M2REG-INDINT-2425-ASM-SET 1-MATH
Suggested solutions

1. $\displaystyle\int \left(5x^4 + 2x^2 - 3\right) \mathrm{d}x = 5 \cdot \frac{1}{5}x^5 + 2 \cdot \frac{1}{3}x^3 - 3x + \text{constant}$

$\displaystyle = x^5 + \frac{2}{3}x^3 - 3x + \text{constant}$ 1A

2. $\displaystyle\int \sec^2 5x \, \mathrm{d}x = \frac{1}{5}\tan 5x + \text{constant}$ 1A

3. $\displaystyle\int \sqrt{x^3\sqrt{x}} \, \mathrm{d}x = \int (x^3 x^{\frac{1}{2}})^{\frac{1}{2}} \, \mathrm{d}x$

$\displaystyle = \int x^{\frac{7}{4}} \, \mathrm{d}x$

$\displaystyle = \frac{4}{11}x^{\frac{11}{4}} + \text{constant}$ 1A

4. $\displaystyle\int \left(3x^2 - \frac{2}{x}\right) \mathrm{d}x = x^3 - 2\ln|x| + \text{constant}$ 1A

5. $\displaystyle\int \sin 3x \cos 7x \, \mathrm{d}x = \int \frac{1}{2}(\sin 10x - \sin 4x) \, \mathrm{d}x$ 1M

$\displaystyle = \frac{1}{8}\cos 4x - \frac{1}{20}\cos 10x + \text{constant}$ 1A

6. $\displaystyle\int \sin 2x \cos 3x \, \mathrm{d}x = \int \frac{1}{2}(\sin 5x - \sin x) \, \mathrm{d}x$ 1M

$\displaystyle = \frac{1}{2}\cos x - \frac{1}{10}\cos 5x + \text{constant}$ 1A

7. $\displaystyle\int \cos x(1 - \tan x) \, \mathrm{d}x = \int (\cos x - \sin x) \, \mathrm{d}x$ 1M

$= \sin x + \cos x + \text{constant}$ 1A

8. $\displaystyle\int \frac{4\cos x - 4\cos x \sin^2 x}{\cot^2 x \sin^2 x} \, \mathrm{d}x = 4\int \frac{\cos x(1 - \sin^2 x)}{\cos^2 x} \, \mathrm{d}x$

$\displaystyle = 4\int \frac{\cos^3 x}{\cos^2 x} \, \mathrm{d}x$ 1M

$\displaystyle = 4\int \cos x \, \mathrm{d}x$

$= 4\sin x + \text{constant}$ 1A

9. $\displaystyle\int \frac{1}{\sin^2 x - 1} \, \mathrm{d}x = \int \left(-\frac{1}{1 - \sin^2 x}\right) \mathrm{d}x$

$\displaystyle = \int \left(-\frac{1}{\cos^2 x}\right) \mathrm{d}x$ 1M

$\displaystyle = \int \left(-\sec^2 x\right) \mathrm{d}x$

$= -\tan x + \text{constant}$ 1A

10. $\int \sin\frac{5x}{2}\cos\frac{3x}{2}\,\mathrm{d}x = \frac{1}{2}\int(\sin 4x + \sin x)\,\mathrm{d}x$ 1M

$= -\frac{1}{8}\cos 4x - \frac{1}{2}\cos x + \text{constant}$ 1A

11. $y = \int (2e^x + 1)\left(e^{-3x} - 2\right)\mathrm{d}x$

$= \int \left(2e^{-2x} - 4e^x + e^{-3x} - 2\right)\mathrm{d}x$ 1M

$= -e^{-2x} - 4e^x - \frac{1}{3}e^{-3x} - 2x + \text{constant}$ 1A

12. $\int \frac{3\sin^2 x + 6\cos^2 x}{\cos^2 x}\,\mathrm{d}x = 3\int(\tan^2 x + 2)\,\mathrm{d}x$

$= 3\int(\sec^2 x + 1)\,\mathrm{d}x$ 1M

$= 3\tan x + 3x + \text{constant}$ 1A

13. (a) $\int 8x^7\,\mathrm{d}x = x^8 + \text{constant}$ 1A

(b) $\int -\frac{1}{6}x^8\,\mathrm{d}x = -\frac{1}{54}x^9 + \text{constant}$ 1A

14. $\int \cos^2 4x\,\mathrm{d}x = \int \frac{1}{2}(\cos 8x + 1)\,\mathrm{d}x$ 1M

$= \frac{1}{16}\sin 8x + \frac{x}{2} + \text{constant}$ 1A

15. (a) $\int \frac{1}{(4x-7)^4}\,\mathrm{d}x = -\frac{1}{12}(4x-7)^{-3} + \text{constant}$ 1A

(b) $\int \frac{2}{\sqrt{2x+9}}\,\mathrm{d}x = 2\sqrt{2x+9} + \text{constant}$ 1A

16. $\int (\sin x - \cos x)^2\,\mathrm{d}x = \int (\sin^2 x - 2\sin x\cos x + \cos^2 x)\,\mathrm{d}x$

$= \int (1 - \sin 2x)\,\mathrm{d}x$ 1M

$= x + \frac{1}{2}\cos 2x + \text{constant}$ 1A

17. $\int x^3\left(x^2 - 4\right)\mathrm{d}x = \int \left(x^5 - 4x^3\right)\mathrm{d}x$ 1M

$= \frac{1}{6}x^6 - 4\cdot\frac{1}{4}x^4 + \text{constant}$

$= \frac{1}{6}x^6 - x^4 + \text{constant}$ 1A

18. (a) $\int \frac{1}{\sqrt{6x+5}}\,\mathrm{d}x = \frac{\sqrt{6x+5}}{3} + \text{constant}$ 1A

(b) $\int \frac{1}{\sqrt{3-2x}}\,\mathrm{d}x = -(3-2x)^{\frac{1}{2}} + \text{constant}$ 1A

19. $\int \left(3e^{-\frac{x}{4}} + 2\right)^2 \mathrm{d}x = \int \left(9e^{-\frac{x}{2}} + 12e^{-\frac{x}{4}} + 4\right) \mathrm{d}x$ 1m

$= -18e^{-\frac{x}{2}} - 48e^{-\frac{x}{4}} + 4x + \text{constant}$ 1A

20. (a) $\int (x-4)^5 \,\mathrm{d}x = \dfrac{(x-4)^6}{6} + \text{constant}$ 1A

(b) $\int (2x+7)^3 \,\mathrm{d}x = \dfrac{(2x+7)^4}{8} + \text{constant}$ 1A

21. $\int \dfrac{3 + 7x^6 \cos^2 x}{\cos^2 x} \,\mathrm{d}x = \int \dfrac{3}{\cos^2 x} \,\mathrm{d}x + 7\int x^6 \,\mathrm{d}x$

$= 3\int \sec^2 x \,\mathrm{d}x + 7\int x^6 \,\mathrm{d}x$ 1M

$= 3\tan x + x^7 + \text{constant}$ 1A

22. (a) $\int \dfrac{2}{3-4x} \,\mathrm{d}x = -\dfrac{1}{2}\ln|3-4x| + \text{constant}$ 1A

(b) $\int e^{-3x-2} \,\mathrm{d}x = -\dfrac{1}{3}e^{-3x-2} + \text{constant}$ 1A

23. $\int \dfrac{5e^{3x} + 4e^{2x}}{2e^{2x}} \,\mathrm{d}x = \int \left(\dfrac{5}{2}e^x + 2\right) \mathrm{d}x$ 1M

$= \dfrac{5}{2}e^x + 2x + \text{constant}$ 1A

24. $\int (3x-1)^2 \,\mathrm{d}x = \int \left(9x^2 - 6x + 1\right) \mathrm{d}x$ 1M

$= 9 \cdot \dfrac{1}{3}x^3 - 6 \cdot \dfrac{1}{2}x^2 + x + \text{constant}$

$= 3x^3 - 3x^2 + x + \text{constant}$ 1A

25. $\int \dfrac{x^2(2x^2 + 3x - 3)}{\sqrt{x}} \,\mathrm{d}x = \int (2x^{\frac{7}{2}} + 3x^{\frac{5}{2}} - 3x^{\frac{3}{2}}) \,\mathrm{d}x$ 1M

$= \dfrac{4}{9}x^{\frac{9}{2}} + \dfrac{6}{7}x^{\frac{7}{2}} - \dfrac{6}{5}x^{\frac{5}{2}} + \text{constant}$ 1A

26. $\int \left(\dfrac{2e^{2x} - 3}{e^{3x}}\right)^2 \mathrm{d}x = \int \dfrac{4e^{4x} - 12e^{2x} + 9}{e^{6x}} \,\mathrm{d}x$

$= \int (4e^{-2x} - 12e^{-4x} + 9e^{-6x}) \,\mathrm{d}x$ 1M

$= -2e^{-2x} + 3e^{-4x} - \dfrac{3}{2}e^{-6x} + \text{constant}$ 1A

27. $\int \dfrac{3e^{2x} + 5e^x - 2}{e^x + 2} \,\mathrm{d}x = \int \dfrac{(3e^x - 1)(e^x + 2)}{e^x + 2} \,\mathrm{d}x$

$= \int (3e^x - 1) \,\mathrm{d}x$ 1M

$= 3e^x - x + \text{constant}$ 1A

28. $y = \int (2+\sqrt{x})^2 \, dx$

$= \int (4 + 4\sqrt{x} + x) \, dx$ 1M

$= 4\int dx + 4\int x^{\frac{1}{2}} \, dx + \int x \, dx$

$= 4x + \frac{8}{3}x^{\frac{3}{2}} + \frac{x^2}{2} + \text{constant}$ 1A

29. $\int \frac{(2x-3)(x^2+3x-4)}{x\sqrt{x}} \, dx = \int \frac{2x^3+3x^2-17x+12}{x^{\frac{3}{2}}} \, dx$

$= \int (2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 17x^{-\frac{1}{2}} + 12x^{-\frac{3}{2}}) \, dx$ 1M

$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} - 34x^{\frac{1}{2}} - 24x^{-\frac{1}{2}} + \text{constant}$ 1A

30. $\int \frac{(3x-1)(x+2)}{\sqrt[3]{x^2}} \, dx = \int \frac{3x^2+5x-2}{x^{\frac{2}{3}}} \, dx$

$= \int \left(3x^{\frac{4}{3}} + 5x^{\frac{1}{3}} - 2x^{-\frac{2}{3}}\right) dx$ 1M

$= \frac{9}{7}x^{\frac{7}{3}} + \frac{15}{4}x^{\frac{4}{3}} - 6x^{\frac{1}{3}} + \text{constant}$ 1A

31. $\int \frac{(2x-1)(x^2+1)}{x^2} \, dx = \int \frac{2x^3-x^2+2x-1}{x^2} \, dx$

$= \int \left(2x - 1 + \frac{2}{x} - \frac{1}{x^2}\right) dx$ 1M

$= x^2 - x + 2\ln|x| + \frac{1}{x} + \text{constant}$ 1A

32. $\int (\sqrt{x} + \sqrt[3]{x})^4 \, dx = \int (x^{\frac{1}{2}} + x^{\frac{1}{3}})^4 \, dx$

$= \int \left(x^2 + 4x^{\frac{11}{6}} + 6x^{\frac{5}{3}} + 4x^{\frac{3}{2}} + x^{\frac{4}{3}}\right) dx$ 1M

$= \frac{1}{3}x^3 + \frac{24}{17}x^{\frac{17}{6}} + \frac{9}{4}x^{\frac{8}{3}} + \frac{8}{5}x^{\frac{5}{2}} + \frac{3}{7}x^{\frac{7}{3}} + \text{constant}$ 1A

33. $\int (2\sqrt{x}-1)(\sqrt[3]{x}+1) \, dx = \int (2x^{\frac{1}{2}} - 1)(x^{\frac{1}{3}} + 1) \, dx$

$= \int (2x^{\frac{5}{6}} + 2x^{\frac{1}{2}} - x^{\frac{1}{3}} - 1) \, dx$ 1M

$= \frac{12}{11}x^{\frac{11}{6}} + \frac{4}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{4}{3}} - x + \text{constant}$ 1A

34. $\int \left(x - \frac{2}{x}\right)^3 dx = \int \left(x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}\right) dx$ 1M

$= \frac{x^4}{4} - 3x^2 + 12\ln|x| + 4x^{-2} + \text{constant}$ 1A

35. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}[(x^4-1)\ln\left(x^2+1\right)] = 4x^3\ln\left(x^2+1\right) + (x^4-1)\dfrac{2x}{x^2+1}$ 1M

$= 4x^3\ln\left(x^2+1\right) + (x^2-1)(x^2+1)\dfrac{2x}{x^2+1}$

$= 2x^3 - 2x + 4x^3\ln\left(x^2+1\right)$ 1A

(b) $\displaystyle\int [2x^3 - 2x + 4x^3\ln\left(x^2+1\right)]\,\mathrm{d}x = (x^4-1)\ln\left(x^2+1\right) + \text{constant}$ 1M

$\displaystyle 2\int (x^3-x)\,\mathrm{d}x + 4\int x^3\ln\left(x^2+1\right)\mathrm{d}x = (x^4-1)\ln\left(x^2+1\right) + \text{constant}$

$\displaystyle\int x^3\ln\left(x^2+1\right)\mathrm{d}x = \frac{1}{4}(x^4-1)\ln\left(x^2+1\right) - \frac{1}{2}\int (x^3-x)\,\mathrm{d}x + \text{constant}$

$= \dfrac{1}{4}(x^4-1)\ln\left(x^2+1\right) - \dfrac{1}{8}x^4 + \dfrac{1}{4}x^2 + \text{constant}$ 1A

36. (a) $\displaystyle\int \frac{2e^{2x}+5e^x-3}{e^x+3}\,\mathrm{d}x = \int (2e^x-1)\,\mathrm{d}x$ 1M

$= 2e^x - x + \text{constant}$ 1A

(b) $\displaystyle\int \frac{e^{4x}-4}{e^{3x}-2e^x}\,\mathrm{d}x = \int \frac{(e^{2x}+2)(e^{2x}-2)}{e^x(e^{2x}-2)}\,\mathrm{d}x$

$\displaystyle= \int e^x\,\mathrm{d}x + 2\int e^{-x}\,\mathrm{d}x$ 1M

$= e^x - 2e^{-x} + \text{constant}$ 1A

37. (a) $\dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} = \tan\left(2\times\dfrac{x}{2}\right)$

$= \tan x$ 1

(b) $\displaystyle\int \frac{\cos x\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}\,\mathrm{d}x = \frac{1}{2}\int \cos x\tan x\,\mathrm{d}x$ 1M

$\displaystyle= \frac{1}{2}\int \sin x\,\mathrm{d}x$

$= -\dfrac{1}{2}\cos x + \text{constant}$ 1A

38. (a) $\displaystyle\int \frac{2x^2-3x+4}{\sqrt{x}}\,\mathrm{d}x = 2\int x^{\frac{3}{2}}\,\mathrm{d}x - 3\int x^{\frac{1}{2}}\,\mathrm{d}x + 4\int x^{-\frac{1}{2}}\,\mathrm{d}x$ 1M

$= \dfrac{4}{5}x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + \text{constant}$ 1A

(b) $\displaystyle\int \frac{x^3+6x-4\sqrt{x}}{\sqrt{x}}\,\mathrm{d}x = \int x^{\frac{5}{2}}\,\mathrm{d}x + 6\int x^{\frac{1}{2}}\,\mathrm{d}x - 4\int \mathrm{d}x$ 1M

$= \dfrac{2}{7}x^{\frac{7}{2}} + 4x^{\frac{3}{2}} - 4x + \text{constant}$ 1A

39. (a) $\displaystyle\int e^{\ln x^{-1}}\,\mathrm{d}x = \int x^{-1}\,\mathrm{d}x$ 1M

$= \ln|x| + \text{constant}$ 1A

(b) $\displaystyle\int 3^{2x}7^x\,\mathrm{d}x = \int 63^x\,\mathrm{d}x$ 1M

$= \dfrac{1}{\ln 63}63^x + \text{constant}$ 1A

40. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x^3 \ln x\right) = x^3\left(\dfrac{1}{x}\right) + (\ln x)(3x^2)$

$= x^2 + 3x^2 \ln x$ 1A

(b) $\displaystyle\int (x^2 + 3x^2 \ln x)\,\mathrm{d}x = x^3 \ln x + \text{constant}$ 1M

$\displaystyle\int x^2\,\mathrm{d}x + 3\int x^2 \ln x\,\mathrm{d}x = x^3 \ln x + \text{constant}$

$\displaystyle 3\int x^2 \ln x\,\mathrm{d}x = x^3 \ln x - \int x^2\,\mathrm{d}x + \text{constant}$

$\displaystyle\int x^2 \ln x\,\mathrm{d}x = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,\mathrm{d}x + \text{constant}$

$\displaystyle = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + \text{constant}$ 1A