REG-LOG-2425-ASM-SET 3-MATH
建議題解

多項選擇題

1. C	2. C	3. D	4. C	5. C
6. A	7. A	8. C	9. B	10. C
11. A	12. D	13. D	14. A	15. C
16. D	17. D	18. C	19. A	20. B
21. B	22. D	23. D	24. A	25. C
26. B	27. B	28. B	29. C	30. D

1. C

$$\begin{aligned} a &= \log \frac{b+c}{b-c} \\ 10^a &= \frac{b+c}{b-c} \\ 10^a(b-c) &= b+c \\ 10^a b - b &= c + 10^a c \\ c &= \frac{b(10^a-1)}{10^a+1} \end{aligned}$$

2. C

$$\begin{aligned} \log(x+y) &= \log x + \log y \\ \log(x+y) &= \log xy \\ x+y &= xy \\ x &= \frac{-y}{1-y} \\ &= \frac{y}{y-1} \end{aligned}$$

3. D

I. ✗。考慮當 $x = 1$ 及 $y = z = 10$ 的情況。可得 $\log x^2 + \log y^2 = \log z^2$ 但 $x^2 + y^2 \neq z^2$。

II. ✓。 $\log x^2 + \log y^2 = \log z^2$

$$\begin{aligned} 2\log x + 2\log y &= 2\log z \\ \log x + \log y &= \log z \end{aligned}$$

III. ✓。$\log x^2 + \log y^2 = \log z^2$

$$\begin{aligned} \log x^2 y^2 &= \log z^2 \\ x^2 y^2 &= z^2 \end{aligned}$$

4. C

$$\log\left(1+x^2\right) = 1 + \log y$$
$$\log\left(1+x^2\right) = \log 10 + \log y$$
$$\log\left(1+x^2\right) = \log(10y)$$
$$1 + x^2 = 10y$$
$$y = \frac{1}{10}(x^2 + 1)$$

5. C

$$\log x^2 - 2 = \log 15x$$
$$\log x^2 - \log 15x = 2$$
$$\log \frac{x^2}{15x} = 2$$
$$\frac{x}{15} = 10^2$$
$$x = 1500$$

6. A

$$\log 10a = 2\log a + 4\log b$$
$$\log 10a = \log\left(a^2b^4\right)$$
$$10a = a^2b^4$$
$$10 = ab^4$$
$$a = \frac{10}{b^4}$$

7. A

$$\frac{\log m + \log n}{\log \frac{m}{n}} = 3$$
$$\log m + \log n = 3(\log m - \log n)$$
$$-2\log m = -4\log n$$
$$\log m = 2\log n$$
$$m = n^2$$

8. C

$$\frac{1}{2}\log x + 1 = \log 5$$
$$\log x^{\frac{1}{2}} = \log \frac{5}{10}$$
$$x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

9. B

$$\log x^2 = 2\log 2 + \log 9$$
$$= \log(4 \times 9)$$
$$x^2 = 36$$
$$x = \pm 6$$

10. C

$$\log(3x + 4) = \log \frac{x}{2} + 1$$
$$\log(3x + 4) = \log \frac{x}{2} + \log 10$$
$$3x + 4 = \frac{x}{2} \times 10$$
$$x = 2$$

11. A

$$\log x^4 = \log 4x^2 + 2$$
$$\log x^4 - \log 4x^2 = 2$$
$$\log \frac{x^4}{4x^2} = 2$$
$$\frac{x^2}{4} = 10^2$$
$$x^2 = 400$$
$$x = 20 \quad \text{或} \quad -20$$

12. D

$$\log_9(x + 2) - 3\log_9 y = \frac{1}{2}$$
$$\log_9 \frac{x + 2}{y^3} = \frac{1}{2}$$
$$\frac{x + 2}{y^3} = 9^{\frac{1}{2}}$$
$$x = 3y^3 - 2$$

13. D

$$\log_2(2x + 1) = \log_4(x + 11)$$
$$\frac{\log(2x + 1)}{\log 2} = \frac{\log(x + 11)}{2\log 2}$$
$$\log(2x + 1)^2 = \log(x + 11)$$
$$(2x + 1)^2 = x + 11$$
$$4x^2 + 3x - 10 = 0$$
$$x = \frac{5}{4} \quad \text{或} \quad -2 \text{（捨去）}$$

14. A

$$2\log y + 1 = 4\log x$$
$$2\log y - 4\log x = -1$$
$$\log \frac{y^2}{x^4} = -1$$
$$\frac{y^2}{x^4} = 10^{-1}$$
$$y^2 = \frac{x^4}{10}$$
$$y = \sqrt{\frac{x^4}{10}}$$

15. C

$$\frac{\log a}{2} = \log \frac{b}{2}$$
$$\log a = \log \left(\frac{b}{2}\right)^2$$
$$a = \frac{b^2}{4}$$

16. D

$$\frac{\log 2x}{\log y} = \frac{2a}{b}$$
$$\log 2x = \frac{2a}{b} \log y$$
$$\log 2x = \log y^{\frac{2a}{b}}$$
$$2x = y^{\frac{2a}{b}}$$
$$x = \frac{1}{2} y^{\frac{2a}{b}}$$

17. D

$$\log \frac{a}{2} = 10 \log b$$
$$\log \frac{a}{2} = \log b^{10}$$
$$\frac{a}{2} = b^{10}$$
$$a = 2b^{10}$$

18. C

$$\log x + \log 2 = 2$$
$$\log 2x = 2$$
$$2x = 10^2$$
$$x = 50$$

19. A

$$\log b - 2\log a = 1$$
$$\log b = 2\log a + \log 10$$
$$\log b = \log\left(10a^2\right)$$
$$b = 10a^2$$

20. B

$$\log_3(2x-1) - \log_3(3x-2) = 2$$
$$\log_3 \frac{2x-1}{3x-2} = 2$$
$$\frac{2x-1}{3x-2} = 3^2$$
$$2x - 1 = 27x - 18$$
$$x = \frac{17}{25}$$

21. B

$$\log(a - 2x) = 3$$
$$a - 2x = 10^3$$
$$x = \frac{1}{2}(a - 1000)$$

22. D

$$\log_2 y - \log_2 x = 6.3 - 2.3$$
$$\log_2 \frac{y}{x} = 4$$
$$\frac{y}{x} = 2^4$$
$$y = 16x$$

23. D

$$\log(x + 1) = -1$$
$$x + 1 = 10^{-1}$$
$$x = -\frac{9}{10}$$

24. A

$$\log_{2x}(4x - 5) = 1$$
$$4x - 5 = (2x)^1$$
$$x = \frac{5}{2}$$

25. C

$\log_5(4x + 3) - 2 = 0$

$\log_5(4x + 3) = 2$

$4x + 3 = 5^2$

$x = \frac{11}{2}$

26. B

$x = y^n$

$\log x = n \log y$

$\frac{\log y}{\log x} = \frac{1}{n}$

$\log_x y = \frac{1}{n}$

27. B

$10^c = ab$

$c = \log ab$

$= \log a + \log b$

28. B

$3^a = 4^b$ 及 $3^a = 12^c$

$a \log 3 = b \log 4$ $\quad a \log 3 = c \log 12$

$\frac{a}{b} = \frac{\log 4}{\log 3}$ $\quad \frac{a}{c} = \frac{\log 12}{\log 3}$

$\frac{a}{b} - \frac{a}{c} = \frac{\log 4}{\log 3} - \frac{\log 12}{\log 3}$

$= \frac{1}{\log 3} \log \frac{4}{12}$

$= \frac{-\log 3}{\log 3}$

$= -1$

29. C

$2^x = 10^z$ 及 $5^y = 10^z$

$x \log 2 = z$ $\quad z = y \log 5$

$\frac{x}{z} = \frac{1}{\log 2}$ $\quad \frac{y}{z} = \frac{1}{\log 5}$

$\frac{x + y}{z} = \frac{1}{\log 2} + \frac{1}{\log 5}$

$= \frac{\log 5 + \log 2}{(\log 2)(\log 5)}$

$= \frac{1}{(\log 2)(\log 5)}$

30. D

$$4^{x+5} + 1 = 11$$

$$4^{x+5} = 10$$

$$(x + 5)\log 4 = \log 10$$

$$x = \frac{\log 10}{\log 4} - 5$$

$$= \frac{1}{\log 4} - 5$$

結構式試題

31. (a) $5 \cdot 4^x - 6 \cdot 3^{2x+1} = 0$

$$5 \cdot 4^x = 6 \cdot 3^{2x+1}$$

$$\log 5 + x \log 4 = \log 6 + (2x + 1) \log 3 \quad \text{1M}$$

$$x(\log 4 - 2 \log 3) = \log 6 + \log 3 - \log 5 \quad \text{1M}$$

$$x = \frac{\log 6 + \log 3 - \log 5}{\log 4 - 2 \log 3}$$

$$\approx -1.58 \quad \text{1A}$$

(b) $3^{5x-1} = 5$

$$(5x - 1) \log 3 = \log 5 \quad \text{1M}$$

$$5x \log 3 = \log 5 + \log 3$$

$$x = \frac{\log 5 + \log 3}{5 \log 3}$$

$$\approx 0.493 \quad \text{1A}$$

32. (a) $3^{x-1} = 7^{4x-3}$

$$(x - 1) \log 3 = (4x - 3) \log 7 \quad \text{1M}$$

$$x(\log 3 - 4 \log 7) = -3 \log 7 + \log 3 \quad \text{1M}$$

$$x = \frac{\log 3 - 3 \log 7}{\log 3 - 4 \log 7}$$

$$\approx 0.709 \quad \text{1A}$$

(b) $2^{2x+3} - 4^{x-2} = 10$

$$4^x(2^3 - 4^{-2}) = 10 \quad \text{1M}$$

$$4^x = \frac{160}{127}$$

$$x \log 4 = \log \frac{160}{127} \quad \text{1M}$$

$$x = \frac{\log \frac{160}{127}}{\log 4}$$

$$\approx 0.167 \quad \text{1A}$$

33. (a) $\log_3(x - 1) + \log_5(x - 1) = 2.5$

$$\frac{\log(x - 1)}{\log 3} + \frac{\log(x - 1)}{\log 5} = 2.5 \quad \text{1M}$$

$$\log(x - 1)\left(\frac{1}{\log 3} + \frac{1}{\log 5}\right) = 2.5$$

$$\log(x - 1) \approx 0.708\,902\,142 \quad \text{1M}$$

$$x - 1 \approx 10^{0.708\,902\,142}$$

$$x \approx 6.12 \quad \text{1A}$$

(b) $7^{x+2} - 7^{x-1} = 342$

$7^x(7^2 - 7^{-1}) = 342$

$7^x = 7$ 1M

$x = 1$ 1A

34. (a) $(\sqrt{3})^{x+2} + (\sqrt{3})^{x+6} = 810$

$(\sqrt{3})^x(3 + 3^3) = 810$ 1M

$(\sqrt{3})^x = 27$

$x \log \sqrt{3} = \log 27$ 1M

$x = \dfrac{\log 27}{\log \sqrt{3}}$

$= 6$ 1A

(b) $(\sqrt[3]{7})^{x+6} - (\sqrt[3]{7})^{x+3} - 14\,406 = 0$

$(\sqrt[3]{7})^x(7^2 - 7) = 14\,406$ 1M

$(\sqrt[3]{7})^x = 343$

$x \log \sqrt[3]{7} = \log 343$ 1M

$x = \dfrac{\log 343}{\log \sqrt[3]{7}}$

$= 9$ 1A

35. (a) $\log_x(2 - x) = 2$

$2 - x = x^2$ 1M

$0 = x^2 + x - 2$ 1A

$= (x + 2)(x - 1)$

$x = -2$（捨去） 或 1（捨去） 1A

方程無解。

(b) $\log_{5\sqrt{2}}(2x^2 - 10x) = 2$

$2x^2 - 10x = (5\sqrt{2})^2$ 1M

$2x^2 - 10x - 50 = 0$

$x = \dfrac{10 \pm \sqrt{10^2 - 4(2)(-50)}}{2(2)}$ 1M

$= \dfrac{5 \pm 5\sqrt{5}}{2}$ 1A

– 試卷完 –