1. $\lim\limits_{h\to 0}\dfrac{1}{h}\left[\dfrac{2x+3+2h}{x+4+h}-\dfrac{2x+3}{x+4}\right]$

$=\lim\limits_{h\to 0}\dfrac{1}{h}\left[\dfrac{(2x+3+2h)(x+4)-(2x+3)(x+4+h)}{(x+4+h)(x+4)}\right]$ 1M

$=\lim\limits_{h\to 0}\dfrac{5h}{h(x+4+h)(x+4)}$

$=\lim\limits_{h\to 0}\dfrac{5}{(x+4+h)(x+4)}$ 2M

$=\dfrac{5}{(x+4)^2}$ 2A

2. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^5+4\right)=\lim\limits_{h\to 0}\dfrac{[(x+h)^5+4]-(x^5+4)}{h}$ 1M

$=\lim\limits_{h\to 0}\dfrac{x^5+5hx^4+10h^2x^3+10h^3x^2+5h^4x+h^5+4-x^5-4}{h}$ 1M

$=\lim\limits_{h\to 0}(5x^4+10hx^3+10h^2x^2+5h^3x+h^4)$ 1M

$=5x^4$ 1A

3. $f(1+h)-f(1)=\dfrac{10(1+h)}{7+3(1+h)^2}-\dfrac{10}{7+3}$

$=\dfrac{(10+10h)-(10+6h+3h^2)}{10+6h+3h^2}$

$=\dfrac{4h-3h^2}{10+6h+3h^2}$ 1

$f'(1)=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$ 1M

$=\lim\limits_{h\to 0}\dfrac{4h-3h^2}{h(10+6h+3h^2)}$

$=\lim\limits_{h\to 0}\dfrac{4-3h}{10+6h+3h^2}$ 1M

$=\dfrac{2}{5}$ 1A

4. $f'(x) = \lim\limits_{h\to 0} \dfrac{\frac{1}{3(x+h)^2+4} - \frac{1}{3x^2+4}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{3[x^2 - (x+h)^2] + (4-4)}{h(3x^2+4)[3(x+h)^2+4]}$ 1M

$= \lim\limits_{h\to 0} \dfrac{-6x + 3h}{(3x^2+4)[3(x+h)^2+4]}$ 1M

$= \dfrac{-6x}{(3x^2+4)^2}$ 1A

5. $\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{x+h}} = \dfrac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x(x+h)}}$

$= \dfrac{\sqrt{x+h} - \sqrt{x}}{\sqrt{x(x+h)}} \times \dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ 1M

$= \dfrac{h}{(x+h)\sqrt{x} + x\sqrt{x+h}}$ 1

$\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{\dfrac{3}{x}} = \lim\limits_{h\to 0} \dfrac{1}{h}\left(\dfrac{\sqrt{3}}{\sqrt{x+h}} - \dfrac{\sqrt{3}}{\sqrt{x}}\right)$ 1M

$= -\sqrt{3} \lim\limits_{h\to 0} \dfrac{1}{h}\left(\dfrac{h}{(x+h)\sqrt{x} + x\sqrt{x+h}}\right)$

$= -\sqrt{3} \lim\limits_{h\to 0} \dfrac{1}{(x+h)\sqrt{x} + x\sqrt{x+h}}$ 1M

$= -\dfrac{\sqrt{3}}{2x\sqrt{x}}$ 1A

6. $f'(2) = \lim\limits_{h\to 0} \dfrac{\frac{2+h}{\sqrt{4+h}} - \frac{2}{\sqrt{4}}}{h}$ 1M

$= \lim\limits_{h\to 0} \left[\dfrac{2+h-\sqrt{4+h}}{h\sqrt{4+h}} \times \dfrac{2+h+\sqrt{4+h}}{2+h+\sqrt{4+h}}\right]$ 1M

$= \lim\limits_{h\to 0} \dfrac{h^2+3h}{h\sqrt{4+h}(2+h+\sqrt{4+h})}$

$= \lim\limits_{h\to 0} \dfrac{h+3}{\sqrt{4+h}(2+h+\sqrt{4+h})}$ 1M

$= \dfrac{3}{8}$ 1A

7. $g(1+h) - g(1) = \dfrac{1}{\sqrt{5(1+h)+4}} - \dfrac{1}{\sqrt{5+4}}$

$= \dfrac{3-\sqrt{5h+9}}{3\sqrt{5h+9}} \times \dfrac{3+\sqrt{5h+9}}{3+\sqrt{5h+9}}$ 1M

$= \dfrac{-5h}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$ 1

$g'(1) = \lim\limits_{h\to 0} \dfrac{g(1+h)-g(1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{-5}{3\sqrt{5h+9}(3+\sqrt{5h+9})}$ 1M

$= \dfrac{-5}{3\sqrt{9}(3+\sqrt{9})}$

$= -\dfrac{5}{54}$ 1A

8. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin 2x) = \lim\limits_{h\to 0}\dfrac{\sin 2(x+h)-\sin 2x}{h}$ 1M

$= \lim\limits_{h\to 0}\left(\dfrac{2}{h}\cos\dfrac{2x+2h+2x}{2}\sin\dfrac{2x+2h-2x}{2}\right)$ 1M

$= 2\lim\limits_{h\to 0}\left(\cos(2x+h)\dfrac{\sin h}{h}\right)$ 1M

$= 2\cos 2x$ 1A

9. $\dfrac{\mathrm{d}}{\mathrm{d}\theta}\sec 6\theta = \lim\limits_{h\to 0}\dfrac{\sec 6(\theta+h)-\sec 6\theta}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{\cos 6\theta-\cos(6\theta+6h)}{h\cos 6\theta\cos(6\theta+6h)}$

$= \lim\limits_{h\to 0}\dfrac{-2\sin(6\theta+3h)\sin(-3h)}{h\cos 6\theta\cos(6\theta+6h)}$ 1M

$= 6\lim\limits_{h\to 0}\dfrac{\sin(-3h)}{-3h}\cdot\dfrac{\sin(6\theta+3h)}{\cos 6\theta\cos(6\theta+6h)}$ 1M

$= 6\times 1\times\dfrac{\sin 6\theta}{\cos^2 6\theta}$ 1M

$= 6\sec 6\theta\tan 6\theta$ 1A

10. (a) $f\left(\dfrac{\pi}{2}+h\right)-f\left(\dfrac{\pi}{2}\right) = -\left(\dfrac{\pi}{2}+h\right)\sin\left(\dfrac{\pi}{2}+h\right)+\dfrac{\pi}{2}\sin\dfrac{\pi}{2}$ 1M

$= -\left(\dfrac{\pi}{2}+h\right)\cos h+\dfrac{\pi}{2}$

$= -\dfrac{\pi}{2}\left[1-2\sin^2\left(\dfrac{h}{2}\right)\right]-h\cos h+\dfrac{\pi}{2}$

$= \pi\sin^2\left(\dfrac{h}{2}\right)-h\cos h$ 1

(b) $f'\left(\dfrac{\pi}{2}\right) = \lim\limits_{h\to 0}\dfrac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h}$ 1M

$= \lim\limits_{h\to 0}\dfrac{\pi\sin^2\left(\frac{h}{2}\right)-h\cos h}{h}$

$= \lim\limits_{h\to 0}\left[\dfrac{\sin\frac{h}{2}}{\left(\frac{h}{2}\right)}\cdot\dfrac{\pi\sin\frac{h}{2}}{2}-\cos h\right]$ 1M

$= 1\cdot 0-1$

$= -1$ 1A

11. $f'(0) = \lim\limits_{h\to 0} \dfrac{e^{2(0+h)} - e^{2(0)}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{e^{2h} - 1}{2h} \cdot 2$ 1M

$= 2$ 1A

12. $f(1+h) = [(1+h)^2 - 1]e^{1+h}$

$= (h^2 + 2h)e^{1+h}$ 1A

$f'(1) = \lim\limits_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(h^2 + 2h)e^{1+h} - 0}{h}$

$= \lim\limits_{h\to 0} (h+2)e^{1+h}$ 1M

$= 2e$ 1A

13. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^3\right) = \lim\limits_{h\to 0} \dfrac{(x+h)^3 - x^3}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(x^3 + 3hx^2 + 3h^2x + h^3) - x^3}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{3x^2 + 3hx + h^2}{1}$ 1M

$= 3x^2$ 1A

14. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2\right) = \lim\limits_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(x^2 + 2xh + h^2) - x^2}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{2x + h}{1}$ 1M

$= 2x$ 1A

15. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^3\right) = \lim\limits_{h\to 0} \dfrac{(x+h)^3 - x^3}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(x^3 + 3hx^2 + 3h^2x + h^3) - x^3}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{3x^2 + 3hx + h^2}{1}$ 1M

$= 3x^2$ 1A

16. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 1\right) = \lim\limits_{h\to 0} \dfrac{[(x+h)^2 + 1] - (x^2 + 1)}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(x^2 + 2hx + h^2) - (x^2)}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{2x + h}{1}$ 1M

$= 2x$ 1A

17. $f'(x) = \lim\limits_{h\to 0} \dfrac{\frac{1}{1+(x+h)} - \frac{1}{1+x}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{-h}{h(1+x+h)(1+x)}$ 2M

$= \lim\limits_{h\to 0} \dfrac{-1}{(1+x+h)(1+x)}$ 1M

$= -\dfrac{1}{(1+x)^2}$ 1A

18. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{x}\right) = \lim\limits_{h\to 0} \dfrac{\frac{1}{x+h} - \frac{1}{x}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{x-(x+h)}{hx(x+h)}$ 1M

$= \lim\limits_{h\to 0} \dfrac{-1}{x(x+h)}$ 1M

$= -\dfrac{1}{x^2}$ 1A

19. (a) $(\sqrt{x+1+h} - \sqrt{x+1})(\sqrt{x+1+h} + \sqrt{x+1}) = (x+1+h) - (x+1)$

$= h$ 1A

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lim\limits_{h\to 0} \dfrac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{\sqrt{x+1+h} - \sqrt{x+1}}{h} \times \dfrac{\sqrt{x+1+h} + \sqrt{x+1}}{\sqrt{x+1+h} + \sqrt{x+1}}$ 1M

$= \lim\limits_{h\to 0} \dfrac{h}{h(\sqrt{x+1+h} + \sqrt{x+1})}$

$= \lim\limits_{h\to 0} \dfrac{1}{\sqrt{x+1+h} + \sqrt{x+1}}$ 1M

$= \dfrac{1}{2\sqrt{x+1}}$ 1A

20. (a) $(\sqrt{2(x+h)} - \sqrt{2x})(\sqrt{2(x+h)} + \sqrt{2x}) = [2(x+h)] - (2x)$

$= 2h$ 1A

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{2x}\right) = \lim\limits_{h\to 0} \dfrac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \dfrac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$ 1M

$= \lim\limits_{h\to 0} \dfrac{2h}{h(\sqrt{2(x+h)} + \sqrt{2x})}$

$= \lim\limits_{h\to 0} \dfrac{2}{\sqrt{2(x+h)} + \sqrt{2x}}$ 1M

$= \dfrac{1}{\sqrt{2x}}$ 1A

21. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right) = \lim\limits_{h\to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$ 1M

$= \lim\limits_{h\to 0} \dfrac{\sqrt{x+h} - \sqrt{x}}{h} \times \dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ 1M

$= \lim\limits_{h\to 0} \dfrac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})}$

$= \lim\limits_{h\to 0} \dfrac{1}{\sqrt{x+h} + \sqrt{x}}$ 1M

$= \dfrac{1}{2\sqrt{x}}$ 1A

22. (a) $\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}} = \dfrac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}$

$= \dfrac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \times \dfrac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$ 1M

$= \dfrac{(x) - (x+h)}{\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$

$= \dfrac{-h}{\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$ 1

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{\sqrt{x}}\right) = \lim\limits_{h \to 0} \dfrac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$ 1M

$= \lim\limits_{h \to 0} \left[\dfrac{1}{h} \cdot \dfrac{-h}{\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}\right]$

$= -\lim\limits_{h \to 0} \dfrac{-1}{\sqrt{x}(\sqrt{x+h})(\sqrt{x} + \sqrt{x+h})}$ 1M

$= -\dfrac{1}{2x\sqrt{x}}$ 1A

23. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x+1}\right) = \lim\limits_{h \to 0} \dfrac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$ 1M

$= \lim\limits_{h \to 0} \dfrac{[(x+h)+1] - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})}$ 1M

$= \lim\limits_{h \to 0} \dfrac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$ 1M

$= \dfrac{1}{2\sqrt{x+1}}$ 1A

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