M2REG-AOD-2425-ASM-SET 3-MATH
Suggested solutions

1. (a) 4 — 1A

(b) $y = \dfrac{x^2 - 5x}{x + 4}$

$= x - 9 + \dfrac{36}{x + 4}$ — 1M

Oblique asymptote is $y = x - 9$. — 1A

(c) $\dfrac{dy}{dx} = 1 - \dfrac{36}{(x + 4)^2}$ — 1A

$= \dfrac{(x + 10)(x - 2)}{(x + 4)^2}$

When $\dfrac{dy}{dx} = 0$, $x = -10$ or 2. — 1A

x	$x < -10$	$-10 < x < -4$	$-4 < x < 2$	$x > 2$
$\dfrac{dy}{dx}$	$+$	$-$	$-$	$+$

1M

There is only one maximum point at $x = -10$.

The claim is agreed. — 1A

2. (a) $g'(x) = \dfrac{-k(2x - 6)}{(x^2 - 6x + 21)^2}$ — 1A

When $g'(x) = 0$, $x = 3$.

x	$x < 3$	$x > 3$
$g'(x)$	$-$	$+$

1M

Minimum point is $\left(3,\ \dfrac{k}{12}\right)$. — 1A

(b) No vertical asymptote.

Horizontal asymptote is $y = 0$. — 1A

(c) $\dfrac{k}{12} = -2$ — 1M

$k = -24$

$g'(x) = \dfrac{48x - 144}{(x^2 - 6x + 21)^2}$

$g''(x) = \dfrac{(x^2 - 6x + 21)^2(48) - (48x - 144)(2)(x^2 - 6x + 21)(2x - 6)}{(x^2 - 6x + 21)^4}$ — 1M

$= \dfrac{-144(x - 1)(x - 5)}{(x^2 - 6x + 21)^3}$

When $g''(x) = 0$, $x = 1$ or 5.

x	$x < 1$	$1 < x < 5$	$x > 5$
$g''(x)$	$-$	$+$	$-$

1M

Points of inflexion are $\left(1,\ -\dfrac{3}{2}\right)$ and $\left(5,\ -\dfrac{3}{2}\right)$. — 1A+1A

3. (a) (i) $f'(x) = \dfrac{(2x-4)(x^2+1)-(x^2-4x+1)(2x)}{(x^2+1)^2}$ 1M

$= \dfrac{4x^2-4}{(x^2+1)^2}$ 1A

(ii) $f'(x) = \dfrac{4(x+1)(x-1)}{(x^2+1)^2}$.

When $f'(x) = 0$, $x = \pm 1$. 1M

x	$x < -1$	$-1 < x < 1$	$x > 1$
$f'(x)$	$+$	$-$	$+$

1M

$f(-1) = 3$ and $f(1) = -1$.

Thus, the maximum and minimum value of $f(x)$ are 3 and -1 respectively. 1

(b) $f''(x) = \dfrac{8x(x^2+1)^2-(4x^2-4)(2)(x^2+1)(2x)}{(x^2+1)^4}$ 1M

$= \dfrac{8x(x^2+1)-4x(4x^2-4)}{(x^2+1)^3}$

$= \dfrac{-8x^3+24x}{(x^2+1)^3}$ 1A

$= \dfrac{-8x(x^2-3)}{(x^2+1)^3}$

When $f''(x) = 0$, $x = 0$ or $\pm\sqrt{3}$.

x	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
$f''(x)$	$+$	$-$	$+$	$-$

1M

$f(-\sqrt{3}) = 1+\sqrt{3}$, $f(0) = 1$ and $f(\sqrt{3}) = 1-\sqrt{3}$

The points of inflexion are $\left(-\sqrt{3},\ 1+\sqrt{3}\right)$, $(0,\ 1)$ and $\left(\sqrt{3},\ 1-\sqrt{3}\right)$. 1A+1A

(c) There are no vertical asymptotes. 1A

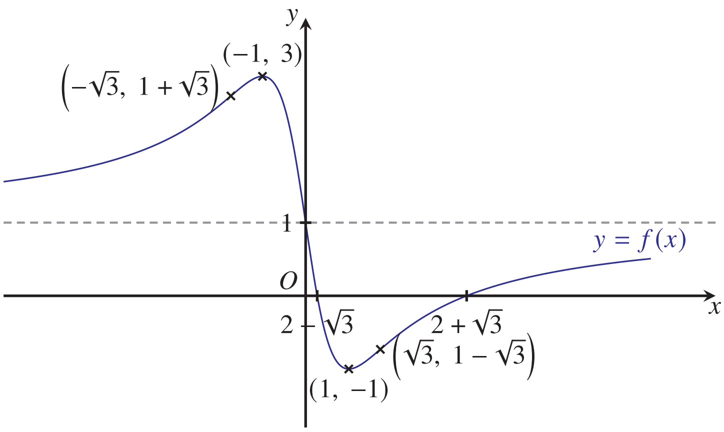
$f(x) = 1 - \dfrac{4x}{x^2+1}$. 1M

Horizontal asymptote is $y = 1$. 1A

(d) (Correct shape) 1A

(Correct points and asymptotes) 1A

(All correct) 1A



4. (a) $f'(x) = \dfrac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$ 1M

$= \dfrac{2-2x^2}{(x^2+1)^2}$ 1A

$f''(x) = \dfrac{(x^2+1)^2(-4x) - (2-2x^2)(2)(x^2+1)(2x)}{(x^2+1)^4}$

$= \dfrac{-4x(x^2+1+2-2x)}{(x^2+1)^3}$

$= \dfrac{4x(x^2-3)}{(x^2+1)^3}$ 1A

(b) When $f'(x) = 0$, $x = \pm 1$. 1A

$f''(1) = -1 < 0$ and $f''(-1) = 1 > 0$ 1M

The maximum point is $(1,\ 1)$ and the minimum point is $(-1,\ -1)$. 1A

(c) When $f''(x) = 0$, $x = 0$ or $\pm\sqrt{3}$.

x	$x < -\sqrt{3}$	$-\sqrt{3} < x < 0$	$0 < x < \sqrt{3}$	$x > \sqrt{3}$
$f''(x)$	$-$	$+$	$-$	$+$

1M

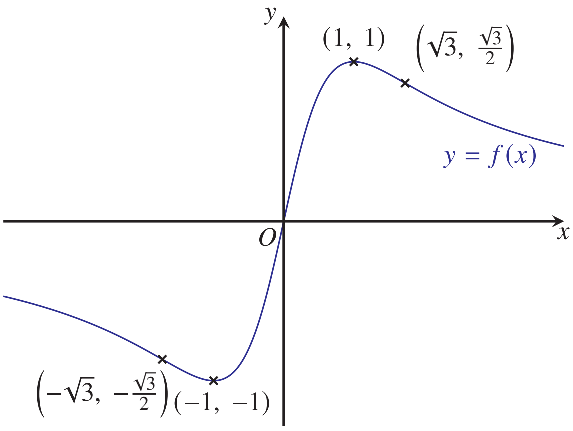
The points of inflexion are $\left(-\sqrt{3},\ -\dfrac{\sqrt{3}}{2}\right)$, $(0,\ 0)$ and $\left(\sqrt{3},\ \dfrac{\sqrt{3}}{2}\right)$. 1A

(d) No vertical asymptote.

Horizontal asymptote is $y = 0$. 1M+1A

(e) (Correct shape) 1A

(All correct) 1A



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