1. $f'(x) = \dfrac{1}{2}(3+x^2)^{-\frac{1}{2}}(2x)$ 1M+1A

$= \dfrac{x}{\sqrt{3+x^2}}$

$f'(-1) = -\dfrac{1}{\sqrt{3+(-1)^2}}$

$= -\dfrac{1}{2}$ 1A

2. $f(x) = x\sqrt{1-x^2}$

$f'(x) = \sqrt{1-x^2} + x \cdot \dfrac{1}{2\sqrt{1-x^2}}(-2x)$ 1+1+1A

$= \dfrac{1-2x^2}{\sqrt{1-x^2}}$

$f'\left(\dfrac{1}{2}\right) = \dfrac{1-\frac{2}{4}}{\sqrt{1-\frac{1}{4}}}$ 1M

$= \dfrac{1}{\sqrt{3}}$ 1A

3. $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x^2}{2x+1}\right) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left[x^2(2x+1)^{-1}\right]$

$= 2x(2x+1)^{-1} + x^2(-1)(2x+1)^{-2}(2)$ 1M+1A

$= (2x+1)^{-2}[2x(2x+1) - 2x^2]$

$= \dfrac{2x(x+1)}{(2x+1)^2}$ 1A

4. $y = \tan\dfrac{x-1}{x+1}$

$\dfrac{dy}{dx} = \sec^2\dfrac{x-1}{x+1}\cdot\dfrac{(x+1)-(x-1)}{(x+1)^2}$ 2M+2A

$= \dfrac{2}{(x+1)^2}\cdot\sec^2\dfrac{x-1}{x+1}$ 1A

5. (a) $\dfrac{d}{dx}\sin\left(x^2+1\right) = \cos\left(x^2+1\right)(2x)$ 1M

$= 2x\cos\left(x^2+1\right)$ 1A

(b) $\dfrac{d}{dx}\dfrac{\sin\left(x^2+1\right)}{x} = \dfrac{x[2x\cos\left(x^2+1\right)] - \sin\left(x^2+1\right)}{x^2}$ 1M

$= \dfrac{2x^2\cos\left(x^2+1\right) - \sin\left(x^2+1\right)}{x^2}$ 1A

6. (a) $\dfrac{d}{dx}\sin^2 x = 2\sin x\cos x$ 1M+1A

(b) $\dfrac{d}{dx}[\sin^2(3x+1)] = 2\sin(3x+1)\cos(3x+1)(3)$ 1M

$= 6\sin(3x+1)\cos(3x+1)$ 1A

7. $\dfrac{d}{dx}\left[\dfrac{\sin(2x+1)}{x}\right] = \dfrac{x\cos(2x+1)(2) - \sin(2x+1)}{x^2}$ 1M+1M

$= \dfrac{2x\cos(2x+1) - \sin(2x+1)}{x^2}$ 1A

8. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos\left(x^3+1\right) = -3x^2\sin\left(x^3+1\right)$ 1M+1A

(b) $\dfrac{\mathrm{d}}{\mathrm{d}x}[x\cos\left(x^3+1\right)] = -3x^3\sin\left(x^3+1\right) + \cos\left(x^3+1\right)$ 1M+1A

9. $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{(3+2\cos kt)k\cos kt - \sin kt(-2k\sin kt)}{(3+2\cos kt)^2}$ 1M+1M+1A

$= \dfrac{3k\cos kt + 2k\cos^2 kt + 2k\sin^2 kt}{(3+2\cos kt)^2}$

$= \dfrac{3k\cos kt + 2k}{(3+2\cos kt)^2}$

當 $t = \dfrac{3\pi}{2k}$ 時，

$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{3k\cos\frac{3\pi}{2} + 2k}{\left(3+2\cos\frac{3\pi}{2}\right)^2}$ 1M

$= \dfrac{2k}{9}$ 2A

10. $f'(x) = \sqrt{x^2+k}\cos 2x + \sin 2x \cdot \dfrac{1}{2}(x^2+k)^{-\frac{1}{2}}(2x)$ 1M+1A

$= 2\sqrt{x^2+k}\cos 2x + \dfrac{x\sin 2x}{\sqrt{x^2+k}}$ 1A

由於 $f'(0) = 1$，

$2\sqrt{k} = 1$ 1M

$k = \dfrac{1}{4}$ 1A

11. $f(x) = (\sin 3x)^{-2}$ 1M

$f'(x) = -2(\sin 3x)^{-3}(3\cos 3x)$ 1M

$= \dfrac{-6\cos 3x}{\sin^3 3x}$ 1A

$f'\left(\dfrac{\pi}{12}\right) = \dfrac{-6\cos\frac{\pi}{4}}{\sin^3\frac{\pi}{4}}$ 1M

$= -12$ 1A

12. (a) $y = \tan(\theta - x)$

$= \dfrac{\tan\theta - \tan x}{1 + \tan\theta\tan x}$ 1A

$= \dfrac{\tan x}{1 + 2\tan^2 x}$ 1A

(b) $\dfrac{dy}{dx} = \dfrac{(1 + 2\tan^2 x)\sec^2 x - \tan x \cdot 4\tan x\sec^2 x}{(1 + 2\tan^2 x)^2}$ 1M

$= \dfrac{\sec^2 x - 2\tan^2 x\sec^2 x}{(1 + 2\tan^2 x)^2}$

當 $\dfrac{dy}{dx} = 0$ 時，

$\sec^2 x - 2\tan^2 x\sec^2 x = 0$

$\sec^2 x(1 - 2\tan^2 x) = 0$

$\tan^2 x = \dfrac{1}{2}$ 1A

$x \approx 0.615$ 1A

13. $x \sin y = 2002$

$\sin y + x \cos y \dfrac{dy}{dx} = 0$ 1M+1A+1A

$\dfrac{dy}{dx} = -\dfrac{\tan y}{x}$ 1A

14. (a) $\dfrac{d}{dx} \sin^3(x^2 + 1) = 3 \sin^2(x^2 + 1) \cos\left(x^2 + 1\right)(2x)$ 1M+1M

$= 6x \sin^2(x^2 + 1) \cos\left(x^2 + 1\right)$ 1A

(b) $xy + y^2 = 2005$

$x \dfrac{dy}{dx} + y + 2y \dfrac{dy}{dx} = 0$ 1A+1A

$\dfrac{dy}{dx} = -\dfrac{y}{x + 2y}$ 1A

15. $y^2 + \sqrt[3]{x}y - 3 = 0$

$2y \dfrac{dy}{dx} + \dfrac{1}{3} x^{-\frac{2}{3}} y + \sqrt[3]{x} \dfrac{dy}{dx} = 0$ 1A+1A

在 $P(8,\ 1)$，

$2(1) \dfrac{dy}{dx} + \dfrac{1}{3}(8)^{-\frac{2}{3}}(1) + 8^{\frac{1}{3}} \dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = -\dfrac{1}{48}$ 1A

16. $3x^2 + 3y^2 - 2xy = 12$

$6x + 6y\dfrac{dy}{dx} - 2y - 2x\dfrac{dy}{dx} = 0$ 1A+1A+1A

$\dfrac{dy}{dx} = \dfrac{y - 3x}{3y - x}$

代 $x = 2$ 及 $y = 0$，

$\dfrac{dy}{dx} = \dfrac{-6}{-2}$

$= 3$ 1A

17. $x + x^2 + x^3 + \ldots + x^{n-1} + x^n = \dfrac{x(x^n - 1)}{x - 1}$

$1 + 2x + 3x^2 + \ldots + (n-1)x^{n-2} + nx^{n-1} = \dfrac{d}{dx}\dfrac{x(x^n - 1)}{x - 1}$ 1A

$= \dfrac{(x-1)[(n+1)x^n - 1] - (x^{n+1} - x)}{(x-1)^2}$ 1M+1A

$= \dfrac{nx^{n+1} - (n+1)x^n + 1}{(x-1)^2}$ 1A

代 $n = 10$ 及 $x = 2$，

$1 + 2(2) + 3(2)^2 + \ldots + 9(2)^8 + 10(2)^9 = \dfrac{10 \times 2^{11} - 11 \times 2^{10} + 1}{(2-1)^2}$ 1M

$= 9217$ 1A

18. $x = 2y + \sin y$

$1 = (2 + \cos y)\dfrac{dy}{dx}$ 1M

$\dfrac{dy}{dx} = \dfrac{1}{2 + \cos y}$

$\dfrac{d^2y}{dx^2} = -(2 + \cos y)^{-2}(-\sin y)\dfrac{dy}{dx}$ 1M

$= \dfrac{\sin y}{(2 + \cos y)^3}$ 1A

19. (a) $\dfrac{dy}{dx} = (x\cos x + \sin x) - \sin x$ 1M

$= x\cos x$ 1A

$\dfrac{d^2y}{dx^2} = -x\sin x + \cos x$ 1A

(b) $x\dfrac{d^2y}{dx^2} + k\dfrac{dy}{dx} + xy = 0$

$x(-x\sin x + \cos x) + kx\cos x + x(x\sin x + \cos x) = 0$ 1M

$(2 + k)x\cos x = 0$

由於對所有實數 x，方程為真，故此 $k = -2$。 1A

20. $\dfrac{dy}{dx} = -\sin(\sin x)\cos x$ 1M+1A

$\dfrac{d^2y}{dx^2} = -\cos x \cdot \cos(\sin x)\cos x - \sin(\sin x)(-\sin x)$ 1M+1A

$= -\cos(\sin x)\cos^2 x + \sin(\sin x)\sin x$ 1A+1A

21. $x = y + \sin y$

$1 = \dfrac{dy}{dx} + \cos y \dfrac{dy}{dx}$ 1M+1A

$\dfrac{dy}{dx} = \dfrac{1}{1 + \cos y}$ 1A

$\dfrac{d^2y}{dx^2} = -(1 + \cos y)^{-2}(-\sin y)\dfrac{dy}{dx}$ 1A

$= \dfrac{\sin y}{(1 + \cos y)^3}$ 1A

22. $y = x \sin 5x$

$\dfrac{dy}{dx} = \sin 5x + x \cos 5x(5)$

$= \sin 5x + 5x \cos 5x$ 2A

$\dfrac{d^2y}{dx^2} = 5 \cos 5x + 5 \cos 5x - 25x \sin 5x$

$= 10 \cos 5x - 25x \sin 5x$ 1A

因此，$\dfrac{d^2y}{dx^2} + 25y = 10 \cos 5x$。 1A

23. $f'(x) = 3 \sin^2 x \cos x$ 1A

$f''(x) = 6 \sin x \cos^2 x - 3 \sin^3 x$ 1M+1A

24. $\dfrac{dy}{dx} = -\dfrac{1}{x^2}\sec^2\dfrac{1}{x}$ 1M+1A

$x^2\dfrac{dy}{dx} + (y^2 + 1) = -\sec^2\dfrac{1}{x} + \tan^2\dfrac{1}{x} + 1$ 1M

$= 0$ 1

對等式兩邊微分，

$2x\dfrac{dy}{dx} + x^2\dfrac{d^2y}{dx^2} + 2y\dfrac{dy}{dx} = 0$ 1A

$x^2\dfrac{d^2y}{dx^2} + 2(x + y)\dfrac{dy}{dx} = 0$

$\dfrac{d^2y}{dx^2} + \dfrac{2(x + y)}{x^2}\dfrac{dy}{dx} = 0$ 1

– 試卷完 –