**M2REG-INDINT-2425-ASM-SET 6-MATH**
**Suggested solutions**

1. $$\int \frac{x^{\frac{3}{2}}-8}{\sqrt{x}-2}\,dx = \int \frac{(x^{\frac{1}{2}}-2)(x+2x^{\frac{1}{2}}+4)}{x^{\frac{1}{2}}-2}\,dx$$
$$= \int (x+2x^{\frac{1}{2}}+4)\,dx \qquad \text{1M}$$
$$= \frac{1}{2}x^2+\frac{4}{3}x^{\frac{3}{2}}+4x+\text{constant} \qquad \text{1A}$$

2. $$\int \frac{3e^{2x}+5e^x-2}{e^x+2}\,dx = \int \frac{(3e^x-1)(e^x+2)}{e^x+2}\,dx$$
$$= \int (3e^x-1)\,dx \qquad \text{1M}$$
$$= 3e^x-x+\text{constant} \qquad \text{1A}$$

3. Let $u=\log_2 x$. Then $du=\frac{1}{x\ln 2}\,dx$. 1M
$$\int \frac{3(\log_2 x)^2+2\log_2 x-4}{x}\,dx = \int (\ln 2)(3u^2+2u-4)\,du \qquad \text{1A}$$
$$= (\ln 2)(u^3+u^2-4u)+\text{constant}$$
$$= (\ln 2)[(\log_2 x)^3+(\log_2 x)^2-4\log_2 x]+\text{constant} \qquad \text{1A}$$

4. Let $u=e^x+1$. Then $du=e^x\,dx$. 1M
$$\int \frac{e^x}{e^{2x}+2e^x+1}\,dx = \int \frac{e^x}{(e^x+1)^2}\,dx$$
$$= \int \frac{du}{u^2}$$
$$= -\frac{1}{u}+\text{constant}$$
$$= -\frac{1}{e^x+1}+\text{constant} \qquad \text{1A}$$

5. Let $u=\ln x$. Then $du=\frac{1}{x}\,dx$. 1M
$$\int \frac{\sqrt{\ln\sqrt{x}}}{x}\,dx = \frac{1}{\sqrt{2}}\int u^{\frac{1}{2}}\,du$$
$$= \frac{\sqrt{2}}{3}u^{\frac{3}{2}}+\text{constant}$$
$$= \frac{\sqrt{2}}{3}(\ln x)^{\frac{3}{2}}+\text{constant} \qquad \text{1A}$$

6. Let $u = 1 - e^{-5x}$. Then $\mathrm{d}u = 5e^{-5x}\,\mathrm{d}x$. 1M

$$\int \frac{\mathrm{d}x}{e^{5x}-1} = \int \frac{e^{-5x}}{1-e^{-5x}}\,\mathrm{d}x$$

$$= \frac{1}{5}\int \frac{\mathrm{d}u}{u} \qquad \text{1M}$$

$$= \frac{1}{5}\ln|u| + \text{constant}$$

$$= \frac{1}{5}\ln|1-e^{-5x}| + \text{constant} \qquad \text{1A}$$

**Alternative solution**

Let $u = e^{5x} - 1$. Then $\mathrm{d}u = 5e^{5x}\,\mathrm{d}x$. 1M

$$\int \frac{\mathrm{d}x}{e^{5x}-1} = \int \frac{1-e^{5x}+e^{5x}}{e^{5x}-1}\,\mathrm{d}x$$

$$= \int \frac{1-e^{5x}}{e^{5x}-1}\,\mathrm{d}x + \int \frac{e^{5x}}{e^{5x}-1}\,\mathrm{d}x$$

$$= -\int \mathrm{d}x + \frac{1}{5}\int \frac{\mathrm{d}u}{u} \qquad \text{1M}$$

$$= -x + \frac{1}{5}\ln|u| + \text{constant}$$

$$= -x + \frac{1}{5}\ln|e^{5x}-1| + \text{constant} \qquad \text{1A}$$

7. Let $= \sqrt{x} - 1$. Then $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$. 1M

$$\int \sqrt{\sqrt{x}-1}\,\mathrm{d}x = \int 2\sqrt{x}\sqrt{\sqrt{x}-1}\cdot\frac{1}{2\sqrt{x}}\,\mathrm{d}x$$

$$= \int 2(u+1)\sqrt{u}\,\mathrm{d}u$$

$$= \int (2u^{\frac{3}{2}} + 2u^{\frac{1}{2}})\,\mathrm{d}u \qquad \text{1M}$$

$$= \frac{4}{5}u^{\frac{5}{2}} + \frac{4}{3}u^{\frac{3}{2}} + \text{constant}$$

$$= \frac{4}{5}(\sqrt{x}-1)^{\frac{5}{2}} + \frac{4}{3}(\sqrt{x}-1)^{\frac{3}{2}} + \text{constant} \qquad \text{1A}$$

8. (a) $\int \frac{x+1}{x+3}\,\mathrm{d}x = \int \left(1 - \frac{2}{x+3}\right)\mathrm{d}x$ 1M

$$= x - 2\ln|x+3| + \text{constant} \qquad \text{1A}$$

(b) Let $u = \sqrt{x}$. Then $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$.

$$\int \frac{\sqrt{x}+1}{x+3\sqrt{x}}\,\mathrm{d}x = 2\int \frac{\sqrt{x}+1}{\sqrt{x}+3}\cdot\frac{1}{2\sqrt{x}}\,\mathrm{d}x$$

$$= 2\int \frac{u+1}{u+3}\,\mathrm{d}u \qquad \text{1M}$$

$$= 2(u - 2\ln|u+3|) + \text{constant}$$

$$= 2\sqrt{x} - 4\ln\left(\sqrt{x}+3\right) + \text{constant} \qquad \text{1A}$$

9. Let $u = 3 + \sin(\ln x)$. Then $\mathrm{d}u = \dfrac{\cos(\ln x)}{x}\,\mathrm{d}x$. 1M

$$\int \frac{\cos(\ln x)}{x[3+\sin(\ln x)]}\,\mathrm{d}x = \int \frac{\mathrm{d}u}{u} \qquad \text{1A}$$
$$= \ln|3+\sin(\ln x)| + \text{constant}$$
$$= \ln(3+\sin(\ln x)) + \text{constant} \qquad \text{1A}$$

10. Let $u = \sin x$. Then $\mathrm{d}u = \cos x\,\mathrm{d}x$. 1M

$$\int \frac{\cos^5 x}{\sin^6 x}\,\mathrm{d}x = \int \frac{(1-u^2)^2}{u^6}\,\mathrm{d}u$$
$$= \int (u^{-6} - 2u^{-4} + u^{-2})\,\mathrm{d}u \qquad \text{1A}$$
$$= -\frac{1}{5u^5} + \frac{2}{3u^3} - \frac{1}{u} + \text{constant}$$
$$= -\frac{1}{5\sin^5 x} + \frac{2}{3\sin^3 x} - \frac{1}{\sin x} + \text{constant} \qquad \text{1A}$$

11. Let $u = \tan x$. Then $\mathrm{d}u = \sec^2 x\,\mathrm{d}x$. 1M

$$\int \frac{\sec^8 x}{\tan^3 x}\,\mathrm{d}x = \int \frac{(1+\tan^2 x)^3}{\tan^3 x}\cdot \sec^2 x\,\mathrm{d}x$$
$$= \int \frac{(1+u^2)^3}{u^3}\,\mathrm{d}u$$
$$= \int \left(u^{-3} + 3u^{-1} + 3u + u^3\right)\mathrm{d}u \qquad \text{1A}$$
$$= -\frac{1}{2\tan^2 x} + 3\ln|\tan x| + \frac{3}{2}\tan^2 x + \frac{1}{4}\tan^4 x + \text{constant} \qquad \text{1A}$$

12. $$\int \frac{2\sin^4 2x}{1-\cos 2x}\,\mathrm{d}x = \int \frac{2\sin^4 2x(1+\cos 2x)}{1-\cos^2 2x}\,\mathrm{d}x \qquad \text{1M}$$
$$= \int 2\sin^2 2x(1+\cos 2x)\,\mathrm{d}x$$
$$= \int (1-\cos 4x)(1+\cos 2x)\,\mathrm{d}x \qquad \text{1M}$$
$$= \int (1-\cos 4x + \cos 2x - \cos 4x\cos 2x)\,\mathrm{d}x$$
$$= \int \left[1 - \cos 4x + \cos 2x - \frac{1}{2}(\cos 6x + \cos 2x)\right]\mathrm{d}x \qquad \text{1M}$$
$$= x + \frac{\sin 2x}{4} - \frac{\sin 4x}{4} - \frac{\sin 6x}{12} + \text{constant} \qquad \text{1A}$$

13. Let $u = 5 - 11x^2$. Then $\mathrm{d}u = -22x\,\mathrm{d}x$. 1M

$$\int x\sqrt{5-11x^2}\,\mathrm{d}x = -\frac{1}{22}\int \sqrt{u}\,\mathrm{d}u$$
$$= -\frac{1}{33}u^{\frac{3}{2}} + \text{constant}$$
$$= -\frac{1}{33}(5-11x^2)^{\frac{3}{2}} + \text{constant} \qquad \text{1A}$$

14. (a) $x^2 - 4x\cos\frac{\pi}{12} + 4 = x^2 - 4x\cos\frac{\pi}{12} + 4\cos^2\frac{\pi}{12} + 4\sin^2\frac{\pi}{12}$

$$= \left(x - 2\cos\frac{\pi}{12}\right)^2 + 4\sin^2\frac{\pi}{12} \qquad \text{1A}$$

(b) Let $x - 2\cos\frac{\pi}{12} = 2\sin\frac{\pi}{12}\tan\theta$. Then $\mathrm{d}x = 2\sin\frac{\pi}{12}\sec^2\theta\,\mathrm{d}\theta$. 1M

$$\begin{aligned}
\int \frac{\sin\frac{\pi}{12}}{x^2 - 4\cos\frac{\pi}{12} + 4}\,\mathrm{d}x &= \int \frac{\sin\frac{\pi}{12}}{\left(x - 2\cos\frac{\pi}{12}\right)^2 + 4\sin^2\frac{\pi}{12}}\,\mathrm{d}x \\
&= \int \frac{\sin\frac{\pi}{12}}{4\sin^2\frac{\pi}{12}\tan^2\theta + 4\sin^2\frac{\pi}{12}} \cdot 2\sin\frac{\pi}{12}\sec^2\theta\,\mathrm{d}\theta \\
&= \frac{1}{2}\int \mathrm{d}\theta && \text{1A} \\
&= \frac{1}{2}\theta + \text{constant} \\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\csc\frac{\pi}{12} - \cot\frac{\pi}{12}\right) + \text{constant} \\
&= \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{6}+\sqrt{2}}{2}x - 2 - \sqrt{3}\right) + \text{constant} && \text{1A}
\end{aligned}$$

15.
$$\begin{aligned}
\int \sec^2 x \ln(\sin x)\,\mathrm{d}x &= \tan x \ln(\sin x) - \int \tan x \cdot \frac{\cos x}{\sin x}\,\mathrm{d}x && \text{1M} \\
&= \tan x \ln(\sin x) - \int \mathrm{d}x \\
&= \tan x \ln(\sin x) - x + \text{constant} && \text{1A}
\end{aligned}$$

16. $\int e^{\sqrt{x}}\,\mathrm{d}x = \int \sqrt{x} \cdot \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$

Let $u = \sqrt{x}$. Then $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$.

$$\begin{aligned}
\int e^{\sqrt{x}}\,\mathrm{d}x &= 2\int u \cdot e^u\,\mathrm{d}u \\
&= 2ue^u - 2\int e^u\,\mathrm{d}u && \text{1M} \\
&= 2ue^u - 2e^u + \text{constant} \\
&= 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + \text{constant} && \text{1A}
\end{aligned}$$

17. (a) (i) Let $u = \ln x$. Then $\mathrm{d}u = \frac{1}{x}\,\mathrm{d}x$. 1M

$$\int \frac{(\ln x)^n}{x}\,\mathrm{d}x = \int u^n\,\mathrm{d}x$$
$$= \frac{u^{n+1}}{n+1} + \text{constant}$$
$$= \frac{(\ln x)^{n+1}}{n+1} + \text{constant} \qquad \text{1A}$$

(ii) $$\int x^a (\ln x)^n\,\mathrm{d}x = \int \left(x^{a+1} \times \frac{(\ln x)^n}{x}\right)\mathrm{d}x \qquad \text{1M}$$
$$= \frac{x^{a+1}(\ln x)^{n+1}}{n+1} - \int \left[(a+1)x^a \times \frac{(\ln x)^{n+1}}{n+1}\right]\mathrm{d}x$$
$$= \frac{x^{a+1}(\ln x)^{n+1}}{n+1} - \frac{a+1}{n+1}\int x^a (\ln x)^{n+1}\,\mathrm{d}x \qquad \text{1}$$

(b) (i) $$\int x^a\,\mathrm{d}x = \frac{x^{a+1}}{a+1} + \text{constant} \qquad \text{1A}$$

(ii) Using the result of (a)(ii), we have

$$\int x^a (\ln x)^{n+1}\,\mathrm{d}x = \frac{x^{a+1}(\ln x)^{n+1}}{a+1} - \frac{n+1}{a+1}\int x^a (\ln x)^n\,\mathrm{d}x\,.$$

$$\int x^a \ln x\,\mathrm{d}x = \frac{x^{a+1}(\ln x)}{a+1} - \frac{1}{a+1}\int x^a\,\mathrm{d}x \qquad \text{1M}$$
$$= \frac{x^{a+1}\ln x}{a+1} - \frac{x^{a+1}}{(a+1)^2} + \text{constant} \qquad \text{1A}$$

$$\int x^a (\ln x)^2\,\mathrm{d}x = \frac{x^{a+1}(\ln x)^2}{a+1} - \frac{2}{a+1}\int x^a (\ln x)\,\mathrm{d}x \qquad \text{1M}$$
$$= \frac{x^{a+1}(\ln x)^2}{a+1} - \frac{2}{a+1}\left[\frac{x^{a+1}\ln x}{a+1} - \frac{x^{a+1}}{(a+1)^2}\right] + \text{constant} \qquad \text{1M}$$
$$= \frac{x^{a+1}(\ln x)^2}{a+1} - \frac{2x^{a+1}\ln x}{(a+1)^2} + \frac{2x^{a+1}}{(a+1)^3} + \text{constant} \qquad \text{1A}$$

 **END OF PAPER**