Solution	Marks

REG-2425-MOCK-SET 10-MATH-CP 1
Suggested solutions

Solution	Marks
1. $\dfrac{3}{5-6k}-\dfrac{4}{9-8k}=\dfrac{3(9-8k)-4(5-6k)}{(5-6k)(9-8k)}$	1M
$=\dfrac{27-24k-20+24k}{(5-6k)(9-8k)}$	1M
$=\dfrac{7}{(5-6k)(9-8k)}$	1A
2. $13h=\dfrac{8k-5}{3k-2}$	
$39hk-26h=8k-5$	1M
$k(39h-8)=26h-5$	1M
$k=\dfrac{26h-5}{39h-8}$	1A
3. (a) $12p^2-25pq+12q^2=(3p-4q)(4p-3q)$	1A
(b) $9p+50pq-12q-24p^2-24q^2$	
$=3(3p-4q)-2(3p-4q)(4p-3q)$	1M
$=(3p-4q)(3-8p+6q)$	1A
4. (a) $\dfrac{2}{3}(x+7)<9-4x$	
$\dfrac{14x}{3}<\dfrac{13}{3}$	1M
$x<\dfrac{13}{14}$	1A
$2x-3\le 0$	
$x\le\dfrac{3}{2}$	
Thus, we have $x<\dfrac{13}{14}$.	1A
(b) 0	1A
5. (a) Let $a=4k$ and $b=5k$, where k is a non-zero constant.	
$\dfrac{7(4k)-6c}{3(5k)-2c}=8$	1M
$c=\dfrac{46k}{5}$	
We have $a:b:c=20:25:46$.	1A
(b) $\dfrac{3a+4b}{6b+5c}=\dfrac{3(4k)+4(5k)}{6(5k)+5\left(\frac{46k}{5}\right)}$	1M
$=\dfrac{8}{19}$	1A

Solution	Marks
6. (a) Marked price $= 120(1 + 25\%)$	1M
$= \$150$	1A
(b) Selling price $= 150(1 - 10\%)$	
$= \$135$	
Percentage profit $= \dfrac{135 - 120}{120} \times 100\%$	1M
$= 12.5\%$	1A
7. (a) $\angle AOB + 144^\circ - 54^\circ = 90^\circ$	1M
Required area $= \dfrac{8 \times 8}{2}$	
$= 32$	1A
(b) The axis of reflectional symmetry passes through O.	
When $\angle AOC < 90^\circ$, $x = 54 + \dfrac{90}{2} = 99$.	1M
When $\angle AOC \geq 90^\circ$, $x = 99 + 180 = 279^\circ$.	
The possible values of x are 99 and 279.	1A
8. (a) $\angle FAE = \angle FAD$ *(given)*	
$AF = AF$ *(common side)*	
$AC \perp DE$ *(given)*	
$\angle AFE = 90^\circ = \angle AFD$	
$\triangle AFD \cong \triangle AFE$ *(ASA)*	

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

Solution	Marks
(b) AD is a median of $\triangle ABC$.	
We have $BD = CD$.	
Since $\triangle AFD \cong \triangle AFE$, we have $AD = AE$.	
Thus, $AE = DC = AD = BD$.	1M
In $\triangle ACD$, we have $\angle DAC = \angle DCA$.	
In $\triangle ABD$, we have $\angle DAB = \angle DBA$.	
Consider $\triangle ABC$.	
$\angle ABC + \angle BAC + \angle ACB = 180^\circ$	
$\angle DBA + (\angle DAB + \angle DAC) + \angle DCA = 180^\circ$	1M
$2\angle DAB + 2\angle DAC = 180^\circ$	
$\angle DAB + \angle DAC = 90^\circ$	
$\angle BAC = 90^\circ$	
$\triangle ABC$ is a right-angled triangle.	1A

Solution	Marks
9. (a) $\dfrac{x+114+90}{360°}=1-\dfrac{16}{45}$	1M+1M
$x+204=232$	
$x=28$	1A
(b) Angle of the basketball sector	
$=360°\times\dfrac{16}{45}$	1M
$=128°$	
$>114°$	
The claim is agreed.	1A
10. (a) Let $P=C+kn$, where k is a non-zero constant.	1A
$\begin{cases}12\,000=C+20k\\13\,050=C+27k\end{cases}$	1M
Solving, we have $C=9000$ and $k=150$.	1A
Required income	
$=9000+150(24)$	
$=\$12\,600$	1A
(b) $9000+150n>14\,000$	1M
$n>33.3$	
The least value of n is 34.	1A
11. (a) Mean $=\dfrac{28+29+\ldots+69}{18}$	
$=42$	1A
Median $=\dfrac{40+42}{2}=41$	1A
(b) (i) 39	1A
(ii) Let x, y and z be the new data, where $x\le y\le z$.	
The median remains unchanged, we have $y=41$.	1A
The mean of x, y and z is also 42.	
Case 1: $x=27$	
Mean $=\dfrac{27+41+z}{3}=42$	
$z=58$	1A
Standard deviation ≈ 10.4	
Case 2: $z=70$	
Mean $=\dfrac{x+41+70}{3}=42$	
$x=15$ (rejected)	
Thus, $x=27$, $y=41$ and $z=58$.	
The standard deviation is 10.4.	1A

Solution	Marks
12. (a) Let $(0,\ a)$ be the coordinates of A, where $a > 0$.	
The coordinates of D are $\left(\frac{a}{k},\ a\right)$.	1M
The coordinates of B and C are $\left(0,\ a-\frac{a}{k}\right)$ and $\left(\frac{a}{k},\ a-\frac{a}{k}\right)$ respectively.	
Slope of $OC = \dfrac{a-\frac{a}{k}-0}{\frac{a}{k}-0}$	1M
$= \dfrac{ak-a}{a}$	
$= k-1$	1A
(b) Slope of $OH = \dfrac{4-0}{-3-0} = -\dfrac{4}{3}$	
Slope of $ON = -\dfrac{4}{3} - 1 = -\dfrac{7}{3}$	1M
The equation of the straight line passing through O and N is $y = -\dfrac{7x}{3}$.	
The equation of L is	
$y - 0 = \dfrac{4-0}{-3+9}(x+9)$	1M
$2x - 3y + 18 = 0$	
The y-intercept of L is 6.	
The coordinates of R are $(0,\ 6)$.	
Solve $\begin{cases} y = -\dfrac{7x}{3} \\ 2x - 3y + 18 = 0 \end{cases}$, we have $x = -2$ and $y = \dfrac{14}{3}$.	
The coordinates of N are $\left(-2,\ \dfrac{14}{3}\right)$.	
y-coordinate of $T = \dfrac{14}{3} - 2 = \dfrac{8}{3}$	1M
$TR : OT = \left(6 - \dfrac{8}{3}\right) : \dfrac{8}{3}$	
$= 5 : 4$	1A

Solution	Marks
13. (a) Volume of the sphere $= \frac{4}{3}\pi(126^3)$	1M
$= 2\,667\,168\pi \text{ cm}^3$	
Let V cm^3 be the volume of the smaller circular cone.	
$V + V\left(\sqrt{\frac{25}{1}}\right)^3 = 2\,667\,168\pi$	1M
$V = 21\,168\pi$	1A
Volume of the smaller circular cone is $21\,168\pi \text{ cm}^3$.	
(b) Let r cm be the base radius of the smaller circular cone.	
$\frac{1}{3}\pi r^2(16) = 21\,168\pi$	1M
$r^3 = 3969$	
$r = 63$	
Total surface area of the smaller circular cone	
$= \pi(63)^2 + \pi(63)\sqrt{63^2 + 16^2}$	1M
$= 8064\pi \text{ cm}^2$	
Required area	
$= 8064 + 8064(25)$	1M
$= 209\,664\pi \text{ cm}^2$	1A

Solution	Marks
14. (a) Let $f(x) = (x^2 + x - 6)(ax + c) + 48x - 26$, where c is a constant.	1M
Compare the coefficients of x and x^2.	
$\begin{cases} 1 = c + a \\ 7 = c - 6a + 48 \end{cases}$	1M
Solving, we have $a = 6$ and $c = -5$.	1A
$f\left(-\frac{1}{2}\right) = \left(\frac{1}{4} + \frac{1}{2} - 6\right)(3 - 5) + 24 - 26 = 0$	1A
(b) $f(x) = 0$	
$(x^2 + x - 6)(6x - 5) + 48x - 26 = 0$	
$6x^3 + x^2 + 7x + 4 = 0$	
$(2x + 1)(3x^2 - x + 4) = 0$	1M
$x = -\frac{1}{2}$ or $3x^2 - x + 4 = 0$	
Consider the equation $3x^2 - x + 4 = 0$.	
$\Delta = (-1)^2 - 4(3)(4)$	1M
$= -47$	
< 0	
The equation $3x^2 - x + 4 = 0$ has no real roots and hence no rational roots.	1M
The equation $f(x) = 0$ has only one rational root $-\frac{1}{2}$.	
The claim is agreed.	1A
15. (a) Let x marks be the score of Peter in the English test.	
$\frac{x - 52}{16} = 0.5$	1M
$x = 60$	1A
(b) Standard score of Peter in the Chinese test	
$= \frac{68 - 66}{8}$	
$= 0.25$	
< 0.5	1A
Peter performs better in the English test than in the Chinese test.	
The claim is not correct.	1A
16. (a) Required probability $= \frac{C_2^{10} C_2^2}{C_4^{12}}$	1M
$= \frac{1}{11}$	1A
(b) Required probability $= 1 - \frac{1}{11}$	1M
$= \frac{10}{11}$	1A

Solution	Marks
17. (a) $(4)^2 + (-8)^2 + 4(4) - 96 = 16 + 64 + 16 - 96 = 0$	
A lies on C.	1
(b) (i) Γ is the angle bisector of $\angle ADB$.	1A
(ii) The coordinates of the centre of C are $(-2,\ 0)$.	
The slope of L_1 is $\frac{3}{4}$.	
The slope of L_2 is $-\frac{4}{3}$.	
Let the equation of L_2 be $y = -\frac{4x}{3} + k$, where k is a constant.	
$x^2 + \left(-\frac{4x}{3} + k\right)^2 + 4x - 96 = 0$	1M
$\frac{25x^2}{9} + \left(-\frac{8k}{3} + 4\right)x + (k^2 - 96) = 0$	
L_2 is a tangent to C.	
$\Delta = \left(-\frac{8k}{3} + 4\right)^2 - 4\left(\frac{25}{9}\right)(k^2 - 96) = 0$	1M
$-4k^2 - \frac{64k}{3} + \frac{3248}{3} = 0$	
$k = 14 \quad \text{or} \quad -\frac{58}{3}$	
Note that the coordinates of E are $(0,\ k)$ and $k > 0$.	
We have $k = 14$.	
Solve $\begin{cases} 3x - 4y - 44 = 0 \\ y = -\frac{4x}{3} + 14 \end{cases}$, we have $x = 12$ and $y = -2$.	
The coordinates of D are $(12,\ -2)$.	1M
Note that Γ passes through D and the centre of C.	
Slope of $\Gamma = \frac{0 + 2}{-2 - 12} = -\frac{1}{7}$	
Required equation is	
$y - 0 = -\frac{1}{7}(x + 2)$	1M
$x + 7y + 2 = 0$	1A

Solution	Marks
18. (a) $\angle BDC = 180^\circ - 37^\circ - 86^\circ = 57^\circ$	
$\dfrac{BD}{\sin 37^\circ} = \dfrac{27}{\sin 57^\circ}$	1M
$BD \approx 19.4\text{ cm}$	
$\angle ADB = 102^\circ - 57^\circ = 45^\circ$	
$AB^2 = 18^2 + BD^2 - 2(18)(BD)\cos 45^\circ$	1M
$AB \approx 14.4\text{ cm}$	1A
(b) (i) Let E be a point on BD such that $AE \perp BD$.	
$DE = AD\cos\angle ADB = 18\cos 45^\circ \approx 12.7\text{ cm}$	1M
$AE = DE\tan 45^\circ \approx 12.7\text{ cm}$	
$BE = BD - DE \approx 6.65\text{ cm}$	
$CE^2 = BE^2 + 27^2 - 2(BE)(27)\cos 86^\circ$	
$CE \approx 27.4\text{ cm}$	
$AC = \sqrt{AE^2 + CE^2} 30.2\text{ cm}$	
Let $s = \dfrac{AB + 27 + AC}{2} \approx 35.8\text{ cm}$.	
Required area	
$= \sqrt{s(s-27)(s-AB)(s-AC)}$	1M
$\approx 194\text{ cm}^2$	1A
(ii) Let h cm be the shortest distance between P and D.	
Consider the volume of the tetrahedron $ABCD$.	
$\dfrac{1}{3}(\text{area of } \triangle ABC)(h) = \dfrac{1}{3}\left(\dfrac{1}{2}(BD)(27)\sin 86^\circ\right)(AE)$	1M
$h \approx 17.1$	
> 17	
The claim is agreed.	1A

Solution	Marks
19. (a) $f(x) = 3x^2 + 6px + 6x + 3p^2 + 3p + q$	
$= 3[x^2 + 2(p+1)x + (p+1)^2 - (p+1)^2] + 3p^2 + 3p + q$	1M
$= 3[x + (p+1)]^2 + q - 3p - 3$	
The coordinates of U are $(-p-1,\ q-3p-3)$.	1A
(b) The graph of $y = f(x)$ is enlarged to $\frac{3}{2}$ times the original along the x-axis;	1A
and translated downwards by 5 units	1A
to become the graph of $y = f\left(\frac{2x}{3}\right) - 5$.	
(c) (i) The coordinates of V are $\left(-\frac{3p+3}{2},\ q-3p-8\right)$.	1M
$(u_x)^2$, $9(p+1)$, $(v_x)^2$ is a geometric sequence.	
$\frac{9(p+1)}{(u_x)^2} = \frac{(v_x)^2}{9(p+1)}$	1M
$\frac{9}{p+1} = \frac{p+1}{4}$	
$(p+1)^2 = 36$	
$p + 1 = \pm 6$	
$p = -7$ or 5	
$2u_y$, $7-q$, $2v_y$ is an arithmetic sequence.	
$(7-q) - 2u_y = 2v_y - (7-q)$	1M
$7 - q - 2(q-3p-3) = 2(q-3p-8) - 7 + q$	
$q = 2p + 6$	
We have $(p,\ q) = (-7,\ -8)$ or $(5,\ 16)$.	
The coordinates of U and V are $(6,\ 10)$ and $(9,\ 5)$ respectively;	1A
or $(-6,\ -2)$ and $(-9,\ -7)$ respectively.	1A
(ii) The coordinates of U and V are $(6,\ 10)$ and $(9,\ 5)$ respectively.	
The coordinates of the mid-point of SV are $\left(\frac{11-2t}{2},\ \frac{11+t}{2}\right)$.	1M
The coordinates of the mid-point of TV are $\left(\frac{12-t}{2},\ t+6\right)$.	
Suppose the x-coordinates of the mid-points are equal.	
$\frac{11-2t}{2} = \frac{12-t}{2}$	1M
$t = -1$	
When $t = -1$, both the coordinates of the mid-point of SU and the mid-point of TV are $\left(\frac{13}{2},\ 5\right)$.	
Since SU and TV bisects each other, $STVU$ is a parallelogram.	
It is possible.	1A

END OF PAPER