

M2REG-INDINT-2425-ASM-SET 5-MATH

Suggested solutions

$$1. a \left(\frac{1}{a} \right)^2 - 3 \left(\frac{1}{a} \right) - 6 = -5 \quad 1M$$

$$-\frac{2}{a} = 1$$

$$a = -2$$

1A

$$f(x) = \int (-2x^2 - 3x - 6) dx \quad 1M$$

$$= -\frac{2x^3}{3} - \frac{3x^2}{2} - 6x + C$$

$$120 = -\frac{2}{3}(-6)^3 - \frac{3}{2}(-6)^2 - 6(-6) + C \quad 1M$$

$$C = -6$$

$$\text{Thus, } f(x) = -\frac{2}{3}x^3 - \frac{3}{2}x^2 - 6x - 6. \quad 1A$$

$$2. \frac{dy}{dx} = \int \frac{8}{e^{2x}} dx \quad 1M$$

$$= -4e^{-2x} + C_1$$

$$y = \int (-4e^{-2x} + C_1) dx$$

$$= 2e^{-2x} + C_1x + C_2$$

1A

When $x = 0$,

$$\begin{cases} -4e^{-2(0)} + C_1 = 1 \\ 2e^{-2(0)} + C_1(0) + C_2 = 7 \end{cases} \quad 1M$$

Solving, we have $C_1 = 5$ and $C_2 = 5$. 1A

Thus, the equation of the curve is $y = \frac{2}{e^{2x}} + 5x + 5$. 1A

$$3. \text{ Let } u = x^2. \text{ Then } du = 2x dx. \quad 1M$$

$$y = \int 2x^3 e^{x^2} dx \quad 1M$$

$$= \int u e^u du$$

$$= u e^u - \int e^u du$$

$$= u e^u - e^u + C$$

1A

$$= x^2 e^{x^2} - e^{x^2} + C$$

$$4 = e^1 - e^1 + C$$

$$C = 4$$

Required equation is $y = x^2 e^{x^2} - e^{x^2} + 4$. 1A

$$4. \frac{dy}{dx} = \int (4e^{2x} - e^{-x}) dx \quad 1M$$

$$= 2e^{2x} + e^{-x} + C$$

$$\text{At } (0, 2), \frac{dy}{dx} = 2 \text{ and}$$

$$2 = 2 + 1 + C \quad 1M$$

$$C = -1$$

$$y = \int (2e^{2x} + e^{-x} - 1) dx$$

$$= e^{2x} - e^{-x} - x + C_1$$

$$\text{When } x = 0, y = 2, \text{ we have } C_1 = 2. \quad 1M$$

$$\text{The equation of the curve is } y = e^{2x} - e^{-x} - \frac{5x}{2} + 2. \quad 1A$$

$$5. \quad (a) \text{ Let } u = x^2, \text{ then } du = 2x dx.$$

$$\int 2x^3 \cos(x^2) dx = \int u \cos u du \quad 1M$$

$$= u \sin u - \int \sin u du \quad 1M$$

$$= x^2 \sin(x^2) + \cos(x^2) + \text{constant} \quad 1A$$

$$(b) \quad (i) y = \int 2x^3 \cos(x^2) dx = x^2 \sin(x^2) + \cos(x^2) + C$$

$$\text{Substitute } (\sqrt{\pi}, -1), \text{ we have } C = 0 \quad 1M$$

$$\text{The equation of } \Gamma \text{ is } y = x^2 \sin(x^2) + \cos(x^2) \quad 1A$$

$$(ii) \text{ Slope of tangent} = 2\pi^{\frac{3}{2}} \cos \pi = -2\pi^{\frac{3}{2}} \quad 1M$$

The equation is

$$y + 1 = -2\pi^{\frac{3}{2}}(x - \sqrt{\pi})$$

$$y = -2\pi^{\frac{3}{2}}x + 2\pi^2 - 1 \quad 1A$$

6. (a) Let $u = 1 + x^3$. Then $du = 3x^2 dx$. 1M

$$\int \frac{x^5}{\sqrt{1+x^3}} dx = \frac{1}{3} \int \frac{u-1}{\sqrt{u}} du \quad 1M$$

$$= \frac{1}{3} \int (u^{\frac{1}{2}} - u^{-\frac{1}{2}}) du$$

$$= \frac{2}{9} u^{\frac{3}{2}} - \frac{2}{3} u^{\frac{1}{2}} + \text{constant}$$

$$= \frac{2}{9} (1+x^3)^{\frac{3}{2}} - \frac{2}{3} (1+x^3)^{\frac{1}{2}} + \text{constant} \quad 1A$$

(b) $y = \int \frac{9x^5}{2\sqrt{1+x^3}} dx$ 1M

$$= \frac{9}{2} \left[\frac{2}{9} (1+x^3)^{\frac{3}{2}} - \frac{2}{3} (1+x^3)^{\frac{1}{2}} \right] + C \quad 1M$$

$$= (1+x^3)^{\frac{3}{2}} - 3\sqrt{1+x^3} + C$$

$$3 = 1 - 3 + C \quad 1M$$

$$C = 5$$

Required equation is $y = (1+x^3)^{\frac{3}{2}} - 3\sqrt{1+x^3} + 5$. 1A

7. (a) Suppose $x > 0$.

$$\begin{aligned} f'(x) &= \frac{2}{x} \left[\left[x^2 - 2(x) \left(\frac{5}{4} \right) + \left(\frac{5}{4} \right)^2 \right] + \frac{55}{16} \right] \\ &= \frac{2}{x} \left[\left(x - \frac{5}{4} \right)^2 + \frac{55}{16} \right] \\ &> 0 \end{aligned} \quad 1\text{M}$$

Thus, $f(x)$ is not a decreasing function. 1A

(b) (i) $y = \int \left(2x - 5 + \frac{10}{x} \right) dx$ 1M

$$= x^2 - 5x + 10 \ln x + C$$

Γ passes through the point $(1, 5)$.

$$5 = 1 - 5 + 0 + C \quad 1\text{M}$$

$$C = 9$$

Required equation is $y = x^2 - 5x + 10 \ln x + 9$. 1A

(ii) $f''(x) = \frac{x(4x - 5) - (2x^2 - 5x + 10)}{x^2}$ 1M

$$= \frac{2(x^2 - 5)}{x^2}$$

When $f''(x) = 0$, $x = \sqrt{5}$.

x	$0 < x < \sqrt{5}$	$x > \sqrt{5}$
$f''(x)$	$-$	$+$

1M

The point of inflexion of Γ is $(\sqrt{5}, 14 - 5\sqrt{5} + 5 \ln 5)$. 1A

(iii) $x > \sqrt{5}$ 1A

$$8. \quad (a) \quad \int x e^x dx = x e^x - \int e^x dx \quad 1M$$

$$= x e^x - e^x + \text{constant} \quad 1A$$

$$(b) \quad \text{Let } u = 4 - x^2. \text{ Then } du = -2x dx.$$

$$y = \int (x^3 - 4x^2) e^{4-x^2} dx \quad 1M$$

$$= \frac{1}{2} \int u e^u du \quad 1A$$

$$= \frac{u e^u}{2} - \frac{e^u}{2} + C$$

$$= \frac{(3 - x^2) e^{4-x^2}}{2} + C$$

The curve passes through (2, 5),

$$5 = \frac{-e^0}{2} + C \quad 1M$$

$$C = \frac{11}{2}$$

$$\text{The equation of the curve is } y = \frac{(3 - x^2) e^{4-x^2}}{2} + \frac{11}{2}. \quad 1A$$

$$9. \quad (a) \quad \text{Let } x = 3 \tan \theta. \text{ Then } dx = 3 \sec^2 \theta d\theta. \quad 1M$$

$$\int \frac{dx}{x^2 + 9} = \int \frac{3 \sec^2 \theta}{9 \tan^2 \theta + 9} d\theta \quad 1M$$

$$= \frac{1}{3} \int d\theta$$

$$= \frac{\theta}{3} + \text{constant}$$

$$= \frac{1}{3} \tan^{-1} \frac{x}{3} + \text{constant} \quad 1A$$

$$(b) \quad y = \int \frac{x^2}{x^2 + 9} dx$$

$$= \int \left(1 - \frac{9}{x^2 + 9} \right) dx \quad 1M$$

$$= x - 3 \tan^{-1} \frac{x}{3} + C, \text{ where } C \text{ is a constant.} \quad 1A$$

$$6 = x - 3 \tan^{-1} 0 + C \quad 1M$$

$$C = 6$$

$$\text{The equation is } y = x - 3 \tan^{-1} \frac{x}{3} + 6. \quad 1A$$