M2REG-INDINT-2425-ASM-SET 2-MATH
Suggested solutions

1. Let $u = 2x - 1$. Then $\mathrm{d}u = 2\,\mathrm{d}x$. 1M

$$\int (x^2 - 1)\sqrt{2x - 1}\,\mathrm{d}x = \int \frac{1}{2}\left[\left(\frac{u+1}{2}\right)^2 - 1\right]\sqrt{u}\,\mathrm{d}u$$
$$= \int \left(\frac{1}{8}u^{\frac{5}{2}} + \frac{1}{4}u^{\frac{3}{2}} - \frac{1}{2}u^{\frac{1}{2}}\right)\mathrm{d}u \qquad \text{1M}$$
$$= \frac{1}{28}u^{\frac{7}{2}} + \frac{1}{10}u^{\frac{5}{2}} - \frac{1}{3}u^{\frac{3}{2}} + \text{constant}$$
$$= \frac{1}{28}(2x-1)^{\frac{7}{2}} + \frac{1}{10}(2x-1)^{\frac{5}{2}} - \frac{1}{3}(2x-1)^{\frac{3}{2}} + \text{constant} \qquad \text{1A}$$

2. Let $u = e^{2x} - e^x + x$. Then $\mathrm{d}u = (2e^{2x} - e^x + 1)\,\mathrm{d}x$. 1M

$$\int (2e^{2x} - e^x + 1)\sqrt{e^{2x} - e^x + x}\,\mathrm{d}x = \int u^{\frac{1}{2}}\,\mathrm{d}u$$
$$= \frac{2}{3}u^{\frac{3}{2}} + \text{constant}$$
$$= \frac{2}{3}(e^{2x} - e^x + x)^{\frac{3}{2}} + \text{constant} \qquad \text{1A}$$

3. Let $u = e^x + 1$. Then $\mathrm{d}u = e^x\,\mathrm{d}x$. 1M

$$\int e^x(e^x + 1)\,\mathrm{d}x = \int u\,\mathrm{d}u$$
$$= \frac{u^2}{2} + \text{constant}$$
$$= \frac{(e^x + 1)^2}{2} + \text{constant} \qquad \text{1A}$$

4. Let $u = x^4 + 1$. Then $\mathrm{d}u = 4x^3\,\mathrm{d}x$. 1M

$$\int \frac{1}{2}x^3(x^4 + 1)^3\,\mathrm{d}x = \frac{1}{8}\int u^3\,\mathrm{d}u$$
$$= \frac{1}{32}u^4 + \text{constant}$$
$$= \frac{1}{32}(x^4 + 1)^4 + \text{constant} \qquad \text{1A}$$

5. (a) $\displaystyle\int (x^2 + 1)\,\mathrm{d}x = \frac{x^3}{3} + x + \text{constant}$ 1A

(b) Let $u = \sqrt{x - 1}$. Then $\mathrm{d}u = \dfrac{1}{2\sqrt{x-1}}\,\mathrm{d}x$. 1M

$$\int \frac{x}{\sqrt{x - 1}}\,\mathrm{d}x = 2\int (u^2 + 1)\,\mathrm{d}u$$
$$= 2\left(\frac{u^3}{3} + u\right) + \text{constant}$$
$$= \frac{2}{3}(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}} + \text{constant} \qquad \text{1A}$$

6. Let $u = e^x - x + 3$. Then $\mathrm{d}u = (e^x - 1)\,\mathrm{d}x$. 1M

$$\int (e^x - 1)\cos(e^x - x + 3)\,\mathrm{d}x = \int \cos u\,\mathrm{d}u$$
$$= \sin u + \text{constant}$$
$$= \sin(e^x - x + 3) + \text{constant} \qquad \text{1A}$$

7. Let $u = \tan x$. Then $\mathrm{d}u = \sec^2 x\,\mathrm{d}x$. 1M

$$\int \frac{(\sin x)^{\frac{1}{3}}}{(\cos x)^{\frac{7}{3}}}\,\mathrm{d}x = \int (\tan x)^{\frac{1}{3}}\sec^2 x\,\mathrm{d}x$$
$$= \int u^{\frac{1}{3}}\,\mathrm{d}u$$
$$= \frac{3}{4}u^{\frac{4}{3}} + \text{constant}$$
$$= \frac{3}{4}(\tan x)^{\frac{4}{3}} + \text{constant} \qquad \text{1A}$$

8. Let $u = x^3$. Then $\mathrm{d}u = 3x^2\,\mathrm{d}x$. 1M

$$\int x^2\sec^2(x^3)\,\mathrm{d}x = \frac{1}{3}\int \sec^2 u\,\mathrm{d}u$$
$$= \frac{1}{3}\tan u + \text{constant}$$
$$= \frac{1}{3}\tan\left(x^3\right) + \text{constant} \qquad \text{1A}$$

9. Let $u = \cos 2x$. Then $\mathrm{d}u = -2\sin 2x\,\mathrm{d}x$. 1M

$$\int \sin 2x\cos^2 2x\,\mathrm{d}x = -\frac{1}{2}\int u^2\,\mathrm{d}u$$
$$= -\frac{1}{6}u^3 + \text{constant}$$
$$= -\frac{1}{6}\cos^3 2x + \text{constant} \qquad \text{1A}$$

10. Let $u = x^2 - 6x + 5$. Then $\mathrm{d}u = (2x - 6)\,\mathrm{d}x$. 1M

$$\int \frac{x-3}{(x^2 - 6x + 5)^3}\,\mathrm{d}x = \frac{1}{2}\int u^{-3}\,\mathrm{d}u$$
$$= -\frac{1}{4}u^{-2} + \text{constant}$$
$$= -\frac{1}{4}(x^2 - 6x + 5)^{-2} + \text{constant} \qquad \text{1A}$$

11. Let $u = 2 + \sin x$. Then $\mathrm{d}u = \cos x\,\mathrm{d}x$. 1M

$$\int \cos x\sqrt{2 + \sin x}\,\mathrm{d}x = \int u^{\frac{1}{2}}\,\mathrm{d}u$$
$$= \frac{2}{3}u^{\frac{3}{2}} + \text{constant}$$
$$= \frac{2}{3}(2 + \sin x)^{\frac{3}{2}} + \text{constant} \qquad \text{1A}$$

12. Let $u = x^3 - 3x - 5$. Then $\mathrm{d}u = (3x^2 - 3)\,\mathrm{d}x$. 1M

$$\int \frac{x^2 - 1}{x^3 - 3x - 5}\,\mathrm{d}x = \frac{1}{3}\int \frac{\mathrm{d}u}{u}$$

$$= \frac{1}{3}\ln|u| + \text{constant}$$

$$= \frac{1}{3}\ln|x^3 - 3x - 5| + \text{constant}$$ 1A

13. Let $u = \tan 3x$. Then $\mathrm{d}u = 3\sec^2 3x\,\mathrm{d}x$. 1M

$$\int \tan^7 3x \sec^2 3x\,\mathrm{d}x = \frac{1}{3}\int u^7\,\mathrm{d}u$$

$$= \frac{1}{24}u^8 + \text{constant}$$

$$= \frac{1}{24}\tan^8 3x + \text{constant}$$ 1A

14. Let $u = \sin x$. Then $\mathrm{d}u = \cos x\,\mathrm{d}x$. 1M

$$\int \frac{\cos x}{\sqrt{\sin x}}\,\mathrm{d}x = \int u^{-\frac{1}{2}}\,\mathrm{d}u$$

$$= 2u^{\frac{1}{2}} + \text{constant}$$

$$= 2\sqrt{\sin x} + \text{constant}$$ 1A

15. Let $u = \tan 2x$. Then $\mathrm{d}u = 2\sec^2 2x\,\mathrm{d}x$. 1M

$$\int \tan^5 2x \sec^2 2x\,\mathrm{d}x = \frac{1}{2}\int u^5\,\mathrm{d}u$$

$$= \frac{1}{12}u^6 + \text{constant}$$

$$= \frac{1}{12}\tan^6 2x + \text{constant}$$ 1A

16. Let $u = x + \frac{1}{x^2}$. Then $\mathrm{d}u = \left(1 - \frac{2}{x^3}\right)\mathrm{d}x$. 1M

$$\int \left(1 - \frac{2}{x^3}\right)\sqrt{x + \frac{1}{x^2}}\,\mathrm{d}x = \int u^{\frac{1}{2}}\,\mathrm{d}u$$

$$= \frac{2}{3}u^{\frac{3}{2}} + \text{constant}$$

$$= \frac{2}{3}\left(x + \frac{1}{x^2}\right)^{\frac{3}{2}} + \text{constant}$$ 1A

17. Let $= \sqrt{x} - 1$. Then $du = \dfrac{1}{2\sqrt{x}}\,dx$. 1M

$$\int \sqrt{\sqrt{x}-1}\,dx = \int 2\sqrt{x}\sqrt{\sqrt{x}-1}\cdot\frac{1}{2\sqrt{x}}\,dx$$

$$= \int 2(u+1)\sqrt{u}\,du$$

$$= \int (2u^{\frac{3}{2}} + 2u^{\frac{1}{2}})\,du \quad \text{1M}$$

$$= \frac{4}{5}u^{\frac{5}{2}} + \frac{4}{3}u^{\frac{3}{2}} + \text{constant}$$

$$= \frac{4}{5}(\sqrt{x}-1)^{\frac{5}{2}} + \frac{4}{3}(\sqrt{x}-1)^{\frac{3}{2}} + \text{constant} \quad \text{1A}$$

18. Let $u = \log_2 x$. Then $du = \dfrac{1}{x\ln 2}\,dx$. 1M

$$\int \frac{3(\log_2 x)^2 + 2\log_2 x - 4}{x}\,dx = \int (\ln 2)(3u^2 + 2u - 4)\,du \quad \text{1A}$$

$$= (\ln 2)(u^3 + u^2 - 4u) + \text{constant}$$

$$= (\ln 2)[(\log_2 x)^3 + (\log_2 x)^2 - 4\log_2 x] + \text{constant} \quad \text{1A}$$

19. Let $u = 1 + \dfrac{3}{x^2}$. Then $du = -\dfrac{6}{x^3}\,dx$.

$$\int \frac{\sqrt{x^2+3}}{x^4}\,dx = -\frac{1}{6}\int \left(\frac{\sqrt{x^2+3}}{x}\right)\left(-\frac{6}{x^3}\right)dx$$

$$= -\frac{1}{6}\int \sqrt{1+\frac{3}{x^2}}\left(-\frac{6}{x^3}\right)dx \quad \text{1M}$$

$$= -\frac{1}{6}\int \sqrt{u}\,du \quad \text{1A}$$

$$= -\frac{1}{9}u^{\frac{3}{2}} + \text{constant}$$

$$= -\frac{1}{9}\left(1+\frac{3}{x^2}\right)^{\frac{3}{2}} + \text{constant} \quad \text{1A}$$

20. (a) $\displaystyle\int \frac{x-1}{x}\,dx = \int \left(1 - \frac{1}{x}\right)dx$ 1M

$$= x - \ln|x| + \text{constant} \quad \text{1A}$$

(b) Let $u = x^2 + 1$. Then $du = 2x\,dx$.

$$\int \frac{x^3}{x^2+1}\,dx = \frac{1}{2}\int \frac{u-1}{u}\,du \quad \text{1M}$$

$$= \frac{1}{2}(u - \ln|u|) + \text{constant} \quad \text{1M}$$

$$= \frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right) + \text{constant} \quad \text{1A}$$

21. Let $u = e^x$. Then $\mathrm{d}u = e^x\,\mathrm{d}x$. 1M

$$\int e^x 5^{e^x}\,\mathrm{d}x = \int 5^u\,\mathrm{d}u$$

$$= \int e^{u\ln 5}\,\mathrm{d}u \qquad \text{1M}$$

$$= \frac{1}{\ln 5}e^{u\ln 5} + \text{constant}$$

$$= \frac{1}{\ln 5}5^{e^x} + \text{constant} \qquad \text{1A}$$

22. (a) $P + \dfrac{Q}{x+4} + \dfrac{R}{2x-5} = \dfrac{P(x+4)(2x-5) + Q(2x-5) + R(x+4)}{(x+4)(2x-5)}$

$$= \frac{2Px^2 + (3P+2Q+R)x + (-20P-5Q+4R)}{(x+4)(2x-5)}$$

We have $2P = 4$, $3P + 2Q + R = 11$ and $-20P - 5Q + 4R = -59$. 1M

Solving, we have $P = 2$, $Q = 3$ and $R = -1$. 1A+1A

(b) $\displaystyle\int \frac{4x^2+11x-59}{2x^2+3x-20}\,\mathrm{d}x = \int \frac{4x^2+11x-59}{(x+4)(2x-5)}\,\mathrm{d}x$

$$= \int \left(2 + \frac{3}{x+4} - \frac{1}{2x-5}\right)\mathrm{d}x \qquad \text{1M}$$

$$= 2x + 3\ln|x+4| - \frac{1}{2}\ln|2x-5| + \text{constant} \qquad \text{1A}$$

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