$

$$3x + 5 = -x^2 + bx + c$$
$$0 = -x^2 + (b - 3)x + (c - 5)$$

x-coordinate of M $= \dfrac{1}{2}\left[-\dfrac{b-3}{-1}\right]$

$= \dfrac{b-3}{2}$

y-coordinate of M $= 3\left(\dfrac{b-3}{2}\right) + 5$

$= \dfrac{19-3b}{2}$

The coordinates of M are $\left(\dfrac{b-3}{2},\ \dfrac{19-3b}{2}\right)$.

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