REG-EXP-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. C	2. B	3. A	4. D	5. C
6. A	7. C	8. A	9. C	10. A
11. A	12. A	13. B	14. C	15. A
16. B	17. C	18. C	19. C	20. D
21. A	22. C	23. A	24. A	25. C
26. C	27. D	28. B	29. A	30. C

1. C

$$\begin{aligned} 27^x &= (3^x)^3 \\ &= a^3 \end{aligned}$$

2. B

$$\begin{aligned} a^2 + \frac{1}{a} &= b + \frac{1}{\sqrt{b}} \\ &= b + \frac{\sqrt{b}}{b} \\ &= \frac{b^2 + \sqrt{b}}{b} \end{aligned}$$

3. A

$$\begin{aligned} x^{-\frac{3}{2}} &= \frac{1}{8} \\ x &= \left(\frac{1}{8}\right)^{-\frac{2}{3}} \\ &= 4 \end{aligned}$$

4. D

$$\begin{aligned} 5^{3x+4} \cdot 5^{\frac{1}{2}} &= 125 \\ 5^{3x+4+\frac{1}{2}} &= 5^3 \\ 3x + \frac{9}{2} &= 3 \\ x &= -\frac{1}{2} \end{aligned}$$

5. C

$$(2^x)(16^y) = (\sqrt{8})^z$$
$$(2^x)(2^{4y}) = 2^{\frac{3z}{2}}$$
$$x + 4y = \frac{3z}{2}$$
$$z = \frac{2}{3}(x + 4y)$$

6. A

$$\frac{64^x \cdot 4^{-2y}}{2^{3x}} = 1$$
$$2^{6x} \cdot 2^{-4y} = 2^{3x}$$
$$6x - 4y = 3x$$
$$\frac{x}{y} = \frac{4}{3}$$

7. C

$$16^x = \frac{1}{\sqrt{8}}$$
$$2^{4x} = 2^{-\frac{3}{2}}$$
$$4x = -\frac{3}{2}$$
$$x = -\frac{3}{8}$$

8. A

$$(2^x)(4^x)(8^x) = 16$$
$$2^{x+2x+3x} = 2^4$$
$$6x = 4$$
$$x = \frac{2}{3}$$

9. C

$$3^{x+2y} = 1 \quad \text{and} \quad 4^x \cdot 8^y = 2$$
$$x + 2y = 0 \qquad 2^{2x+3y} = 2^1$$
$$2x + 3y = 1$$

Solving, we have $x = 2$ and $y = -1$.

10. A

$$4^{x+2y} = 2 \quad \text{and} \quad 3^{x+y} = 1$$
$$2^{2x+4y} = 2^1 \qquad x + y = 0$$
$$2x + 4y = 1$$

Solving, we have $x = -\frac{1}{2}$ and $y = \frac{1}{2}$.

11. A

$2^{x+3y} = 32$ and $2^{3x+y} = \frac{1}{2}$

$x + 3y = 5$ $\qquad$ $3x + y = -1$

Solving, we have $x = -1$ and $y = 2$.

Thus, $x + y = 1$.

12. A

$$8^{x+\frac{1}{3}} + 8^{x-\frac{1}{3}} = 20$$
$$8^x(8^{\frac{1}{3}} + 8^{-\frac{1}{3}}) = 20$$
$$8^x = 8$$
$$x = 1$$

13. B

$$49^{x+1} = 7^{2x-1} + 342$$
$$7^{2x+2} - 7^{2x-1} = 342$$
$$7^{2x}(7^2 - 7^{-1}) = 342$$
$$7^{2x} = 7$$
$$2x = 1$$
$$x = \frac{1}{2}$$

14. C

$$3^x = 36 - 3^{x-1}$$
$$3^x(1 + 3^{-1}) = 36$$
$$3^x = 27$$
$$x = 3$$

15. A

$$7^x + 7^{x+1} = 392$$
$$7^x(1 + 7) = 392$$
$$7^x = 49$$
$$x = 2$$

16. B

$$2^{2x+2} - 3(2^{2x}) = 128$$
$$2^{2x}(2^2 - 3) = 128$$
$$2^{2x} = 128$$
$$2^{2x} = 2^7$$
$$2x = 7$$
$$x = \frac{7}{2}$$

17. C

$$2^{2x+3} - 4^{x+1} = 128$$
$$2^{2x+3} - 2^{2x+2} = 128$$
$$2^{2x}(2^3 - 2^2) = 128$$
$$2^{2x} = 32$$
$$2^{2x} = 2^5$$
$$2x = 5$$
$$x = \frac{5}{2}$$

18. C

$$4^x = 3(4^{x-1}) + 16$$
$$4^x - \frac{3}{4} \cdot 4^x = 16$$
$$4^x \left(1 - \frac{3}{4}\right) = 16$$
$$4^x = 64$$
$$x = 3$$

19. C

$$\text{Required number} = 40(1.4)^3$$
$$\approx 110$$

20. D

$$4\,000\,000 = A(1.08)^0$$
$$A = 4\,000\,000$$
$$\text{Required value} = A(1.08)^2$$
$$= \$4\,665\,600$$

21. A

For the graph of $y = \left(\frac{1}{3}\right)^{-x}$.

x	-1	0	1
y	$\frac{1}{3}$	1	3

The graph passes through $\left(-1,\ \frac{1}{3}\right)$, $(0,\ 1)$ and $(1,\ 3)$.

The answer is A.

22. C

The graph passes through $(-1,\ 4)$ and $(0,\ 1)$.

A. ✗. The graph of $y = 4^x$ does not pass through $(-1,\ 4)$.

B. ✗. The graph of $y = -4^x$ does not pass through $(0,\ 1)$.

C. ✓.

D. ✗. The graph of $y = -\left(\frac{1}{4}\right)^x$ does not pass through $(0,\ 1)$.

23. A

For the graph of $y = 2^x$.

x	-1	0	1
y	$\frac{1}{2}$	1	2

The graph passes through $\left(-1,\ \frac{1}{2}\right)$, $(0,\ 1)$ and $(1,\ 2)$.

The answer is A.

24. A

$a = 3^0 = 1$

$b = 3^a = 3^1 = 3$

25. C

$2 = ka^0$

$k = 2$

26. C

The line $x = 1$ intersects the graph of $y = 2^x$ and $y = 3^x$ at $(1,\ 2)$ and $(1,\ 3)$ respectively.

The answer is therefore C.

27. D

The line $x = 1$ intersects the graph of $y = 2^x$ and the graph of $y = \left(\frac{3}{2}\right)^x$ at $(1,\ 2)$ and $\left(1,\ \frac{3}{2}\right)$ respectively.

Since the y-intercept is 1, the intersection of the required curve and $x = 1$ should lie above the y-intercept.

The answer is D.

28. B

Required graph is $y = 5^{-(-x)} = 5^x$.

29. A

$$a^x = b^{-x}$$
$$a^x b^x = 1$$
$$(ab)^x = 1$$
$$ab = 1$$

30. C

Draw the line $y = 10$.

We have $x = 1.4$.

Conventional Questions

31. $3^x + 3^{x+1} = 108$

$3^x(1 + 3) = 108$

$3^x = 27$ 1M

$3^x = 3^3$ 1M

$x = 3$ 1A

32. $2^{2x+1} + 3(4^x) - 2^{2x-1} = 36$

$2^{2x}(2 + 3 - 2^{-1}) = 36$ 1M

$2^{2x} = 8$ 1M

$2x = 3$

$x = \frac{3}{2}$ 1A

33. (a) $P(1.32)^0 = 20$ 1M

$P = 20$ 1A

(b) Required number $= 20(1.32)^5$ 1M

≈ 80 1A

(c) Ratio of number of bananas $= \frac{P(1.32)^{2t}}{P(1.32)^t}$

$= 1.32^t$ 1M

Since t may not be equal to 2, the ratio may not equal 1.32^2.

The claim is disagreed. 1A

34. (a) Required cost $= 45\,000(20)^{-0.2}$ 1M

$\approx \$24\,700$ 1A

(b) Required percentage decrease $= \frac{45\,000(5)^{-0.2} - 45\,000(20)^{-0.2}}{45\,000(5)^{-0.2}} \times 100\%$ 1M

$\approx 24.2\%$ 1A

35. (a) Suppose Wilkins and Sally meet after r hours from 10:00.

$\frac{30 - 10.5}{r} = \frac{30 - 4}{4}$ 1M

$r = 3$

Wilkins and Sally will meet at 13:00. 1A

(b) When $t = 0$, $x = 0$,

$$0 = \frac{3}{2}(a^0 - k)$$

$$0 = 1 - k$$

$$k = 1$$ 1A

When $t = 3$, $x = 10.5$,

$$10.5 = \frac{3}{2}(a^3 - 1)$$

$$a^3 = 8$$

$$a^3 = 2^3$$ 1M

$$a = 2$$ 1A

(c) When $t = 4$,

$$x = \frac{3}{2}(2^4 - 1)$$

$$= 22.5$$ 1A

Distance between Sally and town M at 14:00 $= \left[30 - (1)\left(\frac{30 - 4}{4}\right)\right]$ km 1M

$= 23.5$ km

> 22.5 km

She cannot meet Wilkins before 14:00. 1A

END OF PAPER