M2REG-INDINT-2425-ASM-SET 4-MATH
Suggested solutions

1. Let $u = x^2 + 1$. Then $du = 2x\,dx$. 1M

$$\int \frac{x(x^2-1)}{\sqrt{x^2+1}}\,dx = \frac{1}{2}\int \frac{u-2}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int (u^{\frac{1}{2}} - 2u^{-\frac{1}{2}})\,du \quad \text{1M}$$

$$= \frac{1}{3}u^{\frac{3}{2}} - 2u^{\frac{1}{2}} + \text{constant}$$

$$= \frac{1}{3}(x^2+1)^{\frac{3}{2}} - 2(x^2+1)^{\frac{1}{2}} + \text{constant} \quad \text{1A}$$

2. (a) $$\frac{A}{3x+2} + \frac{B}{5x-1} = \frac{A(5x-1)+B(3x+2)}{(3x+2)(5x-1)}$$

$$= \frac{(5A+3B)x + (-A+2B)}{(3x+2)(5x-1)}$$

We have $5A + 3B = 12$ and $-A + 2B = -5$. 1M

Solving, we have $A = 3$ and $B = -1$. 1A+1A

(b) $$\int \frac{12x-5}{(3x+2)(5x-1)}\,dx = \int \left(\frac{3}{3x+2} - \frac{1}{5x-1}\right)dx \quad \text{1M}$$

$$= \ln|3x+2| - \frac{1}{5}\ln|5x-1| + \text{constant} \quad \text{1A}$$

3. (a) $$P + \frac{Q}{x+4} + \frac{R}{2x-5} = \frac{P(x+4)(2x-5) + Q(2x-5) + R(x+4)}{(x+4)(2x-5)}$$

$$= \frac{2Px^2 + (3P+2Q+R)x + (-20P-5Q+4R)}{(x+4)(2x-5)}$$

We have $2P = 4$, $3P + 2Q + R = 11$ and $-20P - 5Q + 4R = -59$. 1M

Solving, we have $P = 2$, $Q = 3$ and $R = -1$. 1A+1A

(b) $$\int \frac{4x^2+11x-59}{2x^2+3x-20}\,dx = \int \frac{4x^2+11x-59}{(x+4)(2x-5)}\,dx$$

$$= \int \left(2 + \frac{3}{x+4} - \frac{1}{2x-5}\right)dx \quad \text{1M}$$

$$= 2x + 3\ln|x+4| - \frac{1}{2}\ln|2x-5| + \text{constant} \quad \text{1A}$$

4. (a) $\dfrac{A}{1-x}+\dfrac{B}{1+x}=\dfrac{A(1+x)+B(1-x)}{(1-x)(1+x)}$

$=\dfrac{(A-B)x+(A+B)}{1-x^2}$

We have $A-B=0$ and $A+B=1$. 1M

Solving, we have $A=\dfrac{1}{2}$ and $B=\dfrac{1}{2}$. 1A+1A

(b) Let $u=\cos x$. Then $\mathrm{d}u=-\sin x\,\mathrm{d}x$. 1M

$$\int \frac{1}{\sin x}\,\mathrm{d}x=\int \frac{\sin x}{1-\cos^2 x}\,\mathrm{d}x$$

$$=-\int \frac{\mathrm{d}u}{1-u^2} \quad \text{1A}$$

$$=-\frac{1}{2}\int\left(\frac{1}{1-u}+\frac{1}{1+u}\right)\mathrm{d}u \quad \text{1M}$$

$$=\frac{1}{2}\ln|1-u|-\frac{1}{2}\ln|1+u|+\text{constant}$$

$$=\frac{1}{2}\ln\left|\frac{1-\cos x}{1+\cos x}\right|+\text{constant} \quad \text{1A}$$

(c) Let $3x=\sin\theta$. Then $3\,\mathrm{d}x=\cos\theta\,\mathrm{d}\theta$. 1M

$$\int \frac{\mathrm{d}x}{x\sqrt{1-9x^2}}=\frac{1}{3}\int \frac{\cos\theta}{\frac{\sin\theta}{3}\sqrt{1-\sin^2\theta}}\,\mathrm{d}\theta$$

$$=\int \frac{\mathrm{d}\theta}{\sin\theta}$$

$$=\frac{1}{2}\ln\left|\frac{1-\cos\theta}{1+\cos\theta}\right|+\text{constant} \quad \text{1M}$$

$$=\frac{1}{2}\ln\left|\frac{1-\sqrt{1-9x^2}}{1+\sqrt{1-9x^2}}\right|+\text{constant} \quad \text{1A}$$

5. $$\int (x+2)\sin x\,\mathrm{d}x=-(x+2)\cos x+\int \cos x\,\mathrm{d}x \quad \text{1M}$$

$$=-(x+2)\cos x+\sin x+\text{constant} \quad \text{1A}$$

6. $$\int x\cos(2x+1)\,\mathrm{d}x=\frac{1}{2}x\sin(2x+1)-\frac{1}{2}\int \sin(2x+1)\,\mathrm{d}x \quad \text{1M}$$

$$=\frac{1}{2}x\sin(2x+1)+\frac{1}{4}\cos(2x+1)+\text{constant} \quad \text{1A}$$

7. $$\int x^2\cos(3x-4)\,\mathrm{d}x=\frac{x^2\sin(3x-4)}{3}-\frac{2}{3}\int x\sin(3x-4)\,\mathrm{d}x \quad \text{1M}$$

$$=\frac{x^2\sin(3x-4)}{3}+\frac{2x\cos(3x-4)}{9}-\frac{2}{9}\int \cos(3x-4)\,\mathrm{d}x \quad \text{1M}$$

$$=\frac{x^2\sin(3x-4)}{3}+\frac{2x\cos(3x-4)}{9}-\frac{2\sin(3x-4)}{27}+\text{constant} \quad \text{1A}$$

8. $$\int x^{\frac{1}{4}}\ln x\,\mathrm{d}x=\frac{4}{5}x^{\frac{5}{4}}\ln x-\frac{4}{5}x^{\frac{1}{4}}\,\mathrm{d}x \quad \text{1M}$$

$$=\frac{4}{5}x^{\frac{5}{4}}\ln x-\frac{16}{25}x^{\frac{5}{4}}+\text{constant} \quad \text{1A}$$

9. $\displaystyle\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)\ln x\,\mathrm{d}x=\left(\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}\right)\ln x-\int\left(\frac{2}{3}x^{\frac{1}{2}}-2x^{-\frac{1}{2}}\right)\mathrm{d}x$ 1M

$\displaystyle=\left(\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}\right)\ln x-\frac{4}{9}x^{\frac{3}{2}}+4x^{\frac{1}{2}}+\text{constant}$ 1A

10. $\displaystyle\int x^5(\ln x)^2\,\mathrm{d}x=\frac{x^6(\ln x)^2}{6}-\frac{1}{3}\int x^5\ln x\,\mathrm{d}x$ 1M

$\displaystyle=\frac{x^6(\ln x)^2}{6}-\frac{x^6\ln x}{18}+\frac{1}{18}\int x^5\,\mathrm{d}x$ 1M

$\displaystyle=\frac{x^6(\ln x)^2}{6}-\frac{x^6\ln x}{18}+\frac{x^6}{108}+\text{constant}$ 1A

11. $\displaystyle\int(x+e)e^x\,\mathrm{d}x=(x+e)e^x-\int e^x\,\mathrm{d}x$ 1M

$=(x+e)e^x-e^x+\text{constant}$

$=e^x(x+e-1)+\text{constant}$ 1A

12. $\displaystyle\int xe^{\pi-x}\,\mathrm{d}x=-xe^{\pi-x}+\int e^{\pi-x}\,\mathrm{d}x$ 1M

$=-xe^{\pi-x}-e^{\pi-x}+\text{constant}$

$=-(x+1)e^{\pi-x}+\text{constant}$ 1A

13. $\displaystyle\int(x^2+e^x)e^x\,\mathrm{d}x=\int x^2e^x\,\mathrm{d}x+\int e^{2x}\,\mathrm{d}x$

$\displaystyle=x^2e^x-2\int xe^x\,\mathrm{d}x+\int e^{2x}\,\mathrm{d}x$ 1M

$\displaystyle=x^2e^x-2xe^x+2\int e^x\,\mathrm{d}x+\int e^{2x}\,\mathrm{d}x$ 1M

$\displaystyle=x^2e^x-2xe^x+2e^x+\frac{1}{2}e^{2x}+\text{constant}$ 1A

14. $\displaystyle\int e^x\sin(x+1)\,\mathrm{d}x=e^x\sin(x+1)-\int e^x\cos(x+1)\,\mathrm{d}x$ 1M

$\displaystyle=e^x\sin(x+1)-e^x\cos(x+1)-\int e^x\sin(x+1)\,\mathrm{d}x$ 1M

$\displaystyle2\int e^x\sin(x+1)\,\mathrm{d}x=e^x[\sin(x+1)-\cos(x+1)]+\text{constant}$ 1M

$\displaystyle\int e^x\sin(x+1)\,\mathrm{d}x=\frac{1}{2}e^x[\sin(x+1)-\cos(x+1)]+\text{constant}$ 1A

15. $\displaystyle\int e^{-3x}\cos 4x\,\mathrm{d}x=-\frac{e^{-3x}\cos 4x}{3}-\frac{4}{3}\int e^{-3x}\sin 4x\,\mathrm{d}x$ 1M

$\displaystyle=-\frac{e^{-3x}\cos 4x}{3}+\frac{4e^{-3x}\sin 4x}{9}-\frac{16}{9}\int e^{-3x}\cos 4x\,\mathrm{d}x$ 1M

$\displaystyle\frac{25}{9}\int e^{-3x}\cos 4x\,\mathrm{d}x=-\frac{e^{-3x}\cos 4x}{3}+\frac{4e^{-3x}\sin 4x}{9}+\text{constant}$ 1M

$\displaystyle\int e^{-3x}\cos 4x\,\mathrm{d}x=-\frac{3e^{-3x}\cos 4x}{25}+\frac{4e^{-3x}\sin 4x}{25}+\text{constant}$ 1A

16. $$\int x^2 \cos(\ln x)\, \mathrm{d}x = \frac{x^3 \cos(\ln x)}{3} + \frac{1}{3}\int x^2 \sin(\ln x)\, \mathrm{d}x$$ 1M

$$= \frac{x^3 \cos(\ln x)}{3} + \frac{x^3 \sin(\ln x)}{9} - \frac{1}{9}\int x^2 \cos(\ln x)\, \mathrm{d}x$$ 1M

$$\frac{10}{9}\int x^2 \cos(\ln x)\, \mathrm{d}x = \frac{x^3 \cos(\ln x)}{3} + \frac{x^3 \sin(\ln x)}{9} + \text{constant}$$ 1M

$$\int x^2 \cos(\ln x)\, \mathrm{d}x = \frac{3x^3 \cos(\ln x)}{10} + \frac{x^3 \sin(\ln x)}{10} + \text{constant}$$ 1A

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