M2REG-AOD-2425-ASM-SET 5-MATH
Suggested solutions

1. (a) $\tan\theta = \tan(\angle APO - \angle BPO)$

$$= \frac{\tan\angle APO - \tan\angle BPO}{1 + \tan\angle APO\tan\angle BPO} \qquad \text{1M}$$

$$= \frac{\frac{16}{y} - \frac{10}{y}}{1 + \frac{16}{y}\cdot\frac{10}{y}}$$

$$= \frac{6y}{y^2 + 160} \qquad \text{1A}$$

(b) $$\tan\theta = \frac{6y}{y^2 + 160}$$

$$\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}y} = \frac{6(y^2 + 160) - 6y(2y)}{(y^2 + 160)^2} \qquad \text{1M}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}y} = \frac{6(160 - y^2)\cos^2\theta}{(y^2 + 160)^2} \qquad \text{1A}$$

When $\frac{\mathrm{d}\theta}{\mathrm{d}y} = 0$, $y = \sqrt{160}$. 1A

y	$0 < y < \sqrt{160}$	$y > \sqrt{160}$
$\frac{\mathrm{d}\theta}{\mathrm{d}y}$	$+$	$-$

1M

θ is maximum at $y = \sqrt{160}$.

$$\text{Required value} = \tan^{-1}\frac{6\sqrt{160}}{(\sqrt{160})^2 + 160}$$

$$\approx 0.233\,\text{rad} \qquad \text{1A}$$

2. $h^2 + r^2 = 4^2$ 1M

$$r^2 = 16 - h^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi(16 - h^2)h$$ 1M

$$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{3}\pi(16 - 3h^2)$$ 1M

When $\frac{\mathrm{d}V}{\mathrm{d}h} = 0$, $h = \frac{4}{\sqrt{3}}$ or $-\frac{4}{\sqrt{3}}$ (rejected).

h	$0 < h < \frac{4}{\sqrt{3}}$	$\frac{4}{\sqrt{3}} < h < 4$
$\frac{\mathrm{d}V}{\mathrm{d}h}$	$+$	$-$

1M

V is maximum at $h = \frac{4}{\sqrt{3}}$.

Maximum volume $= \frac{1}{3}\pi\left[16\left(\frac{4}{\sqrt{3}}\right)^2 - \left(\frac{4}{\sqrt{3}}\right)^3\right]$

$$= \frac{128\sqrt{3}\pi}{27}$$ 1A

3. Let the x-coordinate of P be p, and the area of $\triangle MNP$ be A.

$$A = \frac{1}{2}(p)\left(3\sqrt{1 - \frac{p^2}{4}}\right)$$ 1M

$$= \frac{3p}{4}\sqrt{4 - p^2}$$

$$\frac{\mathrm{d}A}{\mathrm{d}p} = \frac{3}{4}(4 - p^2)^{\frac{1}{2}} + \frac{3p}{8}(4 - p^2)^{-\frac{1}{2}}(-2p)$$ 1M

$$= \frac{3}{2}(4 - p^2)^{-\frac{1}{2}}(2 - p^2)$$

When $\frac{\mathrm{d}A}{\mathrm{d}p} = 0$, $p = \sqrt{2}$. 1A

p	$0 < p < \sqrt{2}$	$\sqrt{2} < p < 2$
$\frac{\mathrm{d}A}{\mathrm{d}p}$	$+$	$-$

1M

A is maximum at $p = \sqrt{2}$.

Maximum area $= \frac{3\sqrt{2}}{4}\sqrt{4 - (\sqrt{2})^2}$

$$= \frac{3}{2}$$ 1A

4. (a) Let T hours be the time taken.

$$T = \frac{x}{8} + \frac{\sqrt{(6-x)^2 + 4^2}}{3} \qquad \text{1A}$$

$$= \frac{x}{8} + \frac{\sqrt{x^2 - 12x + 52}}{3}$$

(b) $$\frac{dT}{dx} = \frac{1}{8} + \frac{1}{6}(x^2 - 12x + 52)^{-\frac{1}{2}}(2x - 12) \qquad \text{1M}$$

$$= \frac{1}{8} + \frac{2x - 12}{6(x^2 - 12x + 52)^{\frac{1}{2}}}$$

When $\frac{dT}{dx} = 0$,

$$\frac{1}{8} + \frac{2x - 12}{6\sqrt{x^2 - 12x + 52}} = 0 \qquad \text{1M}$$

$$(8x - 48)^2 = 3^2(x^2 - 12x + 52)$$

$$55x^2 - 660x + 1836 = 0$$

$$x = \frac{330 - 12\sqrt{55}}{55} \quad \text{or} \quad \frac{330 + 12\sqrt{55}}{55} \text{ (rejected)} \qquad \text{1A}$$

x	$0 < x < \frac{330 - 12\sqrt{55}}{55}$	$\frac{330 - 12\sqrt{55}}{55} < x < 6$	
$\frac{dT}{dx}$	$-$	$+$	1M

T attains minimum when $x = \frac{330 - 12\sqrt{55}}{55} \approx 4.38$.

Minimum time taken $= \frac{x}{8} + \frac{\sqrt{x^2 - 12x + 52}}{3}$

$\approx 1.99\text{ h} > 1.5\text{ h}$ 1A

The claim is disagreed. 1A

5. (a) $HB = \sqrt{x^2 + (\sqrt{3})^2}$ 1M

$= \sqrt{x^2 + 3}$ m

$\frac{BC}{BK} = \frac{AB}{HB}$

$\frac{x + y}{x} = \frac{9}{\sqrt{x^2 + 3}}$ 1M

$y = \frac{9x}{\sqrt{x^2 + 3}} - x$ 1

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9(x^2 + 3)^{\frac{1}{2}} - \frac{9x}{2}(x^2 + 3)^{-\frac{1}{2}}(2x)}{x^2 + 3} - 1$ 1M

$= \frac{9[(x^2 + 3) - x^2]}{(x^2 + 3)^{\frac{3}{2}}} - 1$

$= \frac{27}{(x^2 + 3)^{\frac{3}{2}}} - 1$

When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$,

$(x^2 + 3)^{\frac{3}{2}} = 27$

$x^2 + 3 = 9$

$x = \pm\sqrt{6}$

x	$0 < x < \sqrt{6}$	$\sqrt{6} < x < \sqrt{78}$
x	+	−

1M

y attains its maximum at $x = \sqrt{6}$.

Required value $= \frac{9(\sqrt{6})}{\sqrt{(\sqrt{6})^2 + 3}} - \sqrt{6}$

$= 2\sqrt{6}$ 1A

(c) (i) $\frac{AC}{HK} = \frac{AB}{HB}$

$AC = \frac{9\sqrt{3}}{\sqrt{x^2 + 3}}$

$S = \frac{\left(\frac{9\sqrt{3}}{\sqrt{x^2+3}} + \sqrt{3}\right)\left(\frac{9x}{\sqrt{x^2+3}} - x\right)}{2}$

$= \frac{81\sqrt{3}x}{2(x^2 + 3)} - \frac{\sqrt{3}x}{2}$ 1

(ii) $\frac{\mathrm{d}S}{\mathrm{d}x} = \frac{81\sqrt{3}}{2}\left[\frac{(x^2 + 3) - x(2x)}{(x^2 + 3)^2}\right] - \frac{\sqrt{3}}{2}$ 1M

$= \frac{81\sqrt{3}(3 - x^2)}{2(x^2 + 3)^2} - \frac{\sqrt{3}}{2}$

$\left.\frac{\mathrm{d}S}{\mathrm{d}x}\right|_{x=\sqrt{6}} = \frac{81\sqrt{3}[3 - (\sqrt{6})^2]}{2[(\sqrt{6})^2 + 3]^2} - \frac{\sqrt{3}}{2}$

$= -2\sqrt{3} \neq 0$ 1M

Thus, S is not a maximum when y attains its maximum. 1A

6. (a) $100e^{-2.5} = [-(25)^2 + A(25) + B]e^{-\frac{25}{10}}$ 1M

$25A + B = 725$

$75e^{-3} = [-(30)^2 + A(30) + B]e^{-\frac{30}{10}}$

$30A + B = 975$

Solving, we have $A = 50$ and $B = -525$. 1A+1A

(b) (i) $\dfrac{dW(x)}{dx} = (-x^2 + 50x - 525)e^{-\frac{x}{10}}\left(-\dfrac{1}{10}\right) + e^{-\frac{x}{10}}(-2x + 50)$ 1M

$= \dfrac{1}{10}(x^2 - 70x + 1025)e^{-\frac{x}{10}}$ 1A

(ii) When $\dfrac{dW(x)}{dx} = 0$,

$x^2 - 70x + 1025 = 0$ 1M

$x = 35 - 10\sqrt{2}$ or $35 + 10\sqrt{2}$ (rejected) 1A

x	$15 < x < 35 - 10\sqrt{2}$	$35 - 10\sqrt{2} < x < 35$
$\dfrac{dW(x)}{dx}$	+	−

1M

$W(x)$ attains its maximum at $x = 35 - 10\sqrt{2}$.

Maximum weight $= [-(35 - 10\sqrt{2})^2 + 50(35 - 10\sqrt{2}) - 525]e^{-\frac{35-10\sqrt{2}}{10}}$

≈ 10.29 t 1A

(c) $P(x) = \dfrac{1000W(x)}{x}$

$\dfrac{dP(x)}{dx} = 1000\left[\dfrac{xW'(x) - W(x)}{x^2}\right]$ 1M

$\left.\dfrac{dP(x)}{dx}\right|_{x=35-10\sqrt{2}} = 1000\left[\dfrac{(35 - 10\sqrt{2})W'(35 - 10\sqrt{2}) - W(35 - 10\sqrt{2})}{(35 - 10\sqrt{2})^2}\right]$

$= 1000\left[\dfrac{-W(35 - 10\sqrt{2})}{(35 - 10\sqrt{2})^2}\right]$

$\neq 0$ 1M

$P(x)$ does not attains its maximum when $W(x)$ attains its maximum.

The claim is incorrect. 1A

 END OF PAPER