M2REG-AOD-2425-ASM-SET 4-MATH
Suggested solutions

1. (a) Vertical asymptote is $x = 1$. 1A

$$f(x) = \frac{3x^2 - 3x + 1}{x - 1}$$
$$= 3x + \frac{1}{x-1} \quad \text{1M}$$

Oblique asymptote is $y = 3x$. 1A

(b) $f'(x) = 3 - \dfrac{1}{(x-1)^2}$ 1A

$f'(2) = 3 - \dfrac{1}{(2-1)^2}$

$= 2$ 1A

Required equation is

$$y - 7 = 2(x - 2)$$
$$2x - y + 3 = 0 \quad \text{1A}$$

2. (a) $9 = 8a + 4b + 1$ 1M

$\dfrac{dy}{dx} = 3ax^2 + 2bx$

$0 = 3a(4) + 2b(2)$ 1M

Solving, we have $a = -2$ and $b = 6$. 1A

(b) $\dfrac{dy}{dx} = -6x^2 + 12x$

$\dfrac{d^2y}{dx^2} = -12x + 12$

$\left.\dfrac{d^2y}{dx^2}\right|_{(2,\ 9)} = -12(2) + 12 = -12 < 0$ 1M

Thus, $(2,\ 9)$ is a maximum point. The claim is agreed. 1A

(c) When $\dfrac{d^2y}{dx^2} = 0$, $x = 1$.

x	$x < 1$	$x > 1$
$\dfrac{d^2y}{dx^2}$	$+$	$-$

1M

Point of inflexion is $(1,\ 5)$. 1A

3. (a) Vertical asymptote is $x = 4$. 1A

$$f(x) = \frac{x^2 + 2x - 8}{x - 4}$$

$$= x + 6 + \frac{16}{x - 4}$$ 1M

Oblique asymptote is $y = x + 6$. 1A

(b) $$f'(x) = \frac{(x-4)(2x+2) - (x^2 + 2x - 8)}{(x-4)^2}$$ 1M

$$= \frac{x^2 - 8x}{(x-4)^2}$$ 1A

(c) When $f'(x) = 0$, $x = 0$ or 8. 1A

x	$x < 0$	$0 < x < 4$	$4 < x < 8$	$x > 8$
$f'(x)$	$+$	$-$	$-$	$+$

1M

The maximum point is $(0,\ 2)$ and the minimum point is $(8,\ 18)$. 1A+1A

(d) $$\frac{x^2 + 2x - 8}{x - 4} = 0$$

$$x^2 + 2x - 8 = 0$$

$$x = -4 \quad \text{or} \quad 2$$ 1A

Slope of tangent $= \dfrac{(-4)^2 - 8(-4)}{(-4-4)^2}$

$= \dfrac{3}{4}$ 1A

Required equation is

$$y - 0 = \frac{3}{4}(x + 4)$$

$$3x - 4y + 12 = 0$$ 1A

4. (a) $5(1) + 9y - 11 = 0$ 1M

$$y = \frac{2}{3}$$

The point of contact is $\left(1,\ \frac{2}{3}\right)$.

$$\frac{2}{3} = \frac{1+a}{1+b}$$ 1M

$$a = \frac{2b-1}{3}$$

$$\frac{dy}{dx} = \frac{(x^2+bx)-(x+a)(2x+b)}{(x^2+bx)^2}$$ 1M

$$= \frac{-x^2-2ax-ab}{(x^2+bx)^2}$$

$$\frac{-5}{9} = \frac{-1-2a-ab}{(1+b)^2}$$ 1M

$$9 + 18a + 9ab = 5(b+1)^2$$

$$9 + 18\left(\frac{2b-1}{3}\right) + 9\left(\frac{2b-1}{3}\right)b = 5(b^2+2b+1)$$

$$b^2 - b - 2 = 0$$

$b = 2$ or -1 (rejected) 1A

$a = \frac{2(2)-1}{3} = 1$ 1A

(b) $$\frac{dy}{dx} = -\frac{x^2+2x+2}{(x^2+2x)^2}$$

$$-\frac{x^2+2x+2}{(x^2+2x)^2} = 0$$ 1M

$$x^2 + 2x + 2 = 0$$

$\Delta = 2^2 - 4(1)(2) = -4 < 0$

The equation has no real roots.

The curve has no stationary point. 1A

(c) x-intercept $= -1$; y-intercept does not exist. 1A

(d) $$\frac{d^2y}{dx^2} = -\frac{(2x+2)(x^2+2x)^2-(x^2+2x+2)(2)(x^2+2x)(2x+2)}{(x^2+2x)^4}$$ 1M

$$= \frac{2(x+1)(x^2+2x+4)}{(x^2+2x)^3}$$

When $\frac{d^2y}{dx^2} = 0$,

$$(x+1)(x^2+2x+4) = 0$$

$x = -1$ or $x^2 + 2x + 4 = 0$

For $x^2 + 2x + 4 = 0$, $\Delta = 2^2 - 4(1)(4) = -12 < 0$.

The equation has no real roots.

x	$x < -1$	$-1 < x < 0$	$x > 0$	
$\dfrac{d^2y}{dx^2}$	$+$	$-$	$+$	1M

Point of inflexion is $(-1,\ 0)$. 1A

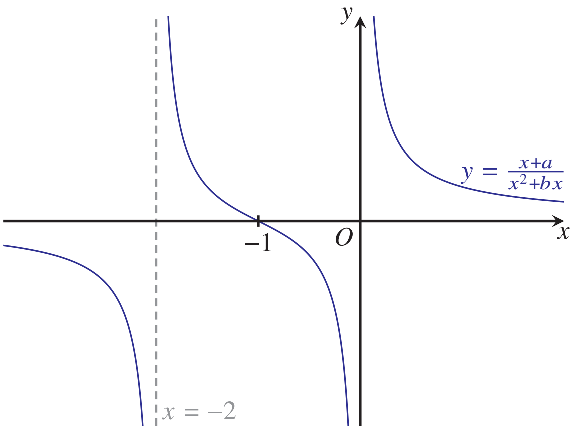
(e) Vertical asymptotes are $x = 0$ and $x = -2$. 1A

$$y = \frac{x+1}{x^2+2x}$$

Horizontal asymptote is $y = 0$. 1A

(f) (Correct shape) 1A

(All correct) 1A



5. (a) x-intercept $= -1$ 1A

y-intercept $= -1$ 1A

(b) $\dfrac{dy}{dx} = 2(x+1)(x-1)^{-1} - (x+1)^2(x-1)^{-2}$ 1M

$= \dfrac{(x-3)(x+1)}{(x-1)^2}$

When $\dfrac{dy}{dx} = 0$, $x = 3$ or -1. 1M

x	$x < -1$	$-1 < x < 1$	$1 < x < 3$	$x > 3$
$\dfrac{dy}{dx}$	+	−	−	+

1M

Maximum point is $(-1,\ 0)$ and minimum point is $(3,\ 8)$. 1A

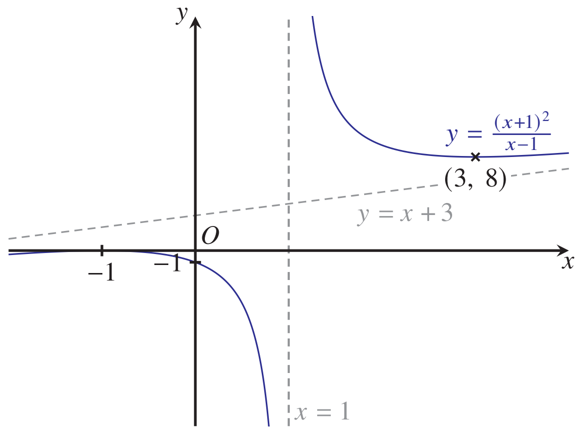
(c) Vertical asymptote is $x = 1$. 1A

$$y = x + 3 + \frac{4}{x-1}$$ 1M

Oblique asymptote is $y = x + 3$. 1A

(d) (Correct shape) 1A

(All correct) 1A



6. (a) Vertical asymptote is $x = 1$. 1A

$$f(x) = x + 2 + \frac{3x - 2}{(x - 1)^2} \qquad \text{1M}$$

Oblique asymptote is $y = x + 2$. 1A

(b) $$f'(x) = 1 + \frac{3(x-1)^2 - (3x-2)(2)(x-1)}{(x-1)^4} \qquad \text{1M}$$

$$= 1 + \frac{-3x+1}{(x-1)^3} \qquad \text{1A}$$

$$1 + \frac{-3x+1}{(x-1)^3} = 0$$

$$(x-1)^3 - 3x + 1 = 0$$

$$x^2(x-3) = 0$$

$$x = 0 \quad \text{or} \quad 3 \qquad \text{1A}$$

(c)

x	$x < 0$	$0 < x < 1$	$1 < x < 3$	$x > 3$
$f'(x)$	$+$	$+$	$-$	$+$

1M

Minimum point is $\left(3,\ \frac{27}{4}\right)$ and no maximum point. 1A

(d) $$f''(x) = \frac{-3(x-1)^3 - (-3x+1)(3)(x-1)^2}{(x-1)^3}$$

$$= \frac{6x}{(x-1)^4} \qquad 1$$

(e) When $f''(x) = 0$, $x = 0$.

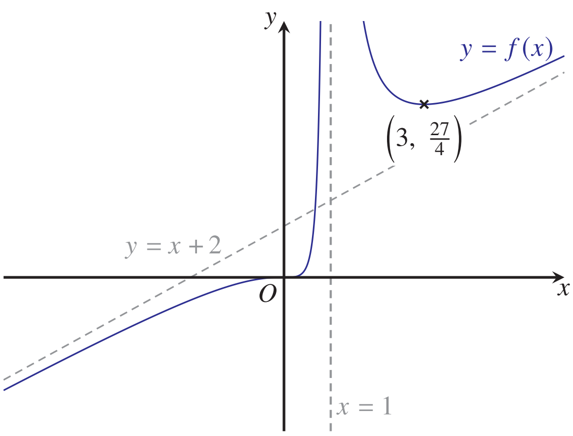
x	$x < 0$	$0 < x < 1$	$x > 1$
$f''(x)$	$-$	$+$	$+$

1M

Point of inflexion is $(0,\ 0)$. 1A

(f) (Correct shape) 1A

(All correct) 1A



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