

M2REG-AOI-2425-ASM-SET 1-MATH

建議題解

$$\begin{aligned}
 1. \quad \text{所求面積} &= \int_1^3 \frac{3}{x+1} dx && 1M \\
 &= \left[3 \ln |x+1| \right]_1^3 && 1M \\
 &= 3 \ln 2 && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (a) \quad \text{設 } u &= x^2 - 2x + 4 \circ \text{ 則 } du = 2(x-1) dx \circ && 1M \\
 \int \frac{x-1}{x^2-2x+4} dx &= \frac{1}{2} \int \frac{du}{u} \\
 &= \frac{1}{2} \ln |u| + \text{常數} \\
 &= \frac{1}{2} \ln |x^2 - 2x + 4| + \text{常數} && 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \text{所求面積} &= - \int_{-1}^1 \frac{x-1}{x^2-2x+4} dx + \int_1^5 \frac{x-1}{x^2-2x+4} dx && 1M \\
 &= - \left[\frac{1}{2} \ln |x^2 - 2x + 4| \right]_{-1}^1 + \left[\frac{1}{2} \ln |x^2 - 2x + 4| \right]_1^5 && 1M \\
 &= \frac{1}{2} \ln 19 + \frac{1}{2} \ln 7 - \ln 3 && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad 3 &= -x(x+4) \\
 0 &= -x^2 - 4x - 3 \\
 x &= -1 \quad \text{或} \quad -3 && 1A \\
 \text{所求面積} &= \int_{-3}^{-1} [-x(x+4) - 3] dx && 1M \\
 &= \int_{-3}^{-1} (-x^2 - 4x - 3) dx \\
 &= \left[-\frac{x^3}{3} - 2x^2 - 3x \right]_{-3}^{-1} && 1M \\
 &= \frac{4}{3} && 1A
 \end{aligned}$$

$$4. \quad (a) \quad \int x \sin x \, dx = -x \cos x + \int \cos x \, dx \quad 1M$$

$$= -x \cos x + \sin x + \text{常數} \quad 1A$$

$$(b) \quad x \sin x = x$$

$$\sin x = 1 \quad \text{或} \quad x = 0$$

$$x = \frac{\pi}{2} \quad \text{或} \quad 0 \quad 1A$$

$$\text{所求面積} = \int_0^{\frac{\pi}{2}} (x - x \sin x) \, dx \quad 1M$$

$$= \left[\frac{x^2}{2} + x \cos x - \sin x \right]_0^{\frac{\pi}{2}} \quad 1M$$

$$= \frac{\pi^2}{8} - 1 \quad 1A$$

$$5. \quad (a) \quad g(x) = (-x)^2 - 4(-x) - 5 \quad 1M$$

$$= x^2 + 4x - 5$$

$$x^2 + 4x - 5 = 0$$

$$x = -5 \quad \text{或} \quad 1$$

$$y = g(x) \text{ 的圖像的 } x \text{ 截距為 } -5 \text{ 及 } 1。 \quad 1A$$

$$(b) \quad \text{所求面積}$$

$$= \int_0^1 [(x^2 + 4x - 5) - (x^2 - 4x - 5)] \, dx - \int_1^5 (x^2 + 4x - 5) \, dx \quad 1M+1M$$

$$= \int_0^1 8x \, dx - \int_1^5 (x^2 + 4x - 5) \, dx$$

$$= \left[4x^2 \right]_0^1 - \left[\frac{x^3}{3} + 2x^2 - 5x \right]_1^5 \quad 1M$$

$$= \frac{220}{3} \quad 1A$$

$$6. \quad g(x) = f\left(x - \frac{\pi}{6}\right) \quad 1M$$

$$= \cos\left(2x - \frac{\pi}{3}\right)$$

$$0 = \cos 2x - \cos\left(2x - \frac{\pi}{3}\right)$$

$$0 = -2 \sin\left(2x - \frac{\pi}{6}\right) \sin \frac{\pi}{6} \quad 1M$$

$$2x - \frac{\pi}{6} = 0 \quad \text{或} \quad \pi$$

$$x = \frac{\pi}{12} \quad \text{或} \quad \frac{7\pi}{12} \quad 1A$$

所求面積

$$\begin{aligned} &= \int_0^{\frac{\pi}{12}} \left[\cos 2x - \cos\left(2x - \frac{\pi}{3}\right) \right] dx + \int_{\frac{\pi}{12}}^{\frac{7\pi}{12}} \left[\cos\left(2x - \frac{\pi}{3}\right) - \cos 2x \right] dx \\ &\quad + \int_{\frac{7\pi}{12}}^{\pi} \left[\cos 2x - \cos\left(2x - \frac{\pi}{3}\right) \right] dx \quad 1M+1M \end{aligned}$$

$$= \left[\frac{1}{2} \sin 2x - \frac{1}{2} \sin\left(2x - \frac{\pi}{3}\right) \right]_0^{\frac{\pi}{12}} + \left[\frac{1}{2} \sin\left(2x - \frac{\pi}{3}\right) - \frac{1}{2} \sin 2x \right]_{\frac{\pi}{12}}^{\frac{7\pi}{12}} + \left[\frac{1}{2} \sin 2x - \frac{1}{2} \sin\left(2x - \frac{\pi}{3}\right) \right]_{\frac{7\pi}{12}}^{\pi} \quad 1M$$

$$= 2 \quad 1A$$

$$7. \quad (a) \quad 1 - \cos x = 2 \sin^2 x$$

$$1 - \cos x - 2(1 - \cos^2 x) = 0 \quad 1M$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$\cos x = -\frac{1}{2} \quad \text{或} \quad 1$$

$$x = \frac{2\pi}{3} \quad \text{或} \quad \frac{4\pi}{3} \quad \text{或} \quad 0 \quad \text{或} \quad 2\pi$$

$$\text{因此，可得 } \alpha = \frac{2\pi}{3} \text{ 及 } \beta = \frac{4\pi}{3}。 \quad 1A+1A$$

(b) 所求面積

$$= \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (1 - \cos x - 2 \sin^2 x) dx \quad 1M$$

$$= \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} [1 - \cos x - (1 - \cos 2x)] dx \quad 1M$$

$$= \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} (\cos 2x - \cos x) dx$$

$$= \left[\frac{\sin 2x}{2} - \sin x \right]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \quad 1M$$

$$= \frac{3\sqrt{3}}{2} \quad 1A$$

8. (a) $\sin\left(2x - \frac{\pi}{3}\right) - \sin 2x = 0$
 $2\cos\left(2x - \frac{\pi}{6}\right)\sin\left(-\frac{\pi}{6}\right) = 0$ 1M
 $2x - \frac{\pi}{6} = \frac{\pi}{2} \quad \text{或} \quad \frac{3\pi}{2}$
 $x = \frac{\pi}{3} \quad \text{或} \quad \frac{5\pi}{6}$ 1A
 P 及 Q 的坐標分別為 $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$ 及 $\left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right)$ 。 1A+1A
- (b) 面積 $= \int_{\frac{\pi}{3}}^{\frac{5\pi}{6}} \left[\sin\left(2x - \frac{\pi}{3}\right) - \sin 2x\right] dx$ 1M
 $= \frac{1}{2} \left[-\cos\left(2x - \frac{\pi}{3}\right) + \cos 2x \right]_{\frac{\pi}{3}}^{\frac{5\pi}{6}}$ 1M
 $= 1$ 1A

9. $x^2 + (3x + 18) = 36$
 $x^2 + 3x - 18 = 0$
 $x = -6 \quad \text{或} \quad 3$ 1A

所求面積

$$= \int_{-6}^3 (\sqrt{36 - x^2} - \sqrt{3x + 18}) dx + \int_{-6}^3 [-\sqrt{3x + 18} - (-\sqrt{36 - x^2})] dx$$
 1M

$$= 2 \int_{-6}^3 (\sqrt{36 - x^2} - \sqrt{3x + 18}) dx$$

設 $x = 6 \sin \theta$ 。則 $dx = 6 \cos \theta d\theta$ 。 1M

所求面積

$$= 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \sqrt{36 - 36 \sin^2 \theta} \cdot 6 \cos \theta d\theta - \frac{4}{9} \left[(3x + 18)^{\frac{3}{2}} \right]_{-6}^3$$

$$= 72 \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} \cos^2 \theta d\theta - 36\sqrt{3}$$

$$= 36 \int_{-\frac{\pi}{2}}^{\frac{\pi}{6}} (1 + \cos 2\theta) d\theta - 36\sqrt{3}$$
 1M

$$= 36 \left[\theta + \frac{\sin 2\theta}{2} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{6}} - 36\sqrt{3}$$
 1M

$$= 24\pi - 27\sqrt{3}$$
 1A

10. (a) $x = (x^2)^2$ 1M

$$0 = x(x^3 - 1)$$

$$x = 0 \quad \text{或} \quad 1$$

P 的坐標為 $(1, 1)$ 。

1A

(b) 所求面積 $= \int_0^1 (\sqrt{x} - x^2) \, dx$ 1M

$$= \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{3} x^3 \right]_0^1$$

1M

$$= \frac{1}{3}$$

1A

11. (a) y 截距 $= \frac{12}{-3} - \frac{12}{3} - 1 = -9$ 1A

$$0 = \frac{12}{x-3} - \frac{12}{x+3} - 1$$

$$(x-3)(x+3) = 12[(x+3) - (x-3)]$$

$$x^2 - 81 = 0$$

$$x = \pm 9$$

x 截距為 9 及 -9。

1A

(b) (i) $f'(x) = -\frac{12}{(x-3)^2} + \frac{12}{(x+3)^2}$ 1A

$$f''(x) = \frac{24}{(x-3)^3} - \frac{24}{(x+3)^3}$$

1A

(ii) 當 $f'(x) = 0$,

$$\frac{12}{(x-3)^2} = \frac{12}{(x+3)^2}$$

1M

$$x-3 = -(x+3) \quad \text{或} \quad x-3 = x+3 \quad (\text{捨去})$$

$$x = 0$$

x	$x < -3$	$-3 < x < 0$	$0 < x < 3$	$x > 3$
$f'(x)$	+	+	-	-

1M

極大點為 (0, -9)，沒有極小點。

1A

當 $f''(x) = 0$,

$$\frac{24}{(x-3)^3} = \frac{24}{(x+3)^3}$$

$$x+3 = x-3$$

無解。

因此，沒有拐點。

1A

(c) $x = -3$ 及 $x = 3$ 均為垂直漸近線。

1A

由於 $y = -1 + \frac{12}{x-3} - \frac{12}{x+3}$, $y = -1$ 為水平漸近線。

1A

(d) $A = \int_9^k \left(\frac{12}{x-3} - \frac{12}{x+3} - 1 - (-1) \right) dx$ 1M

$$= \left[12 \ln |x-3| - 12 \ln |x+3| \right]_9^k$$

1M

$$= 12 \ln \frac{k-3}{6} - 12 \ln \frac{k+3}{12}$$

$$= 12 \ln \frac{k-3}{k+3} + 12 \ln 2$$

由於 $k > 9$, $0 < \frac{k-3}{k+3} < 1$ 及 $\ln \frac{k-3}{k+3} < 0$ 。

可得 $A < 12 \ln 2$ 。

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