

M2REG-DEFINT-2425-ASM-SET 3-MATH**Suggested solutions**

$$\begin{aligned}
1. \quad \int_3^5 (x+1)e^{\frac{x-1}{2}} dx &= 2 \left[(x+1)e^{\frac{x-1}{2}} \right]_3^5 - 2 \int_3^5 e^{\frac{x-1}{2}} dx && 1\text{M}+1\text{A} \\
&= 2(6e^2 - 4e) - 4 \left[e^{\frac{x-1}{2}} \right]_3^5 \\
&= 8e^2 - 4e && 1\text{A}
\end{aligned}$$

$$\begin{aligned}
2. \quad \int_{-1}^0 (3-x)e^{-3x} dx &= -\frac{1}{3} \left[(3-x)e^{-3x} \right]_{-1}^0 - \frac{1}{3} \int_{-1}^0 e^{-3x} dx && 1\text{M}+1\text{A} \\
&= -\frac{1}{3}(3 - 4e^3) + \frac{1}{9} \left[e^{-3x} \right]_{-1}^0 \\
&= -\frac{8}{9} + \frac{11}{9}e^3 && 1\text{A}
\end{aligned}$$

$$\begin{aligned}
3. \quad (a) \quad \int x \cos kx dx &= \frac{x \sin kx}{k} - \frac{1}{k} \int \sin kx dx && 1\text{M} \\
&= \frac{x \sin kx}{k} + \frac{\cos kx}{k^2} + \text{constant} && 1\text{A}
\end{aligned}$$

$$\begin{aligned}
(b) \quad \int_0^{\frac{\pi}{2}} x \cos x \cos 2x dx &= \frac{1}{2} \int_0^{\frac{\pi}{2}} x(\cos 3x + \cos x) dx && 1\text{M} \\
&= \frac{1}{2} \int_0^{\frac{\pi}{2}} x \cos 3x dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} x \cos x dx \\
&= \frac{1}{2} \left[\frac{x \sin 3x}{3} + \frac{\cos 3x}{9} \right]_0^{\frac{\pi}{2}} + \frac{1}{2} \left[x \sin x + \cos x \right]_0^{\frac{\pi}{2}} && 1\text{M} \\
&= \frac{\pi}{6} - \frac{5}{9} && 1\text{A}
\end{aligned}$$

$$\begin{aligned}
4. \quad \int_2^8 \ln \frac{x}{2} dx &= \left[x \ln \frac{x}{2} \right]_2^8 - \int_2^8 dx && 1\text{M}+1\text{A} \\
&= 16 \ln 2 - \left[x \right]_2^8 \\
&= 16 \ln 2 - 6 && 1\text{A}
\end{aligned}$$

$$\begin{aligned}
5. \quad \int_2^4 \left(x - \frac{1}{x^2} \right) \ln x dx &= \left[\left(\frac{x^2}{2} + \frac{1}{x} \right) \ln x \right]_2^4 - \int_2^4 \left(\frac{x}{2} + \frac{1}{x^2} \right) dx && 1\text{M}+1\text{A} \\
&= 14 \ln 2 - \left[\frac{x^2}{4} - \frac{1}{x} \right]_2^4 \\
&= 14 \ln 2 - \frac{13}{4} && 1\text{A}
\end{aligned}$$

6. (a) Let $u = \ln x$. Then $du = \frac{dx}{x}$. 1M

$$\int_1^e \frac{\ln x}{x} dx = \int_0^1 u du$$
1M

$$= \left[\frac{u^2}{2} \right]_0^1$$
1A

$$= \frac{1}{2}$$
1A

(b) $\int_1^e \frac{(1+x^2) \ln x}{x} dx = \int_1^e \frac{\ln x}{x} dx + \int_1^e x \ln x dx$

$$= \frac{1}{2} + \left[\frac{x^2}{2} \ln x \right]_1^e - \int_1^e \left(\frac{x^2}{2} \right) \left(\frac{1}{x} \right) dx$$
1M+1M

$$= \frac{1}{2} + \frac{e^2}{2} - 0 - \frac{1}{2} \int_1^e x dx$$

$$= \frac{1+e^2}{2} - \frac{1}{2} \left[\frac{x^2}{2} \right]_1^e$$

$$= \frac{1+e^2}{2} - \frac{e^2-1}{4}$$

$$= \frac{3+e^2}{4}$$

1A

7. (a) $\int_1^3 e^{2 \ln x} \ln x dx = \int_1^3 x^2 \ln x dx$

$$= \frac{1}{3} \left[x^3 \ln x \right]_1^3 - \frac{1}{3} \int_1^3 x^2 dx$$
1M

$$= 9 \ln 3 - \frac{1}{9} \left[x^3 \right]_1^3$$
1A

$$= 9 \ln 3 - \frac{26}{9}$$
1A

(b) Let $u = \frac{1}{x}$. Then $du = -\frac{1}{x^2} dx$. 1M

$$\int_{\frac{1}{3}}^1 \frac{e^{-2 \ln x} \ln x}{x^2} dx = - \int_3^1 e^{-2 \ln \frac{1}{u}} \ln \frac{1}{u} du$$

$$= - \int_1^3 e^{2 \ln u} \ln u du$$

$$= - \left(9 \ln 3 - \frac{26}{9} \right)$$

$$= \frac{26}{9} - 9 \ln 3$$

1A

$$8. \quad \int_1^{e^{\frac{\pi}{2}}} \sin(\ln x) \, dx = \left[x \sin(\ln x) \right]_1^{e^{\frac{\pi}{2}}} - \int_1^{e^{\frac{\pi}{2}}} \cos(\ln x) \, dx \quad 1M$$

$$= e^{\frac{\pi}{2}} - \left[x \cos(\ln x) \right]_1^{e^{\frac{\pi}{2}}} - \int_1^{e^{\frac{\pi}{2}}} \sin(\ln x) \, dx \quad 1M$$

$$= e^{\frac{\pi}{2}} + 1 - \int_1^{e^{\frac{\pi}{2}}} \sin(\ln x) \, dx$$

$$2 \int_1^{e^{\frac{\pi}{2}}} \sin(\ln x) \, dx = e^{\frac{\pi}{2}} + 1$$

$$\int_1^{e^{\frac{\pi}{2}}} \sin(\ln x) \, dx = \frac{e^{\frac{\pi}{2}}}{2} + \frac{1}{2} \quad 1A$$

$$9. \quad (a) \quad \text{Let } x = 3 \tan \theta. \text{ Then } dx = 3 \sec^2 \theta \, d\theta. \quad 1M$$

$$\int_0^{\sqrt{3}} \frac{1}{x^2 + 9} \, dx = \int_0^{\frac{\pi}{6}} \frac{3 \sec^2 \theta}{9 \tan^2 \theta + 9} \, d\theta \quad 1A$$

$$= \frac{1}{3} \int_0^{\frac{\pi}{6}} d\theta$$

$$= \frac{1}{3} \left[\theta \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{18} \quad 1A$$

$$(b) \quad \int_0^{\sqrt{3}} \ln(x^2 + 9) \, dx = \left[x \ln(x^2 + 9) \right]_0^{\sqrt{3}} - \int_0^{\sqrt{3}} \frac{2x^2}{x^2 + 9} \, dx \quad 1M$$

$$= \sqrt{3} \ln 12 - 2 \int_0^{\sqrt{3}} \left(1 - \frac{9}{x^2 + 9} \right) \, dx \quad 1M$$

$$= \sqrt{3} \ln 12 - 2 \int_0^{\sqrt{3}} dx + 18 \int_0^{\sqrt{3}} \frac{1}{x^2 + 9} \, dx$$

$$= \sqrt{3} \ln 12 - 2 \left[x \right]_0^{\sqrt{3}} + 18 \left(\frac{\pi}{18} \right) \quad 1M$$

$$= \sqrt{3} \ln 12 - 2\sqrt{3} + \pi \quad 1A$$

$$10. \quad \text{Let } u = x^2. \text{ Then } du = 2x \, dx. \quad 1M$$

$$\int_{\sqrt{\frac{\pi}{3}}}^{\sqrt{\pi}} 2x^3 \sin x^2 \, dx = \int_{\frac{\pi}{3}}^{\pi} u \sin u \, du$$

$$= \left[-u \cos u \right]_{\frac{\pi}{3}}^{\pi} + \int_{\frac{\pi}{3}}^{\pi} \cos u \, du \quad 1M$$

$$= \frac{7\pi}{6} + \left[\sin u \right]_{\frac{\pi}{3}}^{\pi}$$

$$= \frac{7\pi}{6} - \frac{\sqrt{3}}{2} \quad 1A$$

11. Let $u = x^3 - 1$. Then $du = 3x^2 dx$. 1M

$$\begin{aligned}\int_{-1}^1 (3x^5 - 3x^2)e^{x^3-1} dx &= \int_{-2}^0 ue^u du \\ &= \left[ue^u \right]_{-2}^0 - \int_{-2}^0 e^u du \\ &= 2e^{-2} - \left[e^u \right]_{-2}^0 \\ &= 3e^{-2} - 1\end{aligned}$$

1M
1A

12. $\int_0^{\frac{\pi}{6}} x^2 \cos 3x dx = \left[\frac{x^2 \sin 3x}{3} \right]_0^{\frac{\pi}{6}} - \frac{2}{3} \int_0^{\frac{\pi}{6}} x \sin 3x dx$ 1M+1A

$$\begin{aligned}&= \frac{\pi^2}{108} + \frac{2}{9} \left[x \cos 3x \right]_0^{\frac{\pi}{6}} - \frac{2}{9} \int_0^{\frac{\pi}{6}} \cos 3x dx \\ &= \frac{\pi^2}{108} + 0 - \frac{2}{27} \left[\sin 3x \right]_0^{\frac{\pi}{6}} \\ &= \frac{\pi^2}{108} - \frac{2}{27}\end{aligned}$$

1A

13. $\int_0^3 \left(\frac{x}{e^{2x}} \right)^2 dx = \int_0^3 x^2 e^{-4x} dx$

$$\begin{aligned}&= -\frac{1}{4} \left[x^2 e^{-4x} \right]_0^3 + \frac{1}{2} \int_0^3 x e^{-4x} dx \\ &= -\frac{9e^{-12}}{4} - \frac{1}{8} \left[x e^{-4x} \right]_0^3 + \frac{1}{8} \int_0^3 e^{-4x} dx \\ &= -\frac{9e^{-12}}{4} - \frac{3e^{-12}}{8} - \frac{1}{32} \left[e^{-4x} \right]_0^3 \\ &= -\frac{85}{32e^{12}} + \frac{1}{32}\end{aligned}$$

1M+1A
1A

14. $\int_0^{\frac{\pi}{4}} e^x \sin 2x dx = \left[e^x \sin 2x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} e^x (2 \cos 2x) dx$ 1M+1M

$$= (e^{\frac{\pi}{4}} - 0) - 2 \left[\left[e^x \cos 2x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} e^x (-2 \sin 2x) dx \right]$$

1M

$$5 \int_0^{\frac{\pi}{4}} e^x \sin 2x dx = e^{\frac{\pi}{4}} - 2(0 - 1)$$

1M

$$\int_0^{\frac{\pi}{4}} e^x \sin 2x dx = \frac{1}{5} (e^{\frac{\pi}{4}} + 2)$$

1A