M2REG-DIFF-2425-ASM-SET 1-MATH
Suggested solutions

1. $\dfrac{dy}{dx} = 12(2x) - 17 + 0$
$= 24x - 17$ 1A

2. $\dfrac{dy}{dx} = -x^{-2} - 4(-2x^{-3}) + 9(-3x^{-4})$
$= -x^{-2} + 8x^{-3} - 27x^{-4}$ 1A

3. $y = \dfrac{1}{x} - \sqrt{9x^3} + \sqrt[4]{x^5} + 4x^{\frac{3}{7}}$
$= x^{-1} - 3x^{\frac{3}{2}} + x^{\frac{5}{4}} + 4x^{\frac{3}{7}}$ 1M
$\dfrac{dy}{dx} = -x^{-2} - \dfrac{9}{2}x^{\frac{1}{2}} + \dfrac{5}{4}x^{\frac{1}{4}} + \dfrac{12}{7}x^{-\frac{4}{7}}$ 1A

4. $y = 4x^3 \cdot \sqrt{x} + 9x^2 \cdot \sqrt[3]{x^4} - 5x \cdot \sqrt[5]{x^2}$
$= 4x^{\frac{7}{2}} + 9x^{\frac{10}{3}} - 5x^{\frac{7}{5}}$ 1M
$\dfrac{dy}{dx} = 14x^{\frac{5}{2}} + 30x^{\frac{7}{3}} - 7x^{\frac{2}{5}}$ 1A

5. $y = x^{-4}(2x^7 - 3x^{-1} + 9)$
$= 2x^3 - 3x^{-5} + 9x^{-4}$ 1M
$\dfrac{dy}{dx} = 6x^2 + 15x^{-6} - 36x^{-5}$ 1A

6. $y = \dfrac{3x^{-1} + 4x^5 - 7x^6}{2x^2}$
$= \dfrac{3}{2}x^{-3} + 2x^3 - \dfrac{7}{2}x^4$ 1M
$\dfrac{dy}{dx} = -\dfrac{9}{2}x^{-4} + 6x^2 - 14x^3$ 1A

7. $y = \dfrac{x^{\frac{3}{2}} - 6x^2 + 3x^{\frac{8}{3}}}{\sqrt[3]{x}}$
$= x^{\frac{7}{6}} - 6x^{\frac{5}{3}} + 3x^{\frac{7}{3}}$ 1M
$\dfrac{dy}{dx} = \dfrac{7}{6}x^{\frac{1}{6}} - 10x^{\frac{2}{3}} + 7x^{\frac{4}{3}}$ 1A

8. $y = \dfrac{x^2 \cdot \sqrt{x^3} - 3x\sqrt{x} + x^{\frac{9}{8}}}{\sqrt[4]{x^{11}}}$
$= x^{\frac{3}{4}} - 3x^{-\frac{5}{4}} + x^{-\frac{13}{8}}$ 1M
$\dfrac{dy}{dx} = \dfrac{3}{4}x^{-\frac{1}{4}} + \dfrac{15}{4}x^{-\frac{9}{4}} - \dfrac{13}{8}x^{-\frac{21}{8}}$ 1A

9. $-15 = p(2)^5 + q(2)^4 + 1$ 1M

$2p + q = -1$

$f'(x) = 5px^4 + 4qx^3$ 1A

$5p(2)^4 + 4q(2)^3 = 0$ 1M

$5p + 2q = 0$

Solving, we have $p = 2$ and $q = -5$. 1A+1A

10. $f(x) = \frac{1}{3}x^3 - \frac{1}{x}$

$= \frac{1}{3}x^3 - x^{-1}$ 1M

$f'(x) = x^2 + x^{-2}$ 1A

$2 = a^2 + a^{-2}$ 1M

$2a^2 = a^4 + 1$

$0 = a^4 - 2a^2 + 1$

$a^2 = 1$

$a = 1 \quad \text{or} \quad -1$ 1A

11. $\frac{dy}{dx} = -5(x - x^3)^{-6}(1 - 3x^2)$ 1M

$= \frac{5(3x^2 - 1)}{(x - x^3)^6}$ 1A

12. $y = (\sqrt[3]{x} + 2)^8$

$= (x^{\frac{1}{3}} + 2)^8$

$\frac{dy}{dx} = 8(x^{\frac{1}{3}} + 2)^7 \left(\frac{1}{3}x^{-\frac{2}{3}}\right)$ 1M

$= \frac{8}{3}x^{-\frac{2}{3}}(x^{\frac{1}{3}} + 2)^7$ 1A

13. $y = \left(\frac{7}{x} - 6x\sqrt{x}\right)^{-12}$

$= (7x^{-1} - 6x^{\frac{3}{2}})^{-12}$

$\frac{dy}{dx} = -12(7x^{-1} - 6x^{\frac{3}{2}})^{-13}(-7x^{-2} - 9x^{\frac{1}{2}})$ 1M

$= \frac{12(7x^{-2} + 9x^{\frac{1}{2}})}{(7x^{-1} - 6x^{\frac{3}{2}})^{13}}$ 1A

14. $y = \frac{1}{(6\sqrt[3]{x} + 5x^{\frac{3}{5}} - 2x^2)^{-4}}$

$= (6x^{\frac{1}{3}} + 5x^{\frac{3}{5}} - 2x^2)^4$

$\frac{dy}{dx} = 4(6x^{\frac{1}{3}} + 5x^{\frac{3}{5}} - 2x^2)^3(2x^{-\frac{2}{3}} + 3x^{-\frac{2}{5}} - 4x)$ 1M+1A

15. $\dfrac{dy}{dx} = 3(2x+1)^2(2)(x-4)^{\frac{1}{5}} + (2x+1)^3\left(\dfrac{1}{5}\right)(x-4)^{-\frac{4}{5}}$ 1M

$= \dfrac{1}{5}(2x+1)^2(x-4)^{-\frac{4}{5}}[30(x-4)+(2x+1)]$

$= \dfrac{(2x+1)^2(32x-119)}{5(x-4)^{\frac{4}{5}}}$ 1A

16. $y = (2x^3-3x)^{\frac{2}{3}}(x^2-1)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = \dfrac{2}{3}(2x^3-3x)^{\frac{-1}{3}}(6x^2-3)(x^2-1)^{\frac{1}{2}} + (2x^3-3x)^{\frac{2}{3}}\left(\dfrac{1}{2}\right)(x^2-1)^{-\frac{1}{2}}(2x)$ 1M

$= (2x^3-3x)^{\frac{-1}{3}}(x^2-1)^{-\frac{1}{2}}\left[2(x^2-1)(2x^2-1)+x(2x^3-3x)\right]$

$= \dfrac{6x^4-9x^2+2}{(x^2-1)^{\frac{1}{2}}(2x^3-3x)^{\frac{1}{3}}}$ 1A

17. $y = (x^2-4x+4)^3(\sqrt{8}-\sqrt{2}x)^8$

$= [(x-2)^2]^3[\sqrt{2}(2-x)]^8$

$= 16(x-2)^{14}$ 1M

$\dfrac{dy}{dx} = 224(x-2)^{13}$ 1A

$\left.\dfrac{dy}{dx}\right|_{x=3} = 224(3-2)^{13}$

$= 224$ 1A

18. $f(x) = \sqrt{(x-4)^3(x^2+4x+16)^3}$

$= \sqrt{[(x-4)(x^2+4x+16)]^3}$

$= (x^3-64)^{\frac{3}{2}}$

$f'(x) = \dfrac{3}{2}(x^3-64)^{\frac{1}{2}}(3x^2)$ 1M

$= \dfrac{9}{2}x^2(x^3-64)^{\frac{1}{2}}$

$18a^2 = \dfrac{9}{2}a^2(a^3-64)^{\frac{1}{2}}$ 1M

$\dfrac{9}{2}a^2[(a^3-64)^{\frac{1}{2}}-4] = 0$

$(a^3-64)^{\frac{1}{2}} = 4 \quad \text{or} \quad a^2 = 0 \text{ (rejected)}$

$a^3 = 80$

$a = 2\sqrt[3]{10}$ 1A

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