REG-LOCUS-2425-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) Slope of $L_1 = \dfrac{2-0}{0-8} = -\dfrac{1}{4}$

Slope of $L_2 = \dfrac{4-0}{0-16} = -\dfrac{1}{4}$

We have $L_1 // L_2$.

Thus, Γ is parallel to L_1. 1A

(b) (i) The coordinates of B are $(0,\ 3)$.

The equation of Γ is

$$\frac{y-3}{x-0} = -\frac{1}{4} \quad \text{1M}$$

$$x + 4y - 12 = 0 \quad \text{1A}$$

(ii) The coordinates of A are $(12,\ 0)$.

Note that $\angle AOB = 90°$.

AB is a diameter of C.

Radius of C

$= \dfrac{1}{2}\sqrt{(12-0)^2 + (0-3)^2}$ 1M

$= \dfrac{1}{2}\sqrt{153}$

Area of C

$= \pi\left(\dfrac{1}{2}\sqrt{153}\right)^2$ 1M

$= \dfrac{153\pi}{4}$ 1A

2. (a) The coordinates of D are $(-5,\ 4)$.

Let $(h,\ k)$ be the coordinates of E.

Denote the locus of P by Γ.

Note that $\Gamma \perp DE$.

$$\frac{-4}{3} \times \frac{k-4}{h+5} = -1 \quad \text{1M}$$

$$3h - 4k + 31 = 0$$

The mid-point of DE lies on Γ.

$$4\left(\frac{-5+h}{2}\right) + 3\left(\frac{4+k}{2}\right) - 17 = 0 \quad \text{1M}$$

$$4h + 3k - 42 = 0$$

Solving, we have $h = 3$ and $k = 10$. 1A

The coordinates of E are $(3,\ 10)$.

(b) Radius of C

$= \sqrt{5^2 + 4^2 - 25}$ 1M

$= 4$

The coordinates of the mid-point of DE are $(-1,\ 7)$.

Distance from mid-point of DE to the centre of C

$= \sqrt{(-1 + 5)^2 + (7 - 4)^2}$

$= 5 > 4$

The minimum distance from P to D is 5, which is greater than the radius of C. 1M

P must lie outside C.

The claim is agreed. 1A

3. (a) The coordinates of G are $(1,\ 6)$.

$AG = \sqrt{(1 - 19)^2 + (6 - 30)^2}$ 1M

$= 30$ 1A

(b) (i) Γ is the perpendicular bisector of AG. 1A

(ii) Radius of $C = \sqrt{1^2 + 6^2 + 252} = 17$ 1M

Note that $AM = AN = GM = GN = 17$.

$MN = 2\sqrt{17^2 - 15^2}$ 1M

$= 16$

Perimeter of $\triangle AMN$

$= 17 + 17 + 16$ 1M

$= 50$ 1A

4. (a) $(x + 1)^2 + (y - 2)^2 = (5 + 1)^2 + (10 - 2)^2$ 1M

$(x + 1)^2 + (y - 2)^2 = 100$ 1A

(b) (i) Note that $(-11 + 1)^2 + (2 - 2)^2 = 100$.

Γ passes through H. 1A

(ii) $AH = \sqrt{(-11 - 5)^2 + (2 - 10)^2} = \sqrt{320} = 8\sqrt{5}$ 1A

(iii) Let M be the mid-point of AH.

The area of $\triangle AHK$ attains its maximum when $MK \perp AH$ and $\angle HAK < 90°$.

$BM = \sqrt{10^2 - \left(\frac{\sqrt{320}}{2}\right)^2} = \sqrt{20}$ 1M

Required area

$= \frac{(\sqrt{320})(\sqrt{20} + 10)}{2}$ 1M

≈ 129

< 130

The claim is agreed. 1A

5. (a) $(x-7)^2+(y+4)^2=(14-7)^2+(-28+4)^2$ 1M

$(x-7)^2+(y+4)^2=625$ 1A

(b) (i) Γ is the perpendicular bisector of GH. 1A

(ii) Let M be the mid-point of GH.

The coordinates of M are $(-2,\ 8)$.

$GM=\sqrt{(7+2)^2+(-4-8)^2}=15<25$ 1M

M lies inside C_1.

Thus, Γ intersects C_1 at two distinct points. 1

(c) Let r_2 be the radius of C_2.

$$GH=25+r_2$$
$$2(15)=25+r_2$$
$$r_2=5$$

We have $GQ=GH+r_2=35$. 1M

$AB=2\sqrt{25^2-15^2}=40$ 1M

Note that $AGBQ$ is a kite.

Required area

$=\dfrac{(40)(35)}{2}$

$=700$ 1A

6. (a) Perpendicular distance from P to $AB=\dfrac{20\times 2}{5}=8$ 1M

Locus of P is a pair of straight lines parallel to AB, maintaining a distance of 8 to AB.

Let θ be the inclination of AB.

$\tan\theta=\dfrac{6-3}{4-0}=\dfrac{3}{4}$. So, $\cos\theta=\dfrac{4}{\sqrt{3^2+4^2}}=\dfrac{4}{5}$.

Let c be the y-intercept of Γ.

$$\frac{4}{5}=\frac{8}{\pm(c-3)}$$ 1M

$$c=-7 \quad \text{or} \quad 13$$

The equation of Γ is $y=\dfrac{3}{4}x-7$ or $y=\dfrac{3}{4}x+13$. 1A

(b) Perpendicular distance from P to $AB=8>5$.

It is not possible to have $AP=AB$ or $BP=AB$.

When $AP=BP$, P lies on perpendicular bisector of AB.

Equation of perpendicular bisector of AB is

$$y-\frac{9}{2}=-\frac{4}{3}(x-2)$$ 1M

$$y=-\frac{4}{3}x+\frac{43}{6}$$

Solve $\begin{cases} y=-\dfrac{4}{3}x+\dfrac{43}{6} \\ y=\dfrac{3}{4}x-7 \end{cases}$ or $\begin{cases} y=-\dfrac{4}{3}x+\dfrac{43}{6} \\ y=\dfrac{3}{4}x+13 \end{cases}$, the coordinates of P are $\left(\frac{34}{5},\ -\frac{19}{10}\right)$ or $\left(-\frac{14}{5},\ \frac{109}{10}\right)$. 1A+1A

7. (a) The coordinates of G are $(4,\ 10)$. 1M

Required equation is

$$(x-4)^2+(y-10)^2=(14-4)^2+(20-10)^2 \quad \text{1M}$$

$$(x-4)^2+(y-10)^2=200 \quad \text{1A}$$

(b) The equation of L_1 is

$$\frac{y-0}{x+6}=\frac{20-0}{14+6} \quad \text{1M}$$

$$y=x+6$$

The coordinates of the three vertices of the bounded region are $(0,\ 6)$, $(0,\ k)$ and $(k-6,\ k)$. 1M

$$\frac{(k-6)(k-6-0)}{2}=200$$

$$k-6=\pm 20$$

$$k=26 \quad \text{or} \quad -14 \text{ (rejected)} \quad \text{1A}$$

(c)

$$\sqrt{(x-4)^2+(y-10)^2}=\sqrt{(y-26)^2} \quad \text{1M+1M}$$

$$x^2+y^2-8x-20y+116=y^2-52y+676 \quad \text{1M}$$

$$x^2-8x+32y-560=0 \quad \text{1A}$$

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