REG-DISP-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. A	3. A	4. A	5. D
6. B	7. D	8. C	9. A	10. A
11. B	12. C	13. B	14. A	15. B
16. A	17. B	18. D	19. A	20. D
21. B	22. D	23. A	24. A	25. B
26. B	27. D	28. C	29. C	30. A

1. D

We have $x = 5$ or $x + 2 = 5$.
When $x + 2 = 5$, $x = 3$ and the mode is 3 and 5, which should be rejected.
Thus, we have $x = 5$, and the numbers are 2, 3, 5, 5, 7, 8.

I. ✓. Mean $= \dfrac{2 + 3 + 5 + 5 + 7 + 8}{6} = 5$

II. ✓. Range $= 8 - 2 = 6$

III. ✓. Inter-quartile range $= 7 - 3 = 4$

2. A

Upper quartile = 59 kg and lower quartile = 46 kg
Inter-quartile range $= 59 - 46$
$= 13$ kg

3. A

$$\frac{4 + 4 + 6 + \ldots + n}{9} = 6$$
$$m + n = 10$$

I. ✓. Since m and n cannot be both greater than 6, the median (the 5th datum) must be 6.

II. ✓. Range is the greatest when the values of m and n are 1 and 9.
Range is the least when $m = n = 5$.
Thus, $5 \le y \le 8$.

III. ✗. When $m = 4$ and $n = 6$, mode = 4 and 6.

4. A

Upper quartile = $(60 + b)$ and lower quartile = $(30 + a)$.
$(60 + b) - (30 + a) \leq 25 \Rightarrow b - a \leq -5$

I. ✓. Since $a \geq 5 + b$, value of a is at least 5, and at most 9 (because it is a single digit integer).

II. ✓. Since $b \leq a - 5$ and $a \leq 9$, value of b is at most $9 - 5 = 4$, and at least 0 (single digit integer).

III. ✗. It is possible that $a = 9$ and $b = 1$, such that the interquartile range is 22 (≤ 25), while $a - b = 8$.

5. D

$$\frac{11 + 5 + 17 + 8 + 14 + 9 + 13 + 10 + x}{9} = 12$$

$$x = 3$$

Arrange the data in ascending order:

3 5 8 9 10 11 13 14 17

Inter-quartile range = $13.5 - 6.5 = 7$

6. B

Denote the data sets $\{x - 2, x - 1, x, x + 2, x + 2, x + 3\}$ and $\{x - 5, x - 4, x - 2, x - 2, x - 1, x\}$ by A and B respectively.

I. ✓. Range of $A = (x + 3) - (x - 2) = 5$
Range of $B = x - (x - 5) = 5$

II. ✗. Median of $A = x + 1$
Median of $B = x - 2$

III. ✓. Inter-quartile range of $A = (x + 2) - (x - 1) = 3$
Inter-quartile range of $B = (x - 1) - (x - 4) = 3$

7. D

Lower quartile = 62 kg.
Required probability = $1 - \frac{4}{24} = \frac{5}{6}$.

8. C

Range = $(40 + k) - (20 + h) \geq 24$

$$k - h \geq 4$$

I. ✗. It is possible that $h = 3$ and $k = 9$ such that range = $26 \geq 24$.

II. ✓. $k \geq 4 + h \geq 4 + 0 = 4$ and $k \leq 9$ obviously.

III. ✓. $k - h \geq 4$ as proved above. Also, $k - h \leq 9 - 0 = 9$

9. A

$a = x - \frac{18}{11}$, $b = x$, $c = x$ and $d = 10$.

I. ✓. $a - b = -\frac{18}{11} < 0$

II. ✗. $b = c = x$

III. ✗. $c - d = x - 10$ which can be positive or negative.

10. A

We have $3 \leq m, n \leq 12$.

I. ✓. No matter how large or how small m and n, the median (5th datum) is always 8.

II. ✗. When $m = n = 3$, mean $= \frac{58}{9} \neq 8$.

III. ✗. When $m = n = 3$, mode $= 3 \neq 8$.

11. B

Range $= (40 + k) - (10 + h) \geq 33 \quad \Rightarrow \quad k - h \geq 3$.

I. ✗. It is possible that $k = 7$ and $h = 4$. The range is 33, satisfying all the conditions.

II. ✓. $k - h \geq 3 \quad \Rightarrow \quad k \geq 3 + h \geq 3$.

III. ✗. It is possible that $k = 9$ and $h = 0$. The range is 39, satisfying all the conditions.

12. C

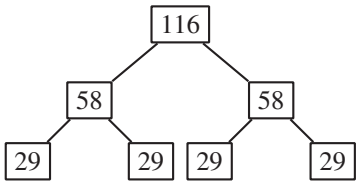
Number of students = 116. We can obtain the median and quartiles using the figure below.

A. ✗. Mode = 7

B. ✗. Median = 7

C. ✓. Lower quartile = 6

D. ✗. Upper quartile = 8



13. B

The upper quartile of the distribution is 210 g.

Required probability $= \frac{7}{24}$

14. A

Arrange the data in ascending order:

16 19 23 24 29 31 33

We have $24 \le x \le 29$ and $\dfrac{x + 24}{2} = 25$.
Thus, we have $x = 26$.
Inter-quartile range $= 30 - 21 = 9$

15. B

Mean $= 5 \quad \Rightarrow \quad 2 + 3 + 4 + \ldots + m + n = 5 \times 9 \quad \Rightarrow \quad m + n = 4$

I. ✗. It is possible that $m = 1$ and $n = 3$. Then mode $= a = 3$.

II. ✓. We have $m < 4$ and $n < 4$. Median $= b =$ 5th datum $= 4$.

III. ✗. It is possible that $m = 1$ and $n = 3$. Then range $= c = 10 - 1 = 9 \ne 8$.

16. A

Median $= \dfrac{(20 + n) + 25}{2} \le 24 \quad$ and $\quad$ Interquartile range $= (30 + n) - (10 + m) \ge 18$

$n \le 3$ $\qquad\qquad$ $n - m \ge -2$

$m - n \le 2$

I. ✓. $m \le n + 2 \le 3 + 2 = 5$ and $m \ge 0$.

II. ✓. From the stem-and-leaf diagram, $n \ge 1$. Combine with $n \le 3$, we have $1 \le n \le 3$.

III. ✗. It is possible that $m = n = 1$ such that all conditions are satisfied.

17. B

Total number of data is 25.
Range $= 17 - 13 = 4$
Inter-quartile range $= 16.5 - 14.5 = 2$

18. D

A. ✗. Mode of the distribution is 8.

B. ✗. Mean $= \dfrac{5(3) + 6(4) + 7(23) + 8(50) + 9(40)}{3 + 4 + 23 + 50 + 40}$

$= 8$

C. ✗. Median of the distribution is 8.

D. ✓. Inter-quartile range $= 9 - 7.5$

$= 1.5$

19. A

By simple calculation, $a = x + 0.875$, $b = x + 0.5$, $c = x$, $d = 4$

I. ✓. $a - c = 0.875 > 0$

II. ✗. $b - d = x - 3.5$, which can be positive or negative.

III. ✗. $d - c = 4 - x$, which can be positive or negative.

20. D

We have $m = n = 5$.

I. ✓.

II. ✓. Mean $= \dfrac{1 + 2 + 5 + 12 + \ldots + 5}{10} = 5.2$

III. ✓. Range $= 12 - 1 = 11$

21. B

Upper quartile = \$40

Angle of sector \$10 $= 360^\circ - 72^\circ - 36^\circ - 90^\circ - 144^\circ = 18^\circ$

Lower quartile $= \dfrac{20 + 30}{2} =$ \$25

Inter-quartile range $= 40 - 25 =$ \$15

22. D

A. ✗. Mode = 30

B. ✗. Median = 30

C. ✗. Lower quartile = 25

D. ✓.

23. A

There are 28 students. Lower quartile = 5 and upper quartile = 6.

Interquartile range $= 6 - 5 = 1$.

24. A

If $x = 0$, IQR = 3; if $x = 8$, IQR = 3.

If $x \geq 9$, IQR < 3. Thus, $0 \leq x \leq 8$.

25. B

I. ✗. m could be 4.

II. ✓.

III. ✗. Maximum value of $x = 6.9 - 3.0 = 3.9 < 34$.

26. B

Inter-quartile range $= 82 - 67 = 15$

27. D

Range = $850 - 50 = 800$

28. C

Lower quartile = $\frac{28 + 30}{2} = 29$ and upper quartile = $\frac{52 + (50 + a)}{2} = \frac{102 + a}{2}$

$$\frac{102 + a}{2} - 29 \leq 24$$

$$a \leq 4$$

According to the stem-and-leaf diagram, $2 \leq a \leq 8$.

Thus, $2 \leq a \leq 4$ and $a = 2$, 3 or 4.

29. C

Lower quartile = 47. Number of students has a weight exceeding lower quartile = 6.

Required probability = $\frac{C_1^6 C_2^4}{C_3^{10}} = \frac{3}{10}$

30. A

Range = $63.5 - 43.5 = 20\,\text{kg}$

Conventional Questions

31. (a) $11 = 28 - \dfrac{(20 + c) + 13}{2}$ 1M

$c = 1$ 1A

(b) $(30 + b) - (10 + a) \geq 25$

$b - a \geq 5$

$23 = \dfrac{(10 + a) + 12 + 13 + \ldots + (30 + b)}{16}$ 1M

$a + b = 6$

We have $a = 0$ and $b = 6$. 1A+1A

32. (a) The median = 31 1A

The mode = 23 1A

(b) (i) Since $0 \leq a \leq 5$ and $7 \leq b \leq 9$,

we have $\begin{cases} a = 0 \\ b = 7 \end{cases}$, $\begin{cases} a = 1 \\ b = 8 \end{cases}$ or $\begin{cases} a = 2 \\ b = 9 \end{cases}$. 1A+1A

(ii) Required probability $= \dfrac{3 + 3 + 3 + 3 + 2 + 9 + 9}{20 \times 13}$ 1M

$= \dfrac{8}{65}$ 1A

33. (a) $(80 + b) - (30 + a) = 57$

$b - a = 7$ 1M

So, $(a,\ b) = (0,\ 7)$ or $(1,\ 8)$. 1A+1A

(b) Interquartile range = $72.5 - 44 = 28.5$ g 1M+1A

(c) Proportion of loaves of bread in Bakery A that exceed 70 g = $\dfrac{4}{17} < \dfrac{1}{4}$. 1M

The claim is disagreed. 1A

34. (a) $47 = (50 + c) - (10 + a)$ 1M

$c - a = 7$

$33 = \dfrac{(10 + a) + 14 + 18 + \ldots + (50 + c)}{18}$ 1M

$a + b + c = 10$

Since $0 \leq a \leq 4$, $0 \leq b \leq 3$ and $7 \leq c \leq 9$, we have

$(a,\ b,\ c) = (0,\ 3,\ 7)$ or $(1,\ 1,\ 8)$. 1A

(b) Required probability $= \dfrac{10}{18}$ 1M

$= \dfrac{5}{9}$ 1A