M2REG-DEFINT-2425-ASM-SET 1-MATH
Suggested solutions

1. $$\int_2^4 \left(x-\frac{1}{x}\right)^3 \mathrm{d}x = \int_2^4 \left[x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}\right] \mathrm{d}x$$ 1M
$$= \left[\frac{x^4}{4} - \frac{3x^2}{2} + 3\ln|x| - \frac{1}{2x^2}\right]_2^4$$
$$= \frac{1341}{32} + 3\ln 2$$ 1A

2. (a) $$\frac{A}{(x-2)^2} + \frac{B}{x^2} = \frac{Ax^2 + B(x-2)^2}{x^2(x-2)^2}$$
$$= \frac{(A+B)x^2 - 4Bx + 4B}{x^2(x-2)^2}$$
We have $A + B = 0$, $-4B = 1$ and $4B = -1$. 1M
Solving, we have $A = \frac{1}{4}$ and $B = -\frac{1}{4}$. 1A

(b) $$\int_4^6 \frac{x-1}{x^2(x-2)^2} \mathrm{d}x = \int_4^6 \left(\frac{1}{4(x-2)^2} - \frac{1}{4x^2}\right) \mathrm{d}x$$ 1M
$$= \left[-\frac{1}{4(x-2)} + \frac{1}{4x}\right]_4^6$$
$$= \frac{1}{24}$$ 1A

3. $$\int_0^2 \frac{1}{\sqrt{2x+1} - \sqrt{2x}} \mathrm{d}x = \int_0^2 \left(\frac{1}{\sqrt{2x+1} - \sqrt{2x}} \times \frac{\sqrt{2x+1} + \sqrt{2x}}{\sqrt{2x+1} + \sqrt{2x}}\right) \mathrm{d}x$$ 1M
$$= \int_0^2 (\sqrt{2x+1} + \sqrt{2x}) \mathrm{d}x$$
$$= \left[\frac{(2x+1)^{\frac{3}{2}}}{3} + \frac{(2x)^{\frac{3}{2}}}{3}\right]_0^2$$ 1A
$$= \frac{5\sqrt{5} + 7}{3}$$ 1A

4. $$\int_1^9 \frac{x-16}{\sqrt{x}-4} \mathrm{d}x = \int_1^9 \frac{(\sqrt{x}+4)(\sqrt{x}-4)}{\sqrt{x}-4} \mathrm{d}x$$ 1M
$$= \int_1^9 (\sqrt{x}+4) \mathrm{d}x$$
$$= \left[\frac{2}{3}x^{\frac{3}{2}} + 4x\right]_1^9$$
$$= \frac{148}{3}$$ 1A

5. $$\int_{-\pi}^{\frac{\pi}{2}} \frac{e^{2x} - \pi e^x - 2\pi^2}{e^x + \pi}\,\mathrm{d}x = \int_{-\pi}^{\frac{\pi}{2}} \frac{e^x(e^x + \pi) - 2\pi(e^x + \pi)}{e^x + \pi}\,\mathrm{d}x$$

$$= \int_{-\pi}^{\frac{\pi}{2}} (e^x - 2\pi)\,\mathrm{d}x \qquad \text{1M}$$

$$= \left[e^x - 2\pi x\right]_{-\pi}^{\frac{\pi}{2}}$$

$$= e^{\frac{\pi}{2}} - e^{-\pi} - 3\pi^2 \qquad \text{1A}$$

6. $$\int_0^2 (3 - e^{-2x})^2\,\mathrm{d}x = \int_0^2 (9 - 6e^{-2x} + e^{-4x})\,\mathrm{d}x \qquad \text{1M}$$

$$= \left[9x + 3e^{-2x} - \frac{1}{4}e^{-4x}\right]_0^2$$

$$= \frac{61}{4} + 3e^{-4} - \frac{1}{4}e^{-8} \qquad \text{1A}$$

7. $$\int_0^{\frac{\pi}{4}} \sin^4 x\,\mathrm{d}x = \frac{1}{4}\int_0^{\frac{\pi}{4}} (1 - \cos 2x)^2\,\mathrm{d}x \qquad \text{1M}$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} (1 - 2\cos 2x + \cos^2 2x)\,\mathrm{d}x$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \left[1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right]\mathrm{d}x \qquad \text{1M}$$

$$= \frac{1}{4}\left[\frac{3x}{2} - \sin 2x + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left(\frac{3\pi}{8} - 1 + 0\right) - \frac{1}{4}(0)$$

$$= \frac{3\pi}{32} - \frac{1}{4} \qquad \text{1A}$$

8. $$\int_0^{\frac{\pi}{6}} \cos 6x \cos 2x\,\mathrm{d}x = \frac{1}{2}\int_0^{\frac{\pi}{6}} (\cos 4x + \cos 8x)\,\mathrm{d}x \qquad \text{1M}$$

$$= \frac{1}{2}\left[\frac{\sin 4x}{4} + \frac{\sin 8x}{8}\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\sqrt{3}}{32} \qquad \text{1A}$$

9. $$\int_0^{\frac{\pi}{8}} \sin 5x \sin 3x\,\mathrm{d}x = \frac{1}{2}\int_0^{\frac{\pi}{8}} (\cos 2x - \cos 8x)\,\mathrm{d}x \qquad \text{1M}$$

$$= \frac{1}{2}\left[\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right]_0^{\frac{\pi}{8}}$$

$$= \frac{\sqrt{2}}{8} \qquad \text{1A}$$

10. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\cos 2x}\,\mathrm{d}x = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+(2\cos^2 x-1)}\,\mathrm{d}x$ 1M

$\displaystyle = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x\,\mathrm{d}x$ 1A

$\displaystyle = \frac{1}{2}\Big[\tan x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\displaystyle = \frac{\sqrt{3}}{3}$ 1A

11. $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{1-\cos 2x}{1+\cos 2x}\,\mathrm{d}x = \int_{0}^{\frac{\pi}{4}} \frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}\,\mathrm{d}x$ 1M

$\displaystyle = \int_{0}^{\frac{\pi}{4}} \tan^2 x\,\mathrm{d}x$

$\displaystyle = \int_{0}^{\frac{\pi}{4}} (\sec^2 x-1)\,\mathrm{d}x$ 1M

$\displaystyle = \Big[\tan x - x\Big]_{0}^{\frac{\pi}{4}}$

$\displaystyle = 1-\frac{\pi}{4}$ 1A

12. $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2(1+\sin x)}\,\mathrm{d}x + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos 2x}{2(1+\sin x)}\,\mathrm{d}x$

$\displaystyle = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1+(1-2\sin^2 x)}{2(1+\sin x)}\,\mathrm{d}x$ 1M

$\displaystyle = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(1+\sin x)(1-\sin x)}{1+\sin x}\,\mathrm{d}x$

$\displaystyle = \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (1-\sin x)\,\mathrm{d}x$ 1A

$\displaystyle = \Big[x+\cos x\Big]_{-\frac{\pi}{6}}^{\frac{\pi}{2}}$

$\displaystyle = \frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ 1A

13. $\displaystyle\int_{1}^{4} \frac{3x}{1+e^x}\,\mathrm{d}x - 3\int_{4}^{1} \frac{xe^x}{1+e^x}\,\mathrm{d}x$

$\displaystyle = \int_{1}^{4} \frac{3x}{1+e^x} + 3\int_{1}^{4} \frac{xe^x}{1+e^x}\,\mathrm{d}x$ 1M

$\displaystyle = \int_{1}^{4} \frac{3x(1+e^x)}{1+e^x}\,\mathrm{d}x$

$\displaystyle = \int_{1}^{4} 3x\,\mathrm{d}x$

$\displaystyle = \left[\frac{3x^2}{2}\right]_{1}^{4}$

$\displaystyle = \frac{45}{2}$ 1A

14. $$\int_0^4 f(x)\,\mathrm{d}x + \int_5^2 f(x)\,\mathrm{d}x = \int_0^2 f(x)\,\mathrm{d}x + \int_2^4 f(x)\,\mathrm{d}x - \int_2^5 f(x)\,\mathrm{d}x \quad \text{1M}$$
$$= \int_0^2 f(x)\,\mathrm{d}x - \left(\int_4^2 f(x)\,\mathrm{d}x + \int_2^5 f(x)\,\mathrm{d}x\right)$$
$$= \int_0^2 f(x)\,\mathrm{d}x - \int_4^5 f(x)\,\mathrm{d}x \quad \text{1M}$$
$$= k \quad \text{1A}$$

15. (a) $$\int_{-1}^1 p(x)\,\mathrm{d}x = \int_{-1}^4 p(x)\,\mathrm{d}x + \int_4^1 p(x)\,\mathrm{d}x$$
$$= \int_{-1}^4 p(x)\,\mathrm{d}x - \int_1^4 p(x)\,\mathrm{d}x \quad \text{1M}$$
$$= 2 \quad \text{1A}$$

(b) $$\int_{-1}^1 5[p(x) - 3q(x)]\,\mathrm{d}x = 5\int_{-1}^1 p(x) - 3\int_{-1}^1 q(x)\,\mathrm{d}x \quad \text{1M}$$
$$= 5(2) - 15(-5)$$
$$= 85 \quad \text{1A}$$

16. (a) $$\frac{1}{4}\int_{-3}^2 f(x)\,\mathrm{d}x = \frac{1}{4}\left[\int_{-3}^{-1.5} f(x)\,\mathrm{d}x + \int_{-1.5}^2 f(x)\,\mathrm{d}x\right] \quad \text{1M}$$
$$= \frac{1}{4}(2+4)$$
$$= \frac{3}{2} \quad \text{1A}$$

(b) $$\int_{-1}^{-1} f(x)\,\mathrm{d}x = 0 \quad \text{1A}$$

(c) $$\int_{-3}^{-1.5}\left[f(t) + \frac{1}{t}\right]\mathrm{d}t = \int_{-3}^{-1.5} f(x)\,\mathrm{d}x + \Big[\ln|t|\Big]_{-3}^{-1.5} \quad \text{1A}$$
$$= 2 - \ln 2 \quad \text{1A}$$

17. (a) $$\int_4^2 \frac{2f(x)}{f(x) - 2g(x)}\,\mathrm{d}x = -2\int_2^4 \frac{f(x)}{f(x) - 2g(x)}\,\mathrm{d}x$$
$$= -2(10)$$
$$= -20 \quad \text{1A}$$

(b) $$\int_2^4 \frac{g(t)}{f(t) - 2g(t)}\,\mathrm{d}t = \int_2^4 \frac{g(x)}{f(x) - 2g(x)}\,\mathrm{d}x$$
$$= -\frac{1}{2}\int_2^4 \left[1 - \frac{f(x)}{f(x) - 2g(x)}\right]\mathrm{d}x \quad \text{1M}$$
$$= -\frac{1}{2}\Big[x\Big]_2^4 + \frac{1}{2}\int_2^4 \frac{f(x)}{f(x) - 2g(x)}\,\mathrm{d}x$$
$$= -\frac{1}{2}(4-2) + \frac{1}{2}(10)$$
$$= 4 \quad \text{1A}$$

END OF PAPER