

M2REG-INDINT-2425-ASM-SET 1-MATH

建議題解

$$\begin{aligned}
 1. \int (5x^4 + 2x^2 - 3) dx &= 5 \cdot \frac{1}{5}x^5 + 2 \cdot \frac{1}{3}x^3 - 3x + \text{常數} \\
 &= x^5 + \frac{2}{3}x^3 - 3x + \text{常數}
 \end{aligned}$$

1A

$$2. \int \sec^2 5x dx = \frac{1}{5} \tan 5x + \text{常數}$$

1A

$$\begin{aligned}
 3. \int \sqrt{x^3} \sqrt{x} dx &= \int (x^3 x^{\frac{1}{2}})^{\frac{1}{2}} dx \\
 &= \int x^{\frac{7}{4}} dx \\
 &= \frac{4}{11} x^{\frac{11}{4}} + \text{常數}
 \end{aligned}$$

1A

$$4. \int \left(3x^2 - \frac{2}{x} \right) dx = x^3 - 2 \ln |x| + \text{常數}$$

1A

$$\begin{aligned}
 5. \int \sin 3x \cos 7x dx &= \int \frac{1}{2} (\sin 10x - \sin 4x) dx \\
 &= \frac{1}{8} \cos 4x - \frac{1}{20} \cos 10x + \text{常數}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 6. \int \sin 2x \cos 3x dx &= \int \frac{1}{2} (\sin 5x - \sin x) dx \\
 &= \frac{1}{2} \cos x - \frac{1}{10} \cos 5x + \text{常數}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 7. \int \cos x (1 - \tan x) dx &= \int (\cos x - \sin x) dx \\
 &= \sin x + \cos x + \text{常數}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 8. \int \frac{4 \cos x - 4 \cos x \sin^2 x}{\cot^2 x \sin^2 x} dx &= 4 \int \frac{\cos x (1 - \sin^2 x)}{\cos^2 x} dx \\
 &= 4 \int \frac{\cos^3 x}{\cos^2 x} dx \\
 &= 4 \int \cos x dx \\
 &= 4 \sin x + \text{常數}
 \end{aligned}$$

1M
1A

$$\begin{aligned}
 9. \int \frac{1}{\sin^2 x - 1} dx &= \int \left(-\frac{1}{1 - \sin^2 x} \right) dx \\
 &= \int \left(-\frac{1}{\cos^2 x} \right) dx \\
 &= \int (-\sec^2 x) dx \\
 &= -\tan x + \text{常數}
 \end{aligned}$$

1M
1A

$$10. \int \sin \frac{5x}{2} \cos \frac{3x}{2} dx = \frac{1}{2} \int (\sin 4x + \sin x) dx \quad 1M$$

$$= -\frac{1}{8} \cos 4x - \frac{1}{2} \cos x + \text{常數} \quad 1A$$

$$11. y = \int (2e^x + 1)(e^{-3x} - 2) dx$$

$$= \int (2e^{-2x} - 4e^x + e^{-3x} - 2) dx \quad 1M$$

$$= -e^{-2x} - 4e^x - \frac{1}{3}e^{-3x} - 2x + \text{常數} \quad 1A$$

$$12. \int \frac{3 \sin^2 x + 6 \cos^2 x}{\cos^2 x} dx = 3 \int (\tan^2 x + 2) dx$$

$$= 3 \int (\sec^2 x + 1) dx \quad 1M$$

$$= 3 \tan x + 3x + \text{常數} \quad 1A$$

$$13. (a) \int 8x^7 dx = x^8 + \text{常數} \quad 1A$$

$$(b) \int -\frac{1}{6}x^8 dx = -\frac{1}{54}x^9 + \text{常數} \quad 1A$$

$$14. \int \cos^2 4x dx = \int \frac{1}{2}(\cos 8x + 1) dx \quad 1M$$

$$= \frac{1}{16} \sin 8x + \frac{x}{2} + \text{常數} \quad 1A$$

$$15. (a) \int \frac{1}{(4x-7)^4} dx = -\frac{1}{12}(4x-7)^{-3} + \text{常數} \quad 1A$$

$$(b) \int \frac{2}{\sqrt{2x+9}} dx = 2\sqrt{2x+9} + \text{常數} \quad 1A$$

$$16. \int (\sin x - \cos x)^2 dx = \int (\sin^2 x - 2 \sin x \cos x + \cos^2 x) dx$$

$$= \int (1 - \sin 2x) dx \quad 1M$$

$$= x + \frac{1}{2} \cos 2x + \text{常數} \quad 1A$$

$$17. \int x^3(x^2 - 4) dx = \int (x^5 - 4x^3) dx \quad 1M$$

$$= \frac{1}{6}x^6 - 4 \cdot \frac{1}{4}x^4 + \text{常數}$$

$$= \frac{1}{6}x^6 - x^4 + \text{常數} \quad 1A$$

$$18. (a) \int \frac{1}{\sqrt{6x+5}} dx = \frac{\sqrt{6x+5}}{3} + \text{常數} \quad 1A$$

$$(b) \int \frac{1}{\sqrt{3-2x}} dx = -(3-2x)^{\frac{1}{2}} + \text{常數} \quad 1A$$

19. $\int \left(3e^{-\frac{x}{4}} + 2\right)^2 dx = \int \left(9e^{-\frac{x}{2}} + 12e^{-\frac{x}{4}} + 4\right) dx$ 1m
 $= -18e^{-\frac{x}{2}} - 48e^{-\frac{x}{4}} + 4x + \text{常數}$ 1A
20. (a) $\int (x-4)^5 dx = \frac{(x-4)^6}{6} + \text{常數}$ 1A
 (b) $\int (2x+7)^3 dx = \frac{(2x+7)^4}{8} + \text{常數}$ 1A
21. $\int \frac{3+7x^6 \cos^2 x}{\cos^2 x} dx = \int \frac{3}{\cos^2 x} dx + 7 \int x^6 dx$
 $= 3 \int \sec^2 x dx + 7 \int x^6 dx$ 1M
 $= 3 \tan x + x^7 + \text{常數}$ 1A
22. (a) $\int \frac{2}{3-4x} dx = -\frac{1}{2} \ln |3-4x| + \text{常數}$ 1A
 (b) $\int e^{-3x-2} dx = -\frac{1}{3} e^{-3x-2} + \text{常數}$ 1A
23. $\int \frac{5e^{3x} + 4e^{2x}}{2e^{2x}} dx = \int \left(\frac{5}{2}e^x + 2\right) dx$ 1M
 $= \frac{5}{2}e^x + 2x + \text{常數}$ 1A
24. $\int (3x-1)^2 dx = \int (9x^2 - 6x + 1) dx$ 1M
 $= 9 \cdot \frac{1}{3}x^3 - 6 \cdot \frac{1}{2}x^2 + x + \text{常數}$
 $= 3x^3 - 3x^2 + x + \text{常數}$ 1A
25. $\int \frac{x^2(2x^2+3x-3)}{\sqrt{x}} dx = \int (2x^{\frac{7}{2}} + 3x^{\frac{5}{2}} - 3x^{\frac{3}{2}}) dx$ 1M
 $= \frac{4}{9}x^{\frac{9}{2}} + \frac{6}{7}x^{\frac{7}{2}} - \frac{6}{5}x^{\frac{5}{2}} + \text{常數}$ 1A
26. $\int \left(\frac{2e^{2x}-3}{e^{3x}}\right)^2 dx = \int \frac{4e^{4x}-12e^{2x}+9}{e^{6x}} dx$
 $= \int (4e^{-2x} - 12e^{-4x} + 9e^{-6x}) dx$ 1M
 $= -2e^{-2x} + 3e^{-4x} - \frac{3}{2}e^{-6x} + \text{常數}$ 1A
27. $\int \frac{3e^{2x}+5e^x-2}{e^x+2} dx = \int \frac{(3e^x-1)(e^x+2)}{e^x+2} dx$
 $= \int (3e^x-1) dx$ 1M
 $= 3e^x - x + \text{常數}$ 1A

$$\begin{aligned}
 28. \quad y &= \int (2 + \sqrt{x})^2 dx \\
 &= \int (4 + 4\sqrt{x} + x) dx
 \end{aligned}
 \tag{1M}$$

$$\begin{aligned}
 &= 4 \int dx + 4 \int x^{\frac{1}{2}} dx + \int x dx \\
 &= 4x + \frac{8}{3}x^{\frac{3}{2}} + \frac{x^2}{2} + \text{常數}
 \end{aligned}
 \tag{1A}$$

$$\begin{aligned}
 29. \quad \int \frac{(2x-3)(x^2+3x-4)}{x\sqrt{x}} dx &= \int \frac{2x^3+3x^2-17x+12}{x^{\frac{3}{2}}} dx \\
 &= \int (2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 17x^{-\frac{1}{2}} + 12x^{-\frac{3}{2}}) dx
 \end{aligned}
 \tag{1M}$$

$$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} - 34x^{\frac{1}{2}} - 24x^{-\frac{1}{2}} + \text{常數}
 \tag{1A}$$

$$\begin{aligned}
 30. \quad \int \frac{(3x-1)(x+2)}{\sqrt[3]{x^2}} dx &= \int \frac{3x^2+5x-2}{x^{\frac{2}{3}}} dx \\
 &= \int (3x^{\frac{4}{3}} + 5x^{\frac{1}{3}} - 2x^{-\frac{2}{3}}) dx
 \end{aligned}
 \tag{1M}$$

$$= \frac{9}{7}x^{\frac{7}{3}} + \frac{15}{4}x^{\frac{4}{3}} - 6x^{\frac{1}{3}} + \text{常數}
 \tag{1A}$$

$$\begin{aligned}
 31. \quad \int \frac{(2x-1)(x^2+1)}{x^2} dx &= \int \frac{2x^3-x^2+2x-1}{x^2} dx \\
 &= \int \left(2x - 1 + \frac{2}{x} - \frac{1}{x^2} \right) dx
 \end{aligned}
 \tag{1M}$$

$$= x^2 - x + 2 \ln |x| + \frac{1}{x} + \text{常數}
 \tag{1A}$$

$$\begin{aligned}
 32. \quad \int (\sqrt{x} + \sqrt[3]{x})^4 dx &= \int (x^{\frac{1}{2}} + x^{\frac{1}{3}})^4 dx \\
 &= \int \left(x^2 + 4x^{\frac{11}{6}} + 6x^{\frac{5}{3}} + 4x^{\frac{3}{2}} + x^{\frac{4}{3}} \right) dx
 \end{aligned}
 \tag{1M}$$

$$= \frac{1}{3}x^3 + \frac{24}{17}x^{\frac{17}{6}} + \frac{9}{4}x^{\frac{8}{3}} + \frac{8}{5}x^{\frac{5}{2}} + \frac{3}{7}x^{\frac{7}{3}} + \text{常數}
 \tag{1A}$$

$$\begin{aligned}
 33. \quad \int (2\sqrt{x}-1)(\sqrt[3]{x}+1) dx &= \int (2x^{\frac{1}{2}}-1)(x^{\frac{1}{3}}+1) dx \\
 &= \int (2x^{\frac{5}{6}} + 2x^{\frac{1}{2}} - x^{\frac{1}{3}} - 1) dx
 \end{aligned}
 \tag{1M}$$

$$= \frac{12}{11}x^{\frac{11}{6}} + \frac{4}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{4}{3}} - x + \text{常數}
 \tag{1A}$$

$$\begin{aligned}
 34. \quad \int \left(x - \frac{2}{x} \right)^3 dx &= \int \left(x^3 - 6x + \frac{12}{x} - \frac{8}{x^3} \right) dx
 \end{aligned}
 \tag{1M}$$

$$= \frac{x^4}{4} - 3x^2 + 12 \ln |x| + 4x^{-2} + \text{常數}
 \tag{1A}$$

35. (a) $\frac{d}{dx}[(x^4 - 1) \ln(x^2 + 1)] = 4x^3 \ln(x^2 + 1) + (x^4 - 1) \frac{2x}{x^2 + 1}$ 1M
 $= 4x^3 \ln(x^2 + 1) + (x^2 - 1)(x^2 + 1) \frac{2x}{x^2 + 1}$
 $= 2x^3 - 2x + 4x^3 \ln(x^2 + 1)$ 1A
- (b) $\int [2x^3 - 2x + 4x^3 \ln(x^2 + 1)] dx = (x^4 - 1) \ln(x^2 + 1) + \text{常數}$ 1M
 $2 \int (x^3 - x) dx + 4 \int x^3 \ln(x^2 + 1) dx = (x^4 - 1) \ln(x^2 + 1) + \text{常數}$
 $\int x^3 \ln(x^2 + 1) dx = \frac{1}{4}(x^4 - 1) \ln(x^2 + 1) - \frac{1}{2} \int (x^3 - x) dx + \text{常數}$
 $= \frac{1}{4}(x^4 - 1) \ln(x^2 + 1) - \frac{1}{8}x^4 + \frac{1}{4}x^2 + \text{常數}$ 1A
36. (a) $\int \frac{2e^{2x} + 5e^x - 3}{e^x + 3} dx = \int (2e^x - 1) dx$ 1M
 $= 2e^x - x + \text{常數}$ 1A
- (b) $\int \frac{e^{4x} - 4}{e^{3x} - 2e^x} dx = \int \frac{(e^{2x} + 2)(e^{2x} - 2)}{e^x(e^{2x} - 2)} dx$
 $= \int e^x dx + 2 \int e^{-x} dx$ 1M
 $= e^x - 2e^{-x} + \text{常數}$ 1A
37. (a) $\frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = \tan\left(2 \times \frac{x}{2}\right)$
 $= \tan x$ 1
- (b) $\int \frac{\cos x \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} dx = \frac{1}{2} \int \cos x \tan x dx$ 1M
 $= \frac{1}{2} \int \sin x dx$
 $= -\frac{1}{2} \cos x + \text{常數}$ 1A
38. (a) $\int \frac{2x^2 - 3x + 4}{\sqrt{x}} dx = 2 \int x^{\frac{3}{2}} dx - 3 \int x^{\frac{1}{2}} dx + 4 \int x^{-\frac{1}{2}} dx$ 1M
 $= \frac{4}{5}x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + \text{常數}$ 1A
- (b) $\int \frac{x^3 + 6x - 4\sqrt{x}}{\sqrt{x}} dx = \int x^{\frac{5}{2}} dx + 6 \int x^{\frac{1}{2}} dx - 4 \int dx$ 1M
 $= \frac{2}{7}x^{\frac{7}{2}} + 4x^{\frac{3}{2}} - 4x + \text{常數}$ 1A
39. (a) $\int e^{\ln x^{-1}} dx = \int x^{-1} dx$ 1M
 $= \ln|x| + \text{常數}$ 1A
- (b) $\int 3^{2x} 7^x dx = \int 63^x dx$ 1M
 $= \frac{1}{\ln 63} 63^x + \text{常數}$ 1A

$$40. \quad (a) \quad \frac{d}{dx}(x^3 \ln x) = x^3 \left(\frac{1}{x} \right) + (\ln x)(3x^2) \\ = x^2 + 3x^2 \ln x$$

1A

$$(b) \quad \int (x^2 + 3x^2 \ln x) dx = x^3 \ln x + \text{常數}$$

1M

$$\int x^2 dx + 3 \int x^2 \ln x dx = x^3 \ln x + \text{常數}$$

$$3 \int x^2 \ln x dx = x^3 \ln x - \int x^2 dx + \text{常數}$$

$$\int x^2 \ln x dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx + \text{常數}$$

$$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + \text{常數}$$

1A