

M2REG-INDINT-2425-ASM-SET 2-MATH

建議題解

1. 設 $u = 2x - 1$ 。則 $du = 2 dx$ 。 1M

$$\begin{aligned}\int (x^2 - 1)\sqrt{2x - 1} dx &= \int \frac{1}{2} \left[\left(\frac{u + 1}{2} \right)^2 - 1 \right] \sqrt{u} du \\ &= \int \left(\frac{1}{8} u^{\frac{5}{2}} + \frac{1}{4} u^{\frac{3}{2}} - \frac{1}{2} u^{\frac{1}{2}} \right) du \\ &= \frac{1}{28} u^{\frac{7}{2}} + \frac{1}{10} u^{\frac{5}{2}} - \frac{1}{3} u^{\frac{3}{2}} + \text{常數} \\ &= \frac{1}{28} (2x - 1)^{\frac{7}{2}} + \frac{1}{10} (2x - 1)^{\frac{5}{2}} - \frac{1}{3} (2x - 1)^{\frac{3}{2}} + \text{常數}\end{aligned}$$

1A

2. 設 $u = e^{2x} - e^x + x$ 。則 $du = (2e^{2x} - e^x + 1) dx$ 。 1M

$$\begin{aligned}\int (2e^{2x} - e^x + 1)\sqrt{e^{2x} - e^x + x} dx &= \int u^{\frac{1}{2}} du \\ &= \frac{2}{3} u^{\frac{3}{2}} + \text{常數} \\ &= \frac{2}{3} (e^{2x} - e^x + x)^{\frac{3}{2}} + \text{常數}\end{aligned}$$

1A

3. 設 $u = e^x + 1$ 。則 $du = e^x dx$ 。 1M

$$\begin{aligned}\int e^x (e^x + 1) dx &= \int u du \\ &= \frac{u^2}{2} + \text{常數} \\ &= \frac{(e^x + 1)^2}{2} + \text{常數}\end{aligned}$$

1A

4. 設 $u = x^4 + 1$ 。則 $du = 4x^3 dx$ 。 1M

$$\begin{aligned}\int \frac{1}{2} x^3 (x^4 + 1)^3 dx &= \frac{1}{8} \int u^3 du \\ &= \frac{1}{32} u^4 + \text{常數} \\ &= \frac{1}{32} (x^4 + 1)^4 + \text{常數}\end{aligned}$$

1A

5. (a) $\int (x^2 + 1) dx = \frac{x^3}{3} + x + \text{常數}$ 1A

- (b) 設 $u = \sqrt{x - 1}$ 。則 $du = \frac{1}{2\sqrt{x - 1}} dx$ 。 1M

$$\begin{aligned}\int \frac{x}{\sqrt{x - 1}} dx &= 2 \int (u^2 + 1) du \\ &= 2 \left(\frac{u^3}{3} + u \right) + \text{常數} \\ &= \frac{2}{3} (x - 1)^{\frac{3}{2}} + 2(x - 1)^{\frac{1}{2}} + \text{常數}\end{aligned}$$

1A

6. 設 $u = e^x - x + 3$ 。則 $du = (e^x - 1) dx$ 。 1M

$$\begin{aligned}\int (e^x - 1) \cos(e^x - x + 3) dx &= \int \cos u du \\ &= \sin u + \text{常數} \\ &= \sin(e^x - x + 3) + \text{常數}\end{aligned}$$

1A

7. 設 $u = \tan x$ 。則 $du = \sec^2 x dx$ 。 1M

$$\begin{aligned}\int \frac{(\sin x)^{\frac{1}{3}}}{(\cos x)^{\frac{7}{3}}} dx &= \int (\tan x)^{\frac{1}{3}} \sec^2 x dx \\ &= \int u^{\frac{1}{3}} du \\ &= \frac{3}{4} u^{\frac{4}{3}} + \text{常數} \\ &= \frac{3}{4} (\tan x)^{\frac{4}{3}} + \text{常數}\end{aligned}$$

1A

8. 設 $u = x^3$ 。則 $du = 3x^2 dx$ 。 1M

$$\begin{aligned}\int x^2 \sec^2(x^3) dx &= \frac{1}{3} \int \sec^2 u du \\ &= \frac{1}{3} \tan u + \text{常數} \\ &= \frac{1}{3} \tan(x^3) + \text{常數}\end{aligned}$$

1A

9. 設 $u = \cos 2x$ 。則 $du = -2 \sin 2x dx$ 。 1M

$$\begin{aligned}\int \sin 2x \cos^2 2x dx &= -\frac{1}{2} \int u^2 du \\ &= -\frac{1}{6} u^3 + \text{常數} \\ &= -\frac{1}{6} \cos^3 2x + \text{常數}\end{aligned}$$

1A

10. 設 $u = x^2 - 6x + 5$ 。則 $du = (2x - 6) dx$ 。 1M

$$\begin{aligned}\int \frac{x-3}{(x^2-6x+5)^3} dx &= \frac{1}{2} \int u^{-3} du \\ &= -\frac{1}{4} u^{-2} + \text{常數} \\ &= -\frac{1}{4} (x^2 - 6x + 5)^{-2} + \text{常數}\end{aligned}$$

1A

11. 設 $u = 2 + \sin x$ 。則 $du = \cos x dx$ 。 1M

$$\begin{aligned}\int \cos x \sqrt{2 + \sin x} dx &= \int u^{\frac{1}{2}} du \\ &= \frac{2}{3} u^{\frac{3}{2}} + \text{常數} \\ &= \frac{2}{3} (2 + \sin x)^{\frac{3}{2}} + \text{常數}\end{aligned}$$

1A

12. 設 $u = x^3 - 3x - 5$ 。則 $du = (3x^2 - 3) dx$ 。 1M

$$\begin{aligned}\int \frac{x^2 - 1}{x^3 - 3x - 5} dx &= \frac{1}{3} \int \frac{du}{u} \\ &= \frac{1}{3} \ln |u| + \text{常數} \\ &= \frac{1}{3} \ln |x^3 - 3x - 5| + \text{常數}\end{aligned}$$

1A

13. 設 $u = \tan 3x$ 。則 $du = 3 \sec^2 3x dx$ 。 1M

$$\begin{aligned}\int \tan^7 3x \sec^2 3x dx &= \frac{1}{3} \int u^7 du \\ &= \frac{1}{24} u^8 + \text{常數} \\ &= \frac{1}{24} \tan^8 3x + \text{常數}\end{aligned}$$

1A

14. 設 $u = \sin x$ 。則 $du = \cos x dx$ 。 1M

$$\begin{aligned}\int \frac{\cos x}{\sqrt{\sin x}} dx &= \int u^{-\frac{1}{2}} du \\ &= 2u^{\frac{1}{2}} + \text{常數} \\ &= 2\sqrt{\sin x} + \text{常數}\end{aligned}$$

1A

15. 設 $u = \tan 2x$ 。則 $du = 2 \sec^2 2x dx$ 。 1M

$$\begin{aligned}\int \tan^5 2x \sec^2 2x dx &= \frac{1}{2} \int u^5 du \\ &= \frac{1}{12} u^6 + \text{常數} \\ &= \frac{1}{12} \tan^6 2x + \text{常數}\end{aligned}$$

1A

16. 設 $u = x + \frac{1}{x^2}$ 。則 $du = \left(1 - \frac{2}{x^3}\right) dx$ 。 1M

$$\begin{aligned}\int \left(1 - \frac{2}{x^3}\right) \sqrt{x + \frac{1}{x^2}} dx &= \int u^{\frac{1}{2}} du \\ &= \frac{2}{3} u^{\frac{3}{2}} + \text{常數} \\ &= \frac{2}{3} \left(x + \frac{1}{x^2}\right)^{\frac{3}{2}} + \text{常數}\end{aligned}$$

1A

17. 設 $u = \sqrt{x} - 1$ 。則 $du = \frac{1}{2\sqrt{x}} dx$ 。 1M

$$\begin{aligned}\int \sqrt{\sqrt{x} - 1} dx &= \int 2\sqrt{x}\sqrt{\sqrt{x} - 1} \cdot \frac{1}{2\sqrt{x}} dx \\ &= \int 2(u + 1)\sqrt{u} du \\ &= \int (2u^{\frac{3}{2}} + 2u^{\frac{1}{2}}) du & 1M \\ &= \frac{4}{5}u^{\frac{5}{2}} + \frac{4}{3}u^{\frac{3}{2}} + \text{常數} \\ &= \frac{4}{5}(\sqrt{x} - 1)^{\frac{5}{2}} + \frac{4}{3}(\sqrt{x} - 1)^{\frac{3}{2}} + \text{常數} & 1A\end{aligned}$$

18. 設 $u = \log_2 x$ 。則 $du = \frac{1}{x \ln 2} dx$ 。 1M

$$\begin{aligned}\int \frac{3(\log_2 x)^2 + 2\log_2 x - 4}{x} dx &= \int (\ln 2)(3u^2 + 2u - 4) du & 1A \\ &= (\ln 2)(u^3 + u^2 - 4u) + \text{常數} \\ &= (\ln 2)[(\log_2 x)^3 + (\log_2 x)^2 - 4\log_2 x] + \text{常數} & 1A\end{aligned}$$

19. 設 $u = 1 + \frac{3}{x^2}$ 。則 $du = -\frac{6}{x^3} dx$ 。

$$\begin{aligned}\int \frac{\sqrt{x^2 + 3}}{x^4} dx &= -\frac{1}{6} \int \left(\frac{\sqrt{x^2 + 3}}{x} \right) \left(-\frac{6}{x^3} \right) dx \\ &= -\frac{1}{6} \int \sqrt{1 + \frac{3}{x^2}} \left(-\frac{6}{x^3} \right) dx & 1M \\ &= -\frac{1}{6} \int \sqrt{u} du & 1A \\ &= -\frac{1}{9}u^{\frac{3}{2}} + \text{常數} \\ &= -\frac{1}{9} \left(1 + \frac{3}{x^2} \right)^{\frac{3}{2}} + \text{常數} & 1A\end{aligned}$$

20. (a) $\int \frac{x-1}{x} dx = \int \left(1 - \frac{1}{x} \right) dx$ 1M
 $= x - \ln|x| + \text{常數}$ 1A

(b) 設 $u = x^2 + 1$ 。則 $du = 2x dx$ 。

$$\begin{aligned}\int \frac{x^3}{x^2 + 1} dx &= \frac{1}{2} \int \frac{u-1}{u} du & 1M \\ &= \frac{1}{2}(u - \ln|u|) + \text{常數} & 1M \\ &= \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2 + 1) + \text{常數} & 1A\end{aligned}$$

21. 設 $u = e^x$ 。則 $du = e^x dx$ 。 1M

$$\begin{aligned}\int e^x 5^{e^x} dx &= \int 5^u du \\ &= \int e^{u \ln 5} du & 1M \\ &= \frac{1}{\ln 5} e^{u \ln 5} + \text{常數} \\ &= \frac{1}{\ln 5} 5^{e^x} + \text{常數} & 1A\end{aligned}$$

$$\begin{aligned}22. \quad (a) \quad P + \frac{Q}{x+4} + \frac{R}{2x-5} &= \frac{P(x+4)(2x-5) + Q(2x-5) + R(x+4)}{(x+4)(2x-5)} \\ &= \frac{2Px^2 + (3P+2Q+R)x + (-20P-5Q+4R)}{(x+4)(2x-5)}\end{aligned}$$

可得 $2P = 4$ 、 $3P + 2Q + R = 11$ 及 $-20P - 5Q + 4R = -59$ 。 1M

求解後，可得 $P = 2$ 、 $Q = 3$ 及 $R = -1$ 。 1A+1A

$$\begin{aligned}(b) \quad \int \frac{4x^2 + 11x - 59}{2x^2 + 3x - 20} dx &= \int \frac{4x^2 + 11x - 59}{(x+4)(2x-5)} dx \\ &= \int \left(2 + \frac{3}{x+4} - \frac{1}{2x-5} \right) dx & 1M \\ &= 2x + 3 \ln |x+4| - \frac{1}{2} \ln |2x-5| + \text{常數} & 1A\end{aligned}$$