M2REG-DIFF-2425-ASM-SET 3-MATH
Suggested solutions

1. $y = \sqrt{(e^x)^3}$

$= e^{\frac{3x}{2}}$

$\frac{dy}{dx} = e^{\frac{3x}{2}}\left(\frac{3}{2}\right)$ 1M

$= \frac{3}{2}e^{\frac{3x}{2}}$ 1A

2. $y = \ln\left(4x^3 + 5x^2\right)$

$\frac{dy}{dx} = \frac{12x^2 + 10x}{4x^3 + 5x^2}$ 1M

$= \frac{12x + 10}{4x^2 + 5x}$ 1A

3. $y = \log_2(2 + e^{2x})$

$= \frac{\ln(2 + e^{2x})}{\ln 2}$

$\frac{dy}{dx} = \frac{1}{\ln 2} \cdot \frac{2e^{2x}}{2 + e^{2x}}$ 1M

$= \frac{2e^{2x}}{(2 + e^{2x})\ln 2}$ 1A

4. $y = \sqrt{x}e^{-3x}$

$= x^{\frac{1}{2}}e^{-3x}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}e^{-3x} + x^{\frac{1}{2}}e^{-3x}(-3)$ 1M

$= e^{-3x}\left(\frac{1}{2\sqrt{x}} - 3\sqrt{x}\right)$ 1A

5. $\ln x + \ln y = 2x^2 + 3x(x + y)$

$\ln x + \ln y = 5x^2 + 3xy$

$\frac{1}{x} + \frac{1}{y} \times \frac{dy}{dx} = 10x + 3y + 3x\frac{dy}{dx}$ 1M

$\left(\frac{1}{y} - 3x\right)\frac{dy}{dx} = 10x + 3y - \frac{1}{x}$

$\frac{1 - 3xy}{y} \times \frac{dy}{dx} = \frac{10x^2 + 3xy - 1}{x}$

$\frac{dy}{dx} = \frac{y(10x^2 + 3xy - 1)}{x(1 - 3xy)}$ 1A

6. $\ln\left(x^2y\right) = 4x^3 - 10y$

$\frac{1}{x^2y}\left(2xy + x^2\frac{dy}{dx}\right) = 12x^2 - 10\frac{dy}{dx}$ 1M

$\left(\frac{1}{y} + 10\right)\frac{dy}{dx} = 12x^2 - \frac{2}{x}$

$\left(\frac{1+10y}{y}\right)\frac{dy}{dx} = \frac{12x^3 - 2}{x}$

$\frac{dy}{dx} = \frac{2y(6x^3-1)}{x(1+10y)}$ 1A

7. $\ln y = 2\ln x + 3\ln(3x-1) - (-x) - \frac{1}{3}\ln\left(9 + 4x^2\right)$ 1M

$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x} + \frac{3(3)}{3x-1} + 1 - \frac{1}{3}\left(\frac{8x}{9+4x^2}\right)$ 1M

$\frac{dy}{dx} = \frac{x^2e^x(3x-1)^3}{\sqrt[3]{9+4x^2}}\left[\frac{2}{x} + 1 + \frac{9}{3x-1} - \frac{8x}{3(9+4x^2)}\right]$ 1A

8. $\ln y = \frac{2}{3}\ln(3x+4) + \frac{4}{3}\ln(5+2x) - \frac{1}{3}\ln\left(2e^{3x} + 3x\right)$ 1M

$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{3}\left(\frac{3}{3x+4}\right) + \frac{4}{3}\left(\frac{2}{5+2x}\right) - \frac{1}{3}\left(\frac{6e^{3x}+3}{2e^{3x}+3x}\right)$ 1M

$\frac{dy}{dx} = \sqrt[3]{\frac{(3x+4)^2(5+2x)^4}{2e^{3x}+3x}}\left[\frac{2}{3x+4} + \frac{8}{3(5+2x)} - \frac{2e^{3x}+1}{2e^{3x}+3x}\right]$ 1A

9. $\ln y = \ln x \ln(x\tan x + e^x)$ 1M

$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x}\ln(x\tan x + e^x) + \ln x \times \frac{\tan x + x\sec^2 x + e^x}{x\tan x + e^x}$ 1M

$\frac{dy}{dx} = (x\tan x + e^x)^{\ln x}\left[\frac{(x\sec^2 x + \tan x + e^x)\ln x}{x\tan x + e^x} + \frac{\ln(x\tan x + e^x)}{x}\right]$ 1A

10. $\sin xy = \ln\left(x^2 + 2\right)$

$\cos xy\left(y + x\frac{dy}{dx}\right) = \frac{2x}{x^2+2}$ 1M

$x\cos xy\frac{dy}{dx} + y\cos xy = \frac{2x}{x^2+2}$

$x\cos xy\frac{dy}{dx} = \frac{2x - (x^2+2)y\cos xy}{x^2+2}$

$\frac{dy}{dx} = \frac{2x - (x^2+2)y\cos xy}{(x^2+2)x\cos xy}$ 1A

11. $\ln(\sin y) = \cos xy + 3$

$\frac{\cos y}{\sin y} \times \frac{dy}{dx} = -\sin xy\left(y + x\frac{dy}{dx}\right)$ 1M

$(\cot y + x\sin xy)\frac{dy}{dx} = -y\sin xy$

$\frac{dy}{dx} = -\frac{y\sin xy}{\cot y + x\sin xy}$ 1A

12. $y^3 = 2\cos(x+y)$

$3y^2\dfrac{dy}{dx} = -2\sin(x+y)\left(1+\dfrac{dy}{dx}\right)$ 1M

$[3y^2 + 2\sin(x+y)]\dfrac{dy}{dx} = -2\sin(x+y)$

$\dfrac{dy}{dx} = -\dfrac{2\sin(x+y)}{3y^2+2\sin(x+y)}$ 1A

13. $\sin x^2y - \cos y = 3x^2$

$\cos x^2y\left(2xy + x^2\dfrac{dy}{dx}\right) + \sin y\dfrac{dy}{dx} = 6x$ 1M

$(x^2\cos x^2y + \sin y)\dfrac{dy}{dx} = 6x - 2xy\cos x^2y$

$\dfrac{dy}{dx} = \dfrac{6x - 2xy\cos x^2y}{x^2\cos x^2y + \sin y}$ 1A