1. $\dfrac{\log_{32} y - 2}{x - \frac{1}{2}} = \dfrac{2 - 0}{\frac{1}{2} + 2}$ 1M

$$\log_{32} y = \frac{4x + 8}{5}$$

$$y = 32^{\frac{4x+8}{5}}$$ 1M

$$y = 2^8 \cdot 2^{4x}$$

$$= 256 \cdot 16^x$$ 1A

2. $\dfrac{\log_{16} y - 1}{x - 0} = \dfrac{0 - 1}{-4 - 0}$ 1M

$$\log_{16} y = \frac{x}{4} + 1$$

$$y = 16^{\frac{x}{4}+1}$$ 1M

$$= 2^{x+4}$$ 1A

3. $\log_4 y = \dfrac{-18 - 0}{-2 - \frac{1}{4}} \left(\log_2 x - \dfrac{1}{4} \right)$ 1M

$$\log_4 y = 8 \log_2 x - 2$$

$$\frac{\log y}{2 \log 2} = 8 \times \frac{\log x}{\log 2} - 2$$ 1M

$$\log y = 16 \log x - 4 \log 2$$

$$\log y = \log \frac{x^{16}}{2^4}$$

$$y = \frac{x^{16}}{16}$$ 1A

4. We have $\alpha + \beta = p$ and $\alpha\beta = p - 2$.

$$\log_4(\alpha + \beta) = \log_2 \alpha + \log_2 \beta$$

$$\frac{\log(\alpha + \beta)}{2 \log 2} = \frac{\log \alpha\beta}{\log 2}$$ 1M

$$\log p = 2 \log(p - 2)$$

$$p = (p - 2)^2$$ 1M

$$0 = p^2 - 5p + 4$$

$p = 4$ or 1 (rejected) 1A

5. Slope $= \dfrac{-1}{3}$

$$\log_2 y = -\frac{1}{3} \log_2 x + 1$$ 1M

$$y = 2^{-\frac{1}{3} \log_2 x + 1}$$

$$y = 2x^{-\frac{1}{3}}$$

Thus, $a = 2$ and $b = -\frac{1}{3}$. 1A+1A

6. (a) Slope of the graph $= \frac{12-6}{3-0} = 2$

$$\log_2 y - 6 = 2(x-0) \quad \text{1M}$$

$$\log_2 y = 2x + 6$$

$$y = 2^{2x+6} \quad \text{1M}$$

$$y = 2^6 \cdot 2^{2x}$$

$$= 64 \cdot 4^x$$

Thus, $a = 64$ and $b = 4$. 1A+1A

(b) $64(4^t) - 64(4^{t-1}) = 786\,432$ 1M

$$4^t(1 - 4^{-1}) = 12\,288$$

$$4^t = 16\,384$$

$$t \log 4 = \log 16\,384 \quad \text{1M}$$

$$t = 7 \quad \text{1A}$$

7. We have

$$\begin{cases} 0 = m^2 - n \\ -12 = m - n \end{cases} \quad \text{1M}$$

$$0 + 12 = m^2 - m \quad \text{1M}$$

$$0 = m^2 - m - 12$$

$$m = 4 \quad \text{or} \quad -3 \text{ (rejected)}$$

When $m = 4$, $n = m + 12 = 16$. 1A

$$4^x - 16 > 2021$$

$$4^x > 2037$$

$$x \log 4 > \log 2037 \quad \text{1M}$$

$$x > 5.50 \quad \text{1A}$$

8. G passes through $(-12,\ 0)$ and $(0,\ 1)$.

$$\begin{cases} 0 = a + \log_b(-12+16) \\ 1 = a + \log_b 16 \end{cases} \quad \text{1M}$$

$$1 - 0 = \log_b 16 - \log_b 4 \quad \text{1M}$$

$$1 = \log_b 4$$

$$b = 4 \quad \text{1A}$$

When $b = 4$, $a = 1 - \log_4 16 = -1$.

$$y = -1 + \log_4(x + 16)$$
$$4^{y+1} = x + 16$$
$$x = 4^{y+1} - 16 \qquad \text{1A}$$

9. Substitute $(0,\ 1)$ and $(1,\ 6)$,

$$\begin{cases} 1 = m - 2n \\ 6 = mn - 2n \end{cases}$$

$6 = (2n + 1)n - 2n$ 1M

$0 = 2n^2 - n - 6$

$n = 2$ or $-\frac{3}{2}$ (rejected) 1A

When $n = 2$, $m = 2(2) + 1 = 5$.

$y = 5 \times 2^x - 4$

$2^x = \frac{y + 4}{5}$

$x = \log_2 \frac{y + 4}{5}$ 1A

10. (a) $6 = ka^0$

$k = 6$ 1A

$54 = 6 \times a^{-2}$

$a^2 = \frac{1}{9}$

$a = \frac{1}{3}$ 1A

(b) $f(x_1) = \frac{3}{f(x_2)}$

$6 \times \frac{1}{3^{x_1}} = \frac{3}{6 \times \frac{1}{3^{x_2}}}$ 1M

$3^{x_1 + x_2} = 12$

$(x_1 + x_2)\log 3 = \log 12$ 1M

$x_1 + x_2 \approx 2.26$ 1A

11. B

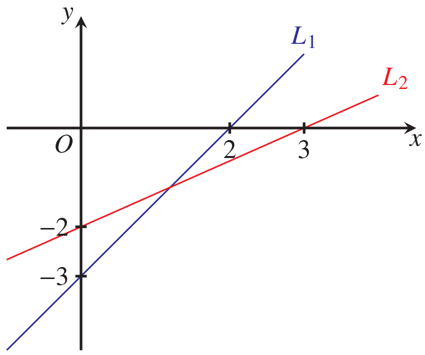
Assign reasonable values to the intercepts.

Use the points $(0,\ -3)$ and $(2,\ 0)$,
we have $a = -3$ and $b = 6$.
Use the points $(0,\ -2)$ and $(3,\ 0)$,
we have $c = -\frac{2}{3}$ and $d = 2$.

I. ✗.

II. ✓.

III. ✗.



12. C

Slope $= \frac{b}{a} > 0$
y-intercept $= -\frac{c}{b} > 0$
The answer is C.

13. A

The equation of L_2 is in the form $3x + 4y + k = 0$, where k is a constant.
y-intercept of L_1 is 2.

$$3(0) + 4(2) + k = 0$$
$$k = -8$$

The equation of L_2 is $3x + 4y - 8 = 0$.

14. D

Slope of $OB = \frac{8-0}{20-0} = \frac{2}{5}$
Slope of $AP = -\frac{5}{2}$
Required equation is

$$\frac{y-7}{x-3} = -\frac{5}{2}$$
$$5x + 2y - 29 = 0$$

15. D

$(-6) + 3y - 6 = 0$
$y = 4$
The coordinates of the intersection are $(-6,\ 4)$.
y-intercept of L_1 is 2.

Suppose the y-intercept of L_2 is k.

$$\frac{k-4}{0+6} \times \frac{-1}{3} = -1$$
$$k = 22$$

Required area $= \frac{1}{2} \times (22-2) \times 6$

$= 60$

16. B

Assign suitable values to the intercepts.

L_1:

Use the points $(1,\ 0)$ and $(0,\ -1.5)$.

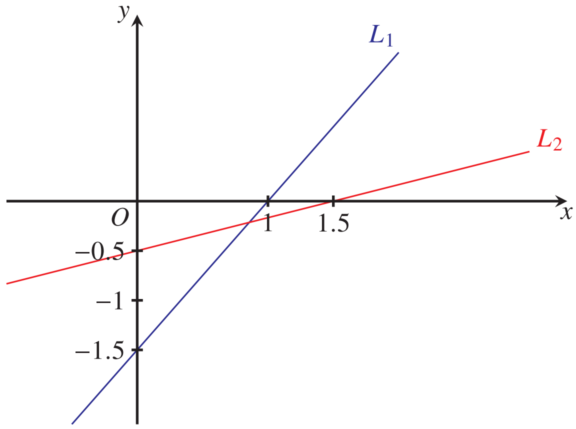
We have $p = -2$ and $q = 3$.

L_2:

Use the points $(1.5,\ 0)$ and $(0,\ -0.5)$.

We have $r = -\frac{2}{3}$ and $s = -1$.

The result follows.



17. D

Equation of L is in the form $x - 2y + k = 0$.

Put $(0,\ 4)$ into $x - 2y + k = 0$.

$$0 - 2(4) + k = 0$$
$$k = 8$$

Required equation is $x - 2y + 8 = 0$.

18. C

Assign suitable values to the intercepts.

(0, 2)

$0 - b^2(2) + b = 0$

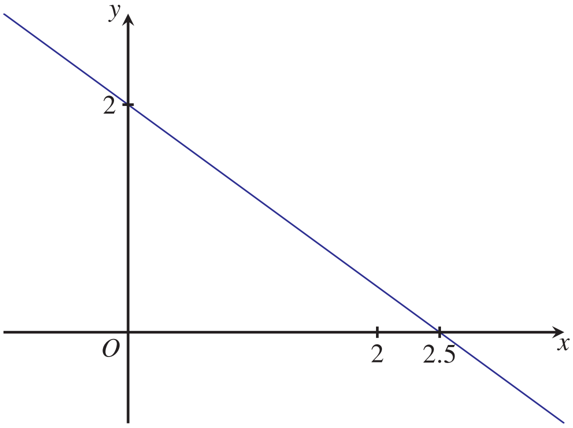
$b = 0.5 \quad \text{or} \quad 0 \text{ (rejected)}$

(2.5, 0)

$2.5a - 0 + 0.5 = 0$

$a = -0.2$

Only statements II and III are true.



19. C

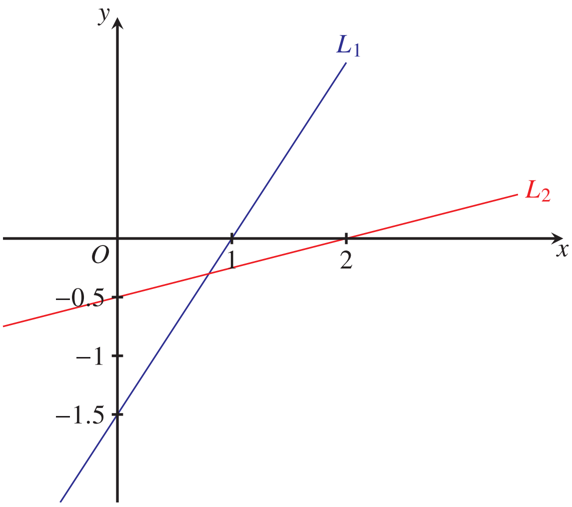
Assign suitable values to the intercepts.

Use the points $(0,\ -1)$ and $(1,\ 0)$,
we have $p = -5$ and $q = 5$.
Use the points $(0,\ -1.5)$ and $(2,\ 0)$,
we have $r = -1.5$ and $s = -3$.

I. ✓.

II. ✗.

III. ✓.



20. A

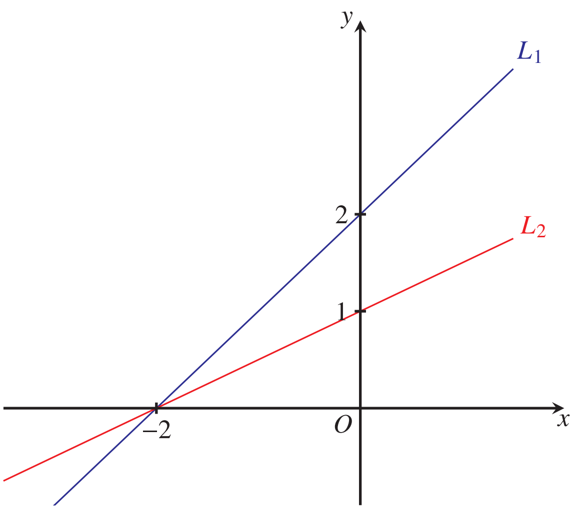
Assign reasonable values to the intercepts.

Use the points $(-2,\ 0)$ and $(0,\ 2)$,
we have $a = -0.5$ and $b = 0.5$.
Use the points $(-2,\ 0)$ and $(0,\ 1)$,
we have $c = -0.5$ and $d = 1$.

I. ✓.

II. ✓.

III. ✗.



21. D

I. ✓. Slope of $L_1 = -\frac{1}{A} < 0$. So, $A > 0$.

II. ✓. Slope of $L_2 = -\frac{1}{C}$.

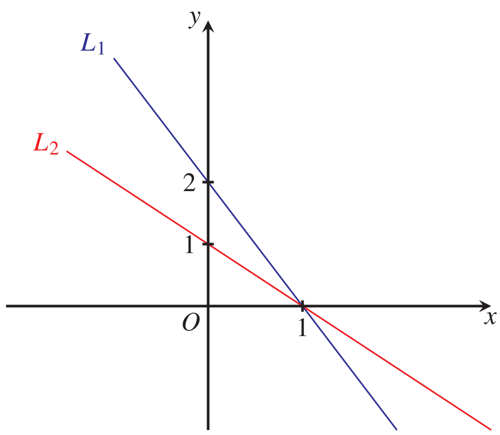
$$\left(\frac{-1}{A}\right)\left(\frac{-1}{C}\right) = -1$$
$$AC = -1$$

III. ✓. x-intercept of $L_1 = B$ and x-intercept of $L_2 = D$.
Thus, $B > D$.

22. A

Assign reasonable values to the intercepts.

Use the points $(1,\ 0)$ and $(0,\ 2)$,
we have $a = -1.5$ and $b = 3$.
Use the points $(1,\ 0)$ and $(0,\ 1)$,
we have $c = -3$ and $d = -3$.



I. ✓.

II. ✓.

III. ✗.

23. D

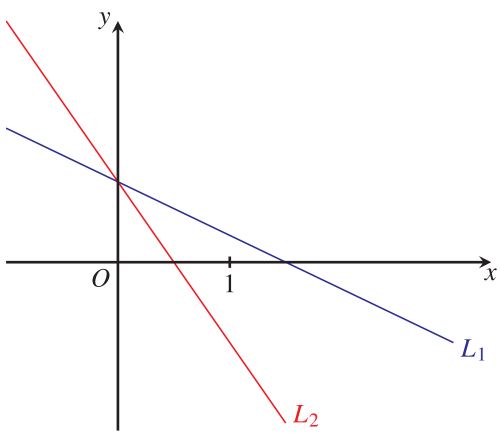
Assign reasonable values to the intercepts.

L_1

Use the points $(0,\ 1)$ and $(1.5,\ 0)$,
we have $a = -\frac{2}{3}$ and $b = -1$.

L_2

Use the points $(0,\ 1)$ and $(0.5,\ 0)$,
we have $d = -2$ and $e = -1$.



I. ✗.

II. ✓.

III. ✓.

IV. ✓.

24. D

$h(0) + k(-8) + 24 = 0$ and $\left(-\frac{h}{k}\right) \times \left(\frac{3}{4}\right) = -1$

$k = 3$

$h = \frac{4k}{3}$

$= 4$

$3x - 4(0) - 15(4) = 0$

$x = 20$

The x-intercept of L_2 is 20.

25. A

Assign reasonable values to the intercepts. Note that the y-intercept of L_2 is $\frac{1}{3}$.

Use the points $(-1,\ 0)$ and $(0,\ 1)$,

we have $a = -1$ and $b = 1$.

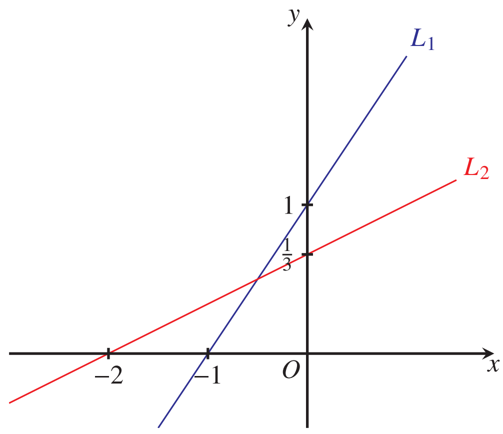
Use the point $(-2,\ 0)$,

we have $c = -0.5$.

I. ✓.

II. ✓.

III. ✗.



26. D

y-intercept of L_2 is 3.

$k(0) - h(3) + 6 = 0$

$h = 2$

L_1 and L_2 are perpendicular to each other.

$\frac{k}{2} \times \frac{-2}{3} = -1$

$k = 3$

The x-intercepts of L_1 and L_2 are -2 and $\frac{9}{2}$ respectively.

Required area $= \frac{\left(\frac{9}{2} + 2\right)(3)}{2}$

$= \frac{39}{4}$

27. B

Assign suitable values to the intercepts.

L_1:

$(1,\ 0) \quad \rightarrow \quad b = 1$

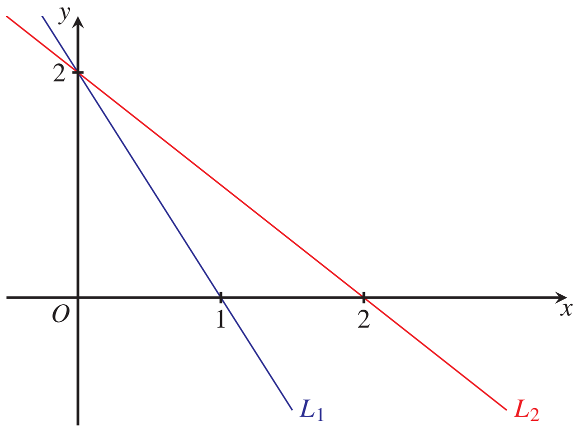
$(0,\ 2) \quad \rightarrow \quad a = \frac{1}{2}$

L_2:

$(2,\ 0) \quad \rightarrow \quad d = 2$

$(0,\ 2) \quad \rightarrow \quad c = 1$

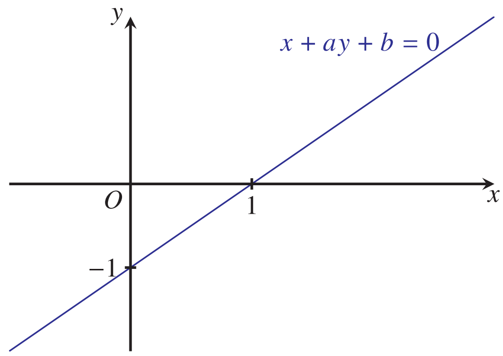
Only statement III is true.



28. D

Assign reasonable values to the intercepts.

Use the points $(1,\ 0)$ and $(0,\ -1)$,
we have $a = -1$ and $b = -1$.
Thus, $a < 0$ and $b < 0$.



29. B

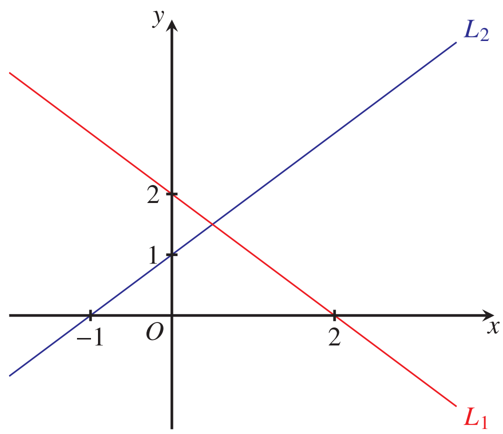
Assign reasonable values to the intercepts.

Use the points $(2,\ 0)$ and $(0,\ 2)$,
we have $a = 1$ and $b = 2$.
Use the points $(-1,\ 0)$ and $(0,\ 1)$,
we have $c = -1$ and $d = 1$.

I. ✓.

II. ✗.

III. ✓.



30. A

Required equation is $2x - 3y + k = 0$, where k is a constant.

$$2(1) - 3(-3) + k = 0$$
$$k = -11$$

Required equation is $2x - 3y - 11 = 0$.

31. C

Note that $\triangle AOB$ is an isosceles triangle.

$$\text{Slope of } L_2 = -(\text{slope of } L_1) = -\left(-\frac{a}{b}\right) = \frac{a}{b}$$

32. D

Two lines are parallel.

$$\frac{-2}{3} = \frac{-6}{k}$$
$$k = 9$$

33. C

L_1 passes through $(-3,\ 0)$.

$$-3a - 0 + 2b = 0$$
$$-3a + 2b = 0$$

The slopes of L_1 and L_2 are equal.

$$\frac{a}{b+1} = -2$$
$$a + 2b = -2$$

Solving, we have $a = -\frac{1}{2}$ and $b = -\frac{3}{4}$.

34. D

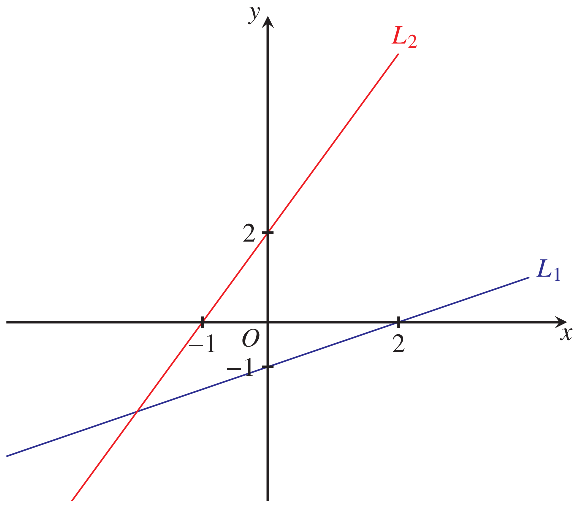
Assign reasonable values to the intercepts.

Use the points $(0, -1)$ and $(2, 0)$,
we have $a = 4$ and $b = 4$.
Use the points $(-1, 0)$ and $(0, 2)$,
we have $c = -2$ and $d = -2$.

I. ✓.

II. ✓.

III. ✓.



35. B

The straight lines are parallel.

$$-\frac{5}{2} = -\frac{10}{-a}$$
$$a = -4$$

36. D

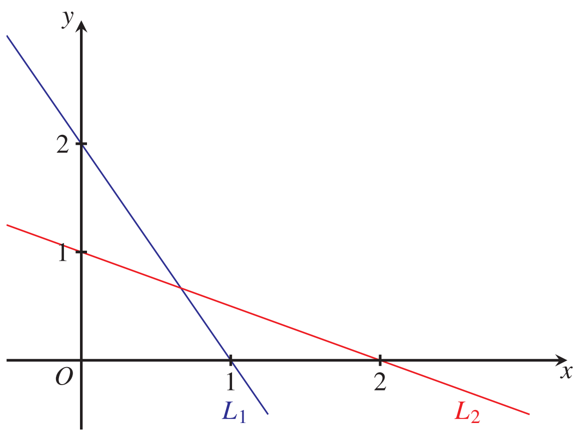
Assign reasonable values to the intercepts.

Use the points $(1, 0)$ and $(0, 2)$,
we have $a = 0.5$ and $b = 1$.
Use the points $(2, 0)$ and $(0, 1)$,
we have $c = 2$ and $d = 2$.

I. ✗.

II. ✓.

III. ✓.



37. D

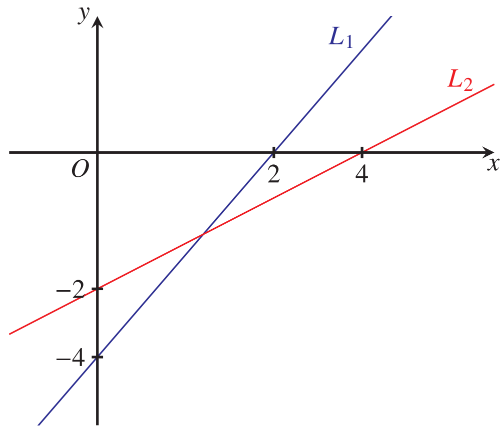
The x-intercepts and y-intercepts of L_2 are 4 and -2 respectively. Assign reasonable values to the intercepts.

Use the points $(2,\ 0)$ and $(0,\ -4)$,
we have $a = -2$ and $b = 1$.

I. ✗.

II. ✓.

III. ✓.



38. A

Solve $\begin{cases} 3x + 4y = 22 \\ 4x - y = -15 \end{cases}$, we have $x = -2$ and $y = 7$.
Required equation is in the form $3x - 5y + k = 0$, where k is a constant.

$$3(-2) - 5(7) + k = 0$$
$$k = 41$$

Required equation is $3x - 5y + 41 = 0$.

39. D

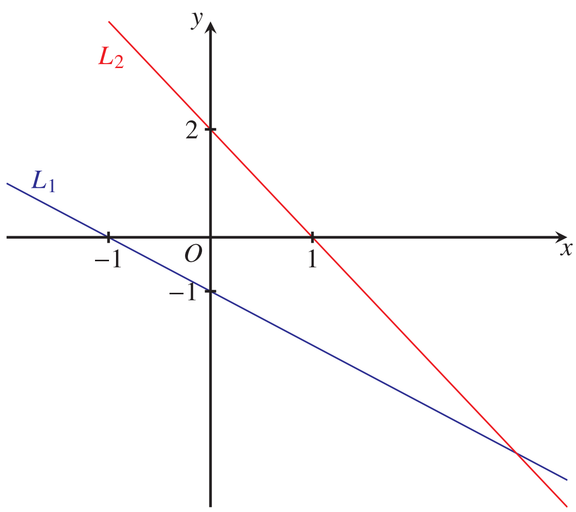
Assign reasonable values to the intercepts.

Use the points $(-1,\ 0)$ and $(0,\ -1)$,
we have $r = 1$ and $s = -1$.
Use the points $(0,\ 2)$ and $(1,\ 0)$,
we have $m = 0.5$ and $n = 1$.

I. ✗.

II. ✓.

III. ✓.



40. C

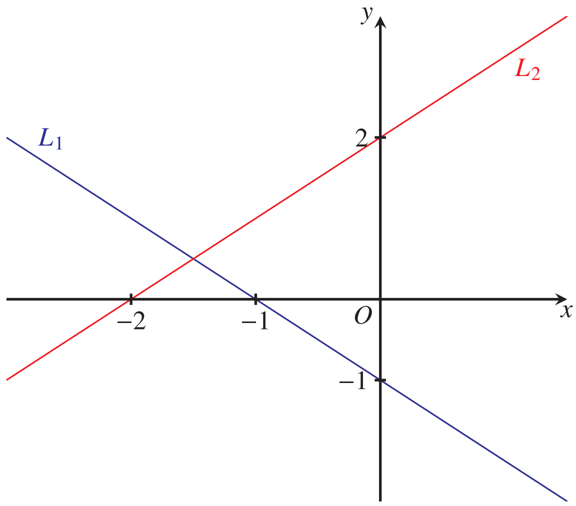
Assign reasonable values to the intercepts.

$\boxed{L_1}$
$(0,\ -1) \rightarrow b = 3$
$(-1,\ 0) \rightarrow a = -3$
$\boxed{L_2}$
$(-2,\ 0) \rightarrow d = -12$
$(0,\ 2) \rightarrow c = -6$

I. ✗.

II. ✓.

III. ✓.



41. $\boxed{\text{D}}$

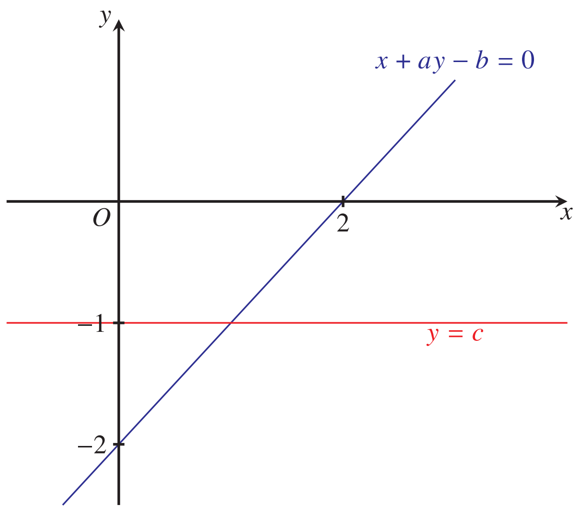
Assign reasonable values to the intercepts.

Use the points $(2,\ 0)$ and $(0,\ -2)$,
we have $a = -1$ and $b = 2$.
Use the point $(0,\ -1)$,
we have $c = -1$.

I. ✗.

II. ✓.

III. ✓.



42. $\boxed{\text{D}}$

The slopes of two straight lines are equal.

$$-\frac{8}{a} = \frac{2}{8-a}$$
$$8a - 64 = 2a$$
$$a = \frac{32}{3}$$

43. $\boxed{\text{B}}$

Compute the x-intercepts, y-intercepts and slopes of two straight lines.

Line	x-intercept	y-intercept	Slope
L_1	$\frac{b}{3}$	$\frac{b}{a}$	$-\frac{3}{a}$
L_2	$-\frac{d}{c}$	$-d$	$-c$

I. ✓.

Consider the slope of L_1.

$$-\frac{3}{a} > 0$$

$$a < 0$$

II. ✗.

Consider the y-intercept of L_2.

$$-d > 0$$

$$d < 0$$

III. ✓.

Consider the x-intercepts of L_1 and L_2.

$$\frac{b}{3} < -\frac{d}{c}$$

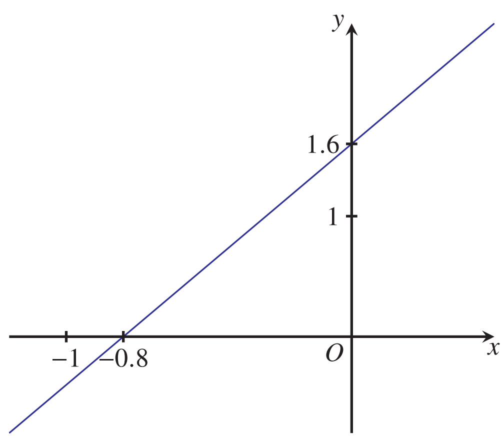
$$bc < -3d \qquad \text{(note that } c > 0\text{)}$$

$$bc + 3d < 0$$

44. A

Assign reasonable values to the intercepts.

Use the points $(-0.8,\ 0)$ and $(0,\ 1.6)$,
we have $a = -5$ and $b = 2.5$.



I. ✓.

II. ✓.

III. ✗.

45. A

Two straight lines have equal slopes.

$$-\frac{a_1}{b_1} = -\frac{a_2}{b_2}$$

$$a_1 b_2 = a_2 b_1$$

46. C

Two straight lines have equal slopes.

$$-\frac{4}{a-4} = -\frac{a+7}{20}$$

$$80 = a^2 + 3a - 28$$

$$0 = a^2 + 3a - 108$$

$$a = -12 \quad \text{or} \quad 9$$

47. D

Compute the x-intercepts, y-intercepts and the slopes of two straight lines.

Line	x-intercept	y-intercept	Slope
L_1	$-\frac{4}{a}$	$\frac{4}{3}$	$\frac{a}{3}$
L_2	$\frac{5}{2}$	$\frac{5}{b}$	$-\frac{2}{b}$

I. ✓.

Slopes of two straight lines are negative.

$$\frac{a}{3} < 0 \quad \text{and} \quad -\frac{2}{b} < 0$$

$$a < 0 \qquad\qquad b > 0$$

Thus, we have $a < 0 < b$.

II. ✓.

Consider the slopes of the two straight lines.

$$\frac{a}{3} < -\frac{2}{b}$$

$$ab < -6 \qquad \text{(note that } b > 0\text{)}$$

III. ✓.

Consider the y-intercepts of two straight lines.

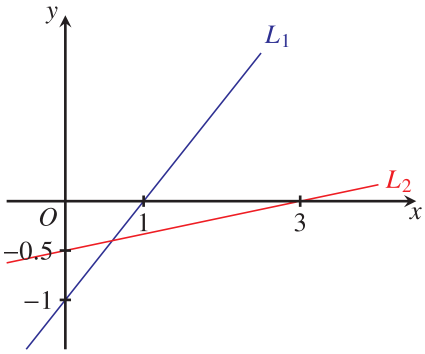
$$\frac{4}{3} > \frac{5}{b}$$

$$b > \frac{15}{4} \qquad \text{(note that } b > 0\text{)}$$

48. D

Assign reasonable values to the intercepts.

Use the points $(1,\ 0)$ and $(0,\ -1)$,
we have $a = 1$ and $b = 1$.
Use the point $(3,\ 0)$ and $(0,\ -0.5)$,
we have $m = 6$ and $n = 3$.



I. ✓.

II. ✓.

III. ✓.

49. D

I. ✓. Slope $= -\frac{a}{5} < 0 \quad \Rightarrow \quad a > 0$

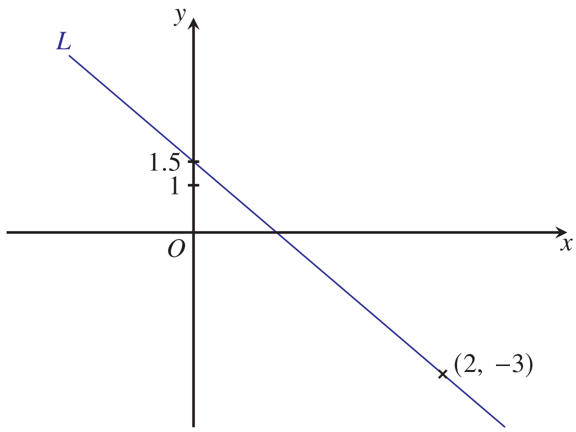
II. ✓. y-intercept $= \frac{b}{5} < -1 \quad \Rightarrow \quad b < -5$

III. ✓. x-intercept $= \frac{b}{a} > -2$ and $a > 0 \quad \Rightarrow \quad b > -2a$

50. D

Assign reasonable values to the y-intercept.

Use the points $(0,\ 1.5)$ and $(2,\ -3)$,
we have $a = \frac{4}{9}$ and $b = -\frac{2}{3}$.



I. ✓.

II. ✓.

III. ✓.

51. D

I. ✗.
The graph of $y = \log_b x$ intersects the straight line $y = 1$ at $(b,\ 1)$.
The x-intercept of the graph of $y = \log_b x$ is 1.
Compare the x-coordinates of two points, we have $b < 1$.

II. ✓.

Let the equation of L be $x = k$, where $k > 1$.

The coordinates of A and B are $(k,\ \log_a k)$ and $(k,\ \log_b k)$ respectively.

$$\begin{aligned} AC &= BC \\ \log_a k &= -\log_b k \\ \frac{\log k}{\log a} &= -\frac{\log k}{\log b} \\ \log b &= -\log a \\ \log ab &= 0 \\ ab &= 1 \end{aligned}$$

III. ✓.

The x-intercepts of two curves are 1.

52. A

$$\log_a 2 = \frac{1}{c} \qquad \text{and} \qquad \log_b 5 = \frac{1}{c}$$

$$\frac{\log 2}{\log a} = \frac{1}{c} \qquad\qquad \frac{\log 5}{\log b} = \frac{1}{c}$$

$$\log a = c \log 2 \qquad\qquad \log b = c \log 5$$

$$\begin{aligned} \log_{ab} 10 &= \frac{\log 10}{\log a + \log b} \\ &= \frac{1}{c(\log 2 + \log 5)} \\ &= \frac{1}{c} \end{aligned}$$

53. A

$$\begin{aligned} \frac{\log_3 y - 0}{\log_9 x - (-1)} &= 10 \\ \log_3 y &= 10 \log_9 x + 10 \\ \frac{\log y}{\log 3} &= 10 \times \frac{\log x}{2 \log 3} + 10 \\ \log y &= 5 \log x + 10 \log 3 \\ \log y &= \log\left(3^{10} x^5\right) \\ y &= 59\,049 x^5 \end{aligned}$$

54. D

$$y = \frac{0-(-1)}{2-0}\log_4 x - 1$$

$$y = \frac{1}{2}\log_4 x - 1$$

$$\log_4 x = 2y + 2$$

$$x = 4^{2y+2}$$

$$x = 2^{4y+4}$$

$$x = 16 \cdot 16^y$$

We have $m = n = 16$.

I. ✗.

II. ✓.

III. ✓.

55. B

$$\log_4 a = \frac{1}{c} \quad \text{and} \quad \log_{25} b = \frac{1}{c}$$

$$\frac{\log a}{2\log 2} = \frac{1}{c} \qquad \frac{\log b}{2\log 5} = \frac{1}{c}$$

$$\log a = \frac{2\log 2}{c} \qquad \log b = \frac{2\log 5}{c}$$

$$\begin{aligned}\log ab &= \log a + \log b \\ &= \frac{2\log 2}{c} + \frac{2\log 5}{c} \\ &= \frac{2\log 10}{c} \\ &= \frac{2}{c}\end{aligned}$$

56. C

$$\log_4 y = \frac{1-0}{0-\left(-\frac{1}{2}\right)}x + 1$$

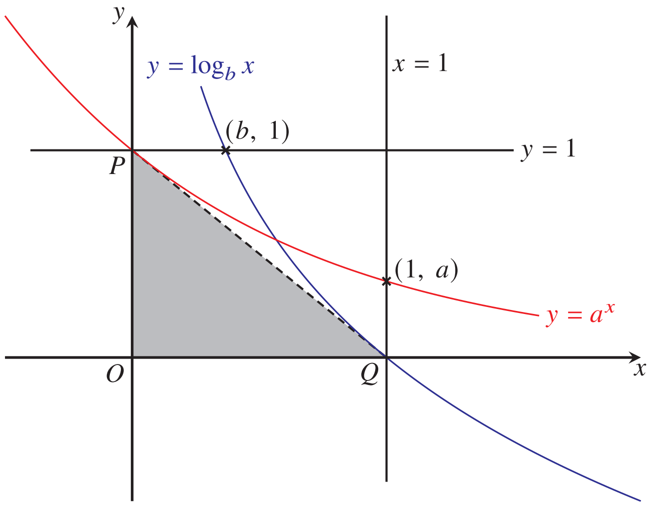
$$\log_4 y = 2x + 1$$

$$y = 4^{2x+1}$$

$$y = 4^{1+2x}$$

57. A

Note that the coordinates of P and Q are $(0,\ 1)$ and $(1,\ 0)$ respectively.



I. ✓. Observe the y-coordinates. We have $0 < a < 1$.

II. ✓. The graph of reflection image of $y = a^x$ is $y = \log_a x$.
We have $a = b$ and $\frac{a}{b} = 1$.

III. ✗. Note that $0 < a < 1$ and $0 < b < 1$.

$$\begin{aligned}\text{Area of } \triangle OPQ &= \frac{(1)(1)}{2} \\ &= \frac{1}{2} \\ &\neq \frac{1}{2}ab\end{aligned}$$

58. A

$$\begin{aligned}\log_8 y - 0 &= \frac{1-0}{-1-2}(\log_8 x - 2) \\ \log_8 y &= -\frac{1}{3}\log_8 x + \frac{2}{3} \\ \frac{\log y}{3\log 2} &= -\frac{1}{3} \times \frac{\log x}{3\log 2} + \frac{2}{3} \\ \log y &= -\frac{1}{3}\log x + 2\log 2 \\ \log y &= \log\left(2^2 x^{-\frac{1}{3}}\right) \\ y &= 4x^{-\frac{1}{3}}\end{aligned}$$

Thus, $k = 4$.

59. B

(4, 0)

$$\log_2 x = 4 \quad \text{and} \quad \log_4 y = 0$$
$$x = 2^4 \qquad\qquad y = 1$$

(0, 5)

$\log_2 x = 0$ and $\log_4 y = 5$

$x = 1$ $\quad$ $y = 4^5$

$y = 2^{10}$

	x^5y^2	x^2y^5	x^4y^2	x^5y^4
$x = 2^4$ and $y = 1$	2^{20}	2^8	2^{16}	2^{20}
$x = 1$ and $y = 2^{10}$	2^{20}	2^{50}	2^{20}	2^{40}

The answer is B.

Slope of the linear graph $= \frac{5-0}{0-4} = -\frac{5}{4}$

$$\log_4 y - 5 = -\frac{5}{4}(\log_2 x - 0)$$

$$5\log_2 x + 4\log_4 y = 20$$

$$\frac{5\log x}{\log 2} + \frac{4\log y}{2\log 2} = 20$$

$$5\log x + 2\log y = 20\log 2$$

$$\log x^5y^2 = \log 2^{20}$$

$$x^5y^2 = 2^{20}$$

60. B

Slope $= \frac{12-0}{0-3} = -4$

The equation of the relationship is

$$\frac{y^3 - 12}{\log_5 x - 0} = -4$$

$$y^3 - 12 = -4\log_5 x$$

Put $y = 2$.

$$2^3 - 12 = -4\log_5 x$$

$$\log_5 x = 1$$

$$x = 5$$

61. A

$$y^2 - 0 = \frac{1-0}{-1-0} \times (\log_2 x - 0)$$
$$y^2 = -\log_2 x$$
$$\log_2 x = -y^2$$
$$x = 2^{-y^2}$$
$$x = \left(\frac{1}{2}\right)^{y^2}$$

62. D

The coordinates of A are $(a,\ 1)$.
The coordinates of B are $(a,\ \log_b a)$.
The coordinates of C are $(a,\ 0)$.
Since $0 < b < a < 1$, we have $\log_b a = \dfrac{\log b}{\log a} < \dfrac{\log a}{\log a} = 1$.
Thus, the point B lies below A.

$$\begin{aligned}\frac{AB}{BC} &= \frac{1-\log_b a}{\log_b a - 0} \\ &= \frac{1}{\log_b a} - 1 \\ &= 1 \div \frac{\log a}{\log b} - 1 \\ &= \frac{\log b}{\log a} - 1 \\ &= \log_a b - 1\end{aligned}$$

63. D

$$\log_3 y = \frac{2-0}{0-(-4)}x + 2$$
$$\log_3 y = \frac{x}{2} + 2$$
$$y = 3^{\frac{x}{2}+2}$$
$$y = 9 \cdot (\sqrt{3})^x$$
Thus, $m = 9$.

64. A

$$\log_4 m - \log_2 n = 1$$
$$\frac{\log m}{2\log 2} - \frac{\log n}{\log 2} = 1$$
$$\log m - 2\log n = 2\log 2$$
$$\log \frac{m}{n^2} = \log 4$$
$$\frac{m}{n^2} = 4$$
$$\frac{n^2}{m} = \frac{1}{4}$$

65. B

$$\log_3 y = \log_{27} a + \log_9 x$$
$$\frac{\log y}{\log 3} = \frac{\log a}{3\log 3} + \frac{\log x}{2\log 3}$$
$$\log y = \frac{1}{3}\log a + \frac{1}{2}\log x$$
$$= \log a^{\frac{1}{3}} x^{\frac{1}{2}}$$
$$y = a^{\frac{1}{3}} x^{\frac{1}{2}}$$

66. A

$$\log_2 y = \frac{5-0}{0-2}\log_8 x + 5$$
$$\log_2 y = -\frac{5}{2}\log_8 x + 5$$
$$\frac{\log y}{\log 2} = -\frac{5}{2} \times \frac{\log x}{3\log 2} + 5$$
$$6\log y = -5\log x + 30\log 2$$
$$\log\left(x^5 y^6\right) = \log 2^{30}$$
$$x^5 y^6 = 2^{30}$$

67. A

When $x = 1$, we have $\log_2 x = 0$, $y = 4$ and $\log_2 y = 2$.
The graph of $\log_2 y$ against $\log_2 x$ passes through the point $(0,\ 2)$.
When $y = 1$, we have $\log_2 y = 0$, $x = 2$ and $\log_2 x = 1$.
The graph of $\log_2 y$ against $\log_2 x$ passes through the point $(1,\ 0)$.
The answer is A.

68. B

The coordinates of A and B are $(0,\ k)$ and $(0,\ q)$ respectively.

$$q = ka^x$$
$$a^x = \frac{q}{k}$$
$$x = \log_a \frac{q}{k}$$

The coordinates of C are $\left(\log_a \frac{q}{k},\ q\right)$.

I. ✓.
The y-coordinate of A is negative.

II. ✗.
The value of ka^x approaches zero when x increases.
Thus, $0 < a < 1$.

III. ✓.

$$\begin{aligned} BC &< OA \\ 0 - \log_a \frac{q}{k} &< 0 - k \\ \log_a \frac{q}{k} &> k \end{aligned}$$

69. A

$$\begin{aligned} \log_2 y - 1 &= \frac{1}{3}(\log_4 x + 3) \\ \log_2 y &= \frac{1}{3}\log_4 x + 2 \\ \frac{\log y}{\log 2} &= \frac{\log x}{3(2\log 2)} + 2 \\ \log y &= \frac{1}{6}\log x + 2\log 2 \\ &= \log 2^2 x^{\frac{1}{6}} \\ y &= 4x^{\frac{1}{6}} \end{aligned}$$

Thus, we have $n = \frac{1}{6}$.

70. A

$$\begin{aligned} \log_2 y &= \frac{3-0}{0-3}\log_2 x + 3 \\ \log_2 y &= -\log_2 x + 3 \\ \frac{\log y}{\log 2} &= -\frac{\log x}{\log 2} + 3 \\ \log y &= -\log x + 3\log 2 \\ \log y &= \log\left(2^3 x^{-1}\right) \\ y &= 8x^{-1} \end{aligned}$$

Thus, $m = 8$ and $n = -1$.

I. ✓.

II. ✗.

III. ✗.

The graph of $y = 8x^{-1}$ is not a straight line.

71. D

$$\begin{aligned} \log_2 y &= \frac{2-0}{0-4}x^3 + 2 \\ \log_2 y &= -\frac{1}{2}x^3 + 2 \end{aligned}$$

Put $x = -2$.

$$\log_2 y = -\frac{1}{2}(-2)^3 + 2$$
$$\log_2 y = 6$$
$$y = 2^6$$
$$y = 64$$

72. B

$$\log_{27} y = \frac{3-0}{0-2}\log_9 x + 3$$
$$\frac{\log y}{3\log 3} = -\frac{3}{2} \times \frac{\log x}{2\log 3} + 3$$
$$4\log y + 9\log x = 36\log 3$$
$$\log\left(x^9 y^4\right) = \log 3^{36}$$
$$x^9 y^4 = 3^{36}$$
$$x^9 y^4 = 9^{18}$$

73. D

$$\log_9 y - 4 = \frac{4-0}{0-(-2)}(x-0)$$
$$\log_9 y = 2x + 4$$
$$y = 9^{2x+4}$$
$$y = 6561 \cdot 81^x$$

Thus, $b = 81$.

74. C

When $x = 0$, we have $y = 4$ and $\log_2 y = 2$.
The coordinates of A are $(0,\ 2)$.

75. A

Consider the point $(0,\ 2)$.

$\log_3 x = 0$ and $\log_3 y = 2$

$x = 1$ $\qquad y = 3^2 = 9$

Consider the point $(4,\ 0)$.

$\log_3 x = 4$ and $\log_3 y = 0$

$x = 3^4 = 81$ $\qquad y = 1$

Check the relation using the values of x and y.

	$x = 1$ & $y = 9$	$x = 81$ & $y = 1$
A.	✓	✓
B.	✗	
C.	✓	✗
D.	✗	

The answer is A.

76. B

Consider the coordinates of points A and B.

$$a^s = k \quad \text{and} \quad b^t = k$$
$$s = \log_a k \qquad t = \log_b k$$

$$\begin{aligned}\frac{s}{t} &= \frac{\log_a k}{\log_b k} \\ &= \frac{\log k}{\log a} \div \frac{\log k}{\log b} \\ &= \frac{\log b}{\log a} \\ &= \log_a b\end{aligned}$$

77. C

I. ✓.

II. ✗.

Take $a = 2$, $b = 0.5$ and $k = 2$.

We have $\log_a k = \log_2 2 = 1$ and $\log_b k = \log_{0.5} 2 = -1$.

III. ✓.

Note that $a > b > 0$ and $k > 1$.

$$\begin{aligned}\frac{a}{b} &> 1 \\ \log\frac{a}{b} &> \log 1 \\ \frac{\log\frac{a}{b}}{\log k} &> 0 \\ \log_k \frac{a}{b} &> 0\end{aligned}$$

78. B

$\log_a 2016 = \dfrac{\log 2016}{\log a}$

Note that $\log 0.5 < 0 < \log 5 < \log 6$.

We have $\log_{0.5} 2016 < 0 < \log_6 2016 < \log_5 2016$.

79. $\boxed{\text{D}}$

$$\log_9 y - 4 = \frac{4-1}{2-(-4)}(\log_9 x - 2)$$

$$\log_9 y - 4 = \frac{1}{2}\log_9 x - 1$$

$$\frac{\log y}{\log 9} - \frac{1}{2} \times \frac{\log x}{\log 9} = 3$$

$$2\log y - \log x = 6\log 9$$

$$\log \frac{y^2}{x} = \log 9^6$$

$$\frac{y^2}{x} = 9^6$$

80. $\boxed{\text{D}}$

$$\log y - 0 = 3(\log x - 2)$$

$$\log y = 3\log x - 6$$

$$y = 10^{\log x^3 - 6}$$

$$y = 10^{-6}x^3$$

$$y = \frac{x^3}{1\,000\,000}$$

81. $\boxed{\text{D}}$

$$x^a y^b = 81^c$$

$$a\log x + b\log y = c\log 81$$

$$\frac{2a\log x}{2\log 3} + \frac{3b\log y}{3\log 3} = \frac{c\log 81}{\log 3}$$

$$2a\log_9 x + 3b\log_{27} y = 4c$$

Put $\log_9 x = 0$, we have $\log_{27} y = \dfrac{4c}{3b}$.

Put $\log_{27} y = 0$, we have $\log_9 x = \dfrac{2c}{a}$.

The coordinates of P and Q are $\left(0,\ \dfrac{4c}{3b}\right)$ and $\left(\dfrac{2c}{a},\ 0\right)$ respectively.

$$OP : OQ = \frac{4c}{3b} : \frac{2c}{a}$$

$$= 2a : 3b$$

82. $\boxed{\text{D}}$

$$0 = \log_a(ax)$$

$$ax = 1$$

$$x = \frac{1}{a}$$

The coordinates of P are $\left(\frac{1}{a},\ 0\right)$.

$$\begin{aligned} 0 &= \log_b(x+b) \quad \text{and} \quad y = \log_b(0+b) \\ x+b &= 1 \qquad\qquad\qquad\quad = 1 \\ x &= 1-b \end{aligned}$$

The coordinates of Q and R are $(1-b,\ 0)$ and $(0,\ 1)$ respectively.

Required area

$= \frac{1}{2} \times \left(\frac{1}{a} - (1-b)\right) \times 1$

$= \frac{1}{2a} + \frac{b}{2} - \frac{1}{2}$

83. D

For the point $(0,\ -3)$.

$$\begin{aligned} \log_9 x &= 0 \quad \text{and} \quad \log_3 y = -3 \\ x &= 1 \qquad\qquad\quad y = 3^{-3} = \frac{1}{27} \end{aligned}$$

Only option D satisfies this pair of x and y.

84. D

Take logarithm on all numbers.

A. $-375 \log 543 \approx -1025.5499$

B. $\frac{1}{4321} \log 867 \approx 0.000\,679\,9$

C. $349 \log \frac{1}{867} \approx -1025.3687$

D. $492 \log \frac{2}{243} \approx -1025.6115$

The answer is D.

85. B

$$\begin{aligned} \log_8 y &= \frac{0-(-3)}{4-0} \log_4 x - 3 \\ \frac{\log y}{3 \log 2} &= \frac{3}{4} \times \frac{\log x}{2 \log 2} - 3 \\ 8 \log y &= 9 \log x - 72 \log 2 \\ \log y^8 &= \log \frac{x^9}{2^{72}} \\ y^8 &= \frac{x^9}{2^{72}} \\ x^9 &= 2^{72} y^8 \end{aligned}$$

86. A

$$\log_8 y - 0 = \frac{1-0}{-1-2}(\log_8 x - 2)$$
$$\frac{\log y}{3\log 2} = -\frac{1}{3} \times \frac{\log x}{3\log 2} + \frac{2}{3}$$
$$\log y = -\frac{1}{3}\log x + 2\log 2$$
$$\log y = \log\left(2^2 x^{-\frac{1}{3}}\right)$$
$$y = 4x^{-\frac{1}{3}}$$

Thus, $k = 4$.

87. C

(0, 2)

$$\log_5 x = 0 \quad \text{and} \quad \log_5 y = 2$$
$$x = 1 \qquad\qquad y = 25$$

Put $x = 1$ and $y = 25$ into $y = kx^a$.

$$25 = k(1^a)$$
$$k = 25$$

88. A

Let m and c be the slope and the intercept on the vertical axis of the graph.

$$\log_5 y = mx + c$$
$$y = 5^{mx+c}$$
$$y = (5^m)^x \cdot 5^c$$

Since $m < 0$, we have $5^m < 1$ and the value of y is decreasing.
Since $c > 1$, we have $5^c > 5$ and the y-intercept of the required graph is greater than 5.
The answer is A.

89. D

Since the value of y is increasing and the base of log is greater than 1,
the value of $\log_2 y$ is also increasing.
When $x = 0$, $y = 4$ and $\log_2 y = \log_2 4 = 2$.
The intercept on the vertical axis of the required graph is 2, which is positive.
The answer is D.

90. C

$\boxed{(0,\ -1)}$

$$\begin{aligned} \log_7 x &= 0 \quad \text{and} \quad \log_7 y = -1 \\ x &= 1 \qquad\qquad\quad y = 7^{-1} \end{aligned}$$

We have $7^{-1} = a(1)^b$, and $a = \dfrac{1}{7}$.

$\boxed{(2,\ 0)}$

$$\begin{aligned} \log_7 x &= 2 \quad \text{and} \quad \log_7 y = 0 \\ x &= 49 \qquad\qquad\quad y = 1 \end{aligned}$$

We have $1 = \dfrac{1}{7}(49)^b$, and $b = \dfrac{1}{2}$.

END OF PAPER