REG-LOG-2425-ASM-SET 4-MATH
建議題解

結構式試題

1. $$\log_3(3x+5)+\log_3(2x+3)=1+\log_3(2x^2+1)$$
$$\log_3(3x+5)+\log_3(2x+3)=\log_3 3+\log_3(2x^2+1)$$
$$\log_3[(3x+5)(2x+3)]=\log_3[3(2x^2+1)] \qquad \text{1M}$$
$$(3x+5)(2x+3)=3(2x^2+1) \qquad \text{1M}$$
$$6x^2+19x+15=6x^2+3$$
$$19x=-12$$
$$x=-\frac{12}{19} \qquad \text{1A}$$

2. $$\log_3(x+3)+1=\log_3(4-2x)$$
$$1=\log_3\frac{4-2x}{x+3} \qquad \text{1M}$$
$$3^1=\frac{4-2x}{x+3} \qquad \text{1M}$$
$$3x+9=4-2x$$
$$x=-1 \qquad \text{1A}$$

3. (a) $$8^{x-4}=5^{2+3x}$$
$$(x-4)\log 8=(2+3x)\log 5 \qquad \text{1M}$$
$$x(\log 8-3\log 5)=2\log 5+4\log 8 \qquad \text{1M}$$
$$x=\frac{2\log 5+4\log 8}{\log 8-3\log 5}$$
$$\approx -4.20 \qquad \text{1A}$$

(b) $$2^{1-2x}\cdot 3^{x+6}=5^{4x-1}$$
$$(1-2x)\log 2+(x+6)\log 3=(4x-1)\log 5 \qquad \text{1M}$$
$$x(\log 3-2\log 2-4\log 5)=-\log 5-\log 2-6\log 3 \qquad \text{1M}$$
$$x=\frac{-\log 5-\log 2-6\log 3}{\log 3-2\log 2-4\log 5}$$
$$\approx 1.32 \qquad \text{1A}$$

4. (a) $$6^{2x+3}=11^{1-4x}$$
$$(2x+3)\log 6=(1-4x)\log 11 \qquad \text{1M}$$
$$x(2\log 6+4\log 11)=\log 11-3\log 6 \qquad \text{1M}$$
$$x=\frac{\log 11-3\log 6}{2\log 6+4\log 11}$$
$$\approx -0.226 \qquad \text{1A}$$

(b)
$$3^{2x+1} \cdot 5^{x-3} = 12^{4-3x}$$
$$(2x+1)\log 3 + (x-3)\log 5 = (4-3x)\log 12 \quad \text{1M}$$
$$x(2\log 3 + \log 5 + 3\log 12) = 4\log 12 + 3\log 5 - \log 3 \quad \text{1M}$$
$$x = \frac{4\log 12 + 3\log 5 - \log 3}{2\log 3 + \log 5 + 3\log 12}$$
$$\approx 1.21 \quad \text{1A}$$

5. (a)
$$3^{2x} = 6^{x-2}$$
$$2x\log 3 = (x-2)\log 6 \quad \text{1M}$$
$$x(2\log 3 - \log 6) = -2\log 6 \quad \text{1M}$$
$$x = \frac{-2\log 6}{2\log 3 - \log 6}$$
$$\approx -8.84 \quad \text{1A}$$

(b)
$$7^{1-x} - 5^{\frac{x}{2}-3} = 0$$
$$7^{1-x} = 5^{\frac{x}{2}-3}$$
$$(1-x)\log 7 = \left(\frac{x}{2} - 3\right)\log 5 \quad \text{1M}$$
$$\log 7 - x\log 7 = \frac{x}{2}\log 5 - 3\log 5$$
$$\log 7 + 3\log 5 = x\left(\log 7 + \frac{1}{2}\log 5\right) \quad \text{1M}$$
$$x = \frac{\log 7 + 3\log 5}{\log 7 + \frac{1}{2}\log 5}$$
$$\approx 2.46 \quad \text{1A}$$

6. (a)
$$10(5^x) = 3^{2x+1}$$
$$\log 10 + x\log 5 = (2x+1)\log 3 \quad \text{1M}$$
$$x(\log 5 - 2\log 3) = \log 3 - 1 \quad \text{1M}$$
$$x = \frac{\log 3 - 1}{\log 5 - 2\log 3}$$
$$\approx 2.05 \quad \text{1A}$$

(b)
$$2(4.6^{2-x}) = 7^{x-3}$$
$$\log 2 + (2-x)\log 4.6 = (x-3)\log 7 \quad \text{1M}$$
$$x(-\log 4.6 - \log 7) = -3\log 7 - \log 2 - 2\log 4.6 \quad \text{1M}$$
$$x \approx 2.76 \quad \text{1A}$$

7. (a) The value of the car at the end of 2012 $= \$224\,000 \times (1 - 2.5\%)^3$ 1M

$= \$207\,616.5$ 1A

(b) Suppose the value of the car will just drop below \$200 000 after t years.

$$224\,000(1-2.5\%)^t < 200\,000 \quad \text{1M}$$

$$0.975^t < \frac{25}{28}$$

$$t\log 0.975 < \log\frac{25}{28} \quad \text{1M}$$

$$t > 4.5 \quad \text{1A}$$

The value of the car will just drop below \$200 000 at the end of 2014. 1A

8. (a) When $x = 20$, $N = 10\,000\log\frac{50}{20} \approx 3980$ 1M

The total sales is 3980 when the selling price is \$20. 1A

(b) $$6000 = 10\,000\log\frac{50}{x} \quad \text{1M}$$

$$\log\frac{50}{x} = 0.6$$

$$\frac{50}{x} = 10^{0.6} \quad \text{1M}$$

$$x \approx 12.6$$

The selling price should be \$12.6. 1A

9. Put $K = -3.2$ and $M = 7.6$,

$$7.6 = \frac{2}{3}\log E - 3.2$$

$$\log E = 16.2$$

$$E = 10^{16.2} \quad \text{1M}$$

$$\approx 1.58\times 10^{16}$$

The energy released by the earthquake is 1.58×10^{16} J. 1A

10. Let I be the sound intensity before playing the music, then the sound intensity after playing the music is $2I$.

Let β dB be the sound intensity level after playing the music.

$$\begin{cases} 45 = 10\log\dfrac{I}{I_0} \\ \beta = 10\log\dfrac{2I}{I_0} \end{cases} \quad \text{1M}$$

Therefore,

$$\begin{aligned}\beta - 45 &= 10\left(\log\frac{2I}{I_0} - \log\frac{I}{I_0}\right)\\ &= 10\log\left(\frac{2I}{I_0}\times\frac{I_0}{I}\right) && \text{1M}\\ &= 10\log 2\\ \beta &= 45 + 10\log 2\\ &\approx 48.0\end{aligned}$$

The sound intensity level after playing the music is 48.0 dB. 1A

11. Let I_1 and I_2 be the sound intensities measured on the road before and after building noise absorber respectively.

$$\begin{cases} 75 = 10\log\dfrac{I_1}{I_0} \\ 65 = 10\log\dfrac{I_2}{I_0} \end{cases}$$

Therefore,

$$\begin{aligned}75 - 65 &= 10\left[\log\frac{I_1}{I_0} - \log\frac{I_2}{I_0}\right] && \text{1M}\\ 1 &= \log\frac{I_1}{I_2} && \text{1M}\\ \frac{I_1}{I_2} &= 10 && \text{1M}\end{aligned}$$

The sound intensity measured on the road before building noise absorber is 10 times that after building noise absorber. 1A

12. (a) Suppose the number of bacteria will become $2x$ after n hours.

$$\begin{aligned}x(1+5\%)^n &= 2x && \text{1M}\\ 1.05^n &= 2\\ \log 1.05^n &= \log 2 && \text{1M}\\ n\log 1.05 &= \log 2 && \text{1M}\\ n &\approx 14.2\end{aligned}$$

The number of bacteria will become $2x$ after 14.2 hours. 1A

(b) Suppose the number of bacteria will increased by 50% after t hours.

$$\begin{aligned}x(1+5\%)^t &= x(1+50\%) && \text{1M}\\ \log 1.05^t &= \log 1.05\\ t\log 1.05 &= \log 1.05\\ t &\approx 8.31\end{aligned}$$

The number of bacteria will be increased by 50% after 8.31 hours. 1A

13. (a) Put $N = \frac{1}{3}$ into $N = \log_8 \frac{E}{32}$,

$$\frac{1}{3} = \log_8 \frac{E}{32}$$

$$\frac{E}{32} = 8^{\frac{1}{3}}$$

$$E = 64 \qquad \text{1M}$$

Put $E = 64$ and $M = 3$ into $M = \log_a E$,

$$3 = \log_a 64$$

$$64 = a^3$$

$$a = 4 \qquad \text{1A}$$

(b) Put $E = 2576$ into $M = \log_4 E$,

$$\begin{aligned} M &= \log_4 256 \\ &= \log_4 4^4 \qquad \text{1M} \\ &= 4 \end{aligned}$$

The magnitude of the explosion is 4 on Scale A. 1A

14. Let E_1 units and E_2 units be the energy released before and after the amendment respectively.

$$\begin{cases} 7.8 = \frac{2}{3}\log E_1 + 2.9 \\ 8 = \frac{2}{3}\log E_2 + 2.9 \end{cases}$$

$$8 - 7.8 = \frac{2}{3}\log E_2 + 2.9 - \frac{2}{3}\log E_1 - 2.9 \qquad \text{1M}$$

$$0.2 = \frac{2}{3}(\log E_2 - \log E_1)$$

$$0.3 = \log \frac{E_2}{E_1}$$

$$\frac{E_2}{E_1} = 10^{0.3} \qquad \text{1A}$$

$$\begin{aligned} \text{Percentage increase} &= \frac{E_2 - E_1}{E_1} \times 100\% \qquad \text{1M} \\ &= \left(\frac{E_2}{E_1} - 1\right) \times 100\% \\ &\approx 99.5\% \qquad \text{1A} \end{aligned}$$