

M2REG-DIFF-2425-ASM-SET 4-MATH

建議題解

$$1. \quad \frac{dy}{dx} = \frac{1}{2}(5x^2 + 2x)^{-\frac{1}{2}}(10x + 2) \quad 1M$$

$$= (5x + 1)(5x^2 + 2x)^{-\frac{1}{2}} \quad 1A$$

$$\frac{d^2y}{dx^2} = 5(5x^2 + 2x)^{-\frac{1}{2}} + (5x + 1) \left(-\frac{1}{2}\right) (5x^2 + 2x)^{-\frac{3}{2}}(10x + 2)$$

$$= (5x^2 + 2x)^{-\frac{3}{2}} [5(5x^2 + 2x) - (5x + 1)^2]$$

$$= -(5x^2 + 2x)^{-\frac{3}{2}} \quad 1A$$

$$2. \quad \frac{dy}{dx} = 2xe^{x^2} + x^2e^{x^2}(2x) \quad 1M$$

$$= 2e^{x^2}(x + x^3) \quad 1A$$

$$\frac{d^2y}{dx^2} = 2e^{x^2}(2x)(x + x^3) + 2e^{x^2}(1 + 3x^2)$$

$$= 2e^{x^2}[(2x^2 + 2x^4) + (1 + 3x^2)]$$

$$= 2e^{x^2}(2x^4 + 5x^2 + 1) \quad 1A$$

$$3. \quad \frac{dy}{dx} = e^{-\frac{x}{2}} \left(-\frac{1}{2}\right) \cos \frac{x}{2} + e^{-\frac{x}{2}} \left(-\sin \frac{x}{2}\right) \left(\frac{1}{2}\right) \quad 1M$$

$$= -\frac{1}{2}e^{-\frac{x}{2}} \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) \quad 1A$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2}e^{-\frac{x}{2}} \left(-\frac{1}{2}\right) \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) - \frac{1}{2}e^{-\frac{x}{2}} \left(-\frac{1}{2} \sin \frac{x}{2} + \frac{1}{2} \cos \frac{x}{2}\right)$$

$$= \frac{1}{4}e^{-\frac{x}{2}} \left[\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right) + \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)\right]$$

$$= \frac{1}{2}e^{-\frac{x}{2}} \sin \frac{x}{2} \quad 1A$$

$$4. \quad \frac{dy}{dx} = \cos x \ln(\cos x) + \sin x \cdot \frac{-\sin x}{\cos x} \quad 1M$$

$$= \cos x \ln(\cos x) - \sin x \tan x \quad 1A$$

$$\frac{d^2y}{dx^2} = -\sin x \ln(\cos x) + \cos x \cdot \frac{-\sin x}{\cos x} - \cos x \tan x - \sin x \sec^2 x$$

$$= -\sin x [\ln(\cos x) + 1 + 1 + \sec^2 x]$$

$$= -\sin x [\ln(\cos x) + \sec^2 x + 2] \quad 1A$$

$$5. \quad (a) \quad \frac{dy}{dx} = 5 \left(\frac{2x+3}{2x-1} \right)^4 \cdot \frac{2(2x-1) - (2x+3)(2)}{(2x-1)^2} \quad 1M$$

$$= 5 \left(\frac{2x+3}{2x-1} \right)^4 \cdot \frac{-8}{(2x-1)^2}$$

$$= \frac{-40(2x+3)^4}{(2x-1)^6} \quad 1$$

$$(b) \quad \frac{dy}{dx} = -40(2x+3)^4(2x-1)^{-6}$$

$$\frac{d^2y}{dx^2} = -160(2x+3)^3(2)(2x-1)^{-6} - 40(2x+3)^4(-6)(2x-1)^{-7}(2) \quad 1M$$

$$= 160(2x+3)^3(2x-1)^{-7} [-2(2x-1) + 3(2x+3)]$$

$$= 160(2x+3)^3(2x-1)^{-7}(2x+11) \quad 1A$$

$$6. \quad (a) \quad y = \ln(1+x^2) - \ln(1-x^2)$$

$$\frac{dy}{dx} = \frac{2x}{1+x^2} - \frac{-2x}{1-x^2} \quad 1M$$

$$= \frac{2x(1-x^2) + 2x(1+x^2)}{(1-x^2)(1+x^2)}$$

$$= \frac{4x}{1-x^4} \quad 1$$

$$(b) \quad \frac{d^2y}{dx^2} = \frac{4(1-x^4) - 4x(-4x^3)}{(1-x^4)^2} \quad 1M$$

$$= \frac{4(3x^4+1)}{(1-x^4)^2} \quad 1A$$

$$7. \quad \frac{dy}{dx} = e^{-4x} + xe^{-4x}(-4) \quad 1M$$

$$= e^{-4x}(1-4x) \quad 1A$$

$$\frac{d^2y}{dx^2} = e^{-4x}(-4)(1-4x) + e^{-4x}(-4)$$

$$= -4e^{-4x}[(1-4x) + 1]$$

$$= 8e^{-4x}(2x-1) \quad 1A$$

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 4y$$

$$= [8e^{-4x}(2x-1)] + 3[e^{-4x}(1-4x)] - 4xe^{-4x}$$

$$= e^{-4x} [8(2x-1) + 3(1-4x) - 4x]$$

$$= -5e^{-4x} \quad 1$$

$$8. \quad \frac{dy}{dx} = \frac{1}{x} \cdot \frac{1}{x^2} + \ln x(-2x^{-3}) \quad 1M$$

$$= x^{-3}(1 - 2 \ln x) \quad 1A$$

$$\frac{d^2y}{dx^2} = -3x^{-4}(1 - 2 \ln x) + x^{-3} \left(-\frac{2}{x} \right)$$

$$= -x^{-4} [3(1 - 2 \ln x) + 2]$$

$$= -x^{-4}(5 - 6 \ln x) \quad 1A$$

$$x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 4y$$

$$= x^2 [-x^{-4}(5 - 6 \ln x)] + 5x [x^{-3}(1 - 2 \ln x)] + 4x^{-2} \ln x$$

$$= x^{-2} [-(5 - 6 \ln x) + 5(1 - 2 \ln x) + 4 \ln x]$$

$$= 0 \quad 1$$

$$9. \quad \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} \ln x + x^{\frac{3}{2}} \cdot \frac{1}{x} \quad 1M$$

$$= \frac{1}{2}x^{\frac{1}{2}}(3 \ln x + 2) \quad 1A$$

$$\frac{d^2y}{dx^2} = \frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 2) + \frac{1}{2}x^{\frac{1}{2}} \left(\frac{3}{x} \right)$$

$$= \frac{1}{4}x^{-\frac{1}{2}} [(3 \ln x + 2) + 6]$$

$$= \frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 8) \quad 1A$$

$$4x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + 9y$$

$$= 4x^2 \left[\frac{1}{4}x^{-\frac{1}{2}}(3 \ln x + 8) \right] - 8x \left[\frac{1}{2}x^{\frac{1}{2}}(3 \ln x + 2) \right] + 9x^{\frac{3}{2}} \ln x$$

$$= x^{\frac{3}{2}} [(3 \ln x + 8) - 4(3 \ln x + 2) + 9 \ln x]$$

$$= 0 \quad 1$$

$$10. \quad \frac{dy}{dx} = -2x^{-3} \cos x + x^{-2}(-\sin x) \quad 1M$$

$$= -x^{-3}(2 \cos x + x \sin x) \quad 1A$$

$$\frac{d^2y}{dx^2} = 3x^{-4}(2 \cos x + x \sin x) - x^{-3} [-2 \sin x + \sin x + x \cos x]$$

$$= x^{-4} [3(2 \cos x + x \sin x) - x(-2 \sin x + \sin x + x \cos x)]$$

$$= x^{-4}(4x \sin x - x^2 \cos x + 6 \cos x) \quad 1A$$

$$x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + (x^2 + 2)y$$

$$= x^2 [x^{-4}(4x \sin x - x^2 \cos x + 6 \cos x)] + 4x [-x^{-3}(2 \cos x + x \sin x)] + (x^2 + 2)x^{-2} \cos x$$

$$= x^{-2} [(4x \sin x - x^2 \cos x + 6 \cos x) - 4(2 \cos x + x \sin x) + (x^2 + 2) \cos x]$$

$$= 0 \quad 1$$

$$11. \quad \frac{dy}{dx} = 9x^2 - 8x \quad 1A$$

$$\frac{d^2y}{dx^2} = 18x - 8 \quad 1A$$

$$\begin{aligned} 0 &= x^2 \frac{d^2y}{dx^2} + kx \frac{dy}{dx} + my \\ &= x^2(18x - 8) + kx(9x^2 - 8x) + m(3x^3 - 4x^2) \\ &= (18 + 9k + 3m)x^3 + (-8 - 8k - 4m)x^2 \end{aligned}$$

$$\text{可得 } 18 + 9k + 3m = 0 \text{ 及 } -8 - 8k - 4m = 0 \circ \quad 1M$$

$$\text{求解後，可得 } k = -4 \text{ 及 } m = 6 \circ \quad 1A$$

$$12. \quad (a) \quad \frac{dy}{dx} = e^x \sin x + e^x \cos x \quad 1M$$

$$= e^x(\sin x + \cos x) \quad 1A$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= e^x(\sin x + \cos x) + e^x(\cos x - \sin x) \\ &= 2e^x \cos x \quad 1A \end{aligned}$$

$$(b) \quad 0 = \frac{d^2y}{dx^2} - k \frac{dy}{dx} + 2y$$

$$\begin{aligned} 0 &= 2e^x \cos x - k e^x(\sin x + \cos x) + 2e^x \sin x \\ &= e^x[(2 - k) \cos x + (-k + 2) \sin x] \end{aligned}$$

$$\text{可得 } 2 - k = 0 \circ \quad 1M$$

$$\text{因此，} k = 2 \circ \quad 1A$$

$$13. \quad (a) \quad (i) \quad \frac{dy}{dx} = k \left(\frac{x+1}{x-1} \right)^{k-1} \cdot \frac{1(x-1) - (x+1)(1)}{(x-1)^2} \quad 1M$$

$$= k \left(\frac{x+1}{x-1} \right)^k \left(\frac{x+1}{x-1} \right)^{-1} \cdot \frac{-2}{(x-1)^2}$$

$$= ky \frac{-2}{(x+1)(x-1)}$$

$$= \frac{2ky}{1-x^2} \quad 1$$

$$(ii) \quad \frac{d^2y}{dx^2} = \frac{2k \frac{dy}{dx} - 2ky(-2x)}{(1-x^2)^2}$$

$$= \frac{2k}{1-x^2} \frac{dy}{dx} + \frac{4kxy}{(1-x^2)^2} \quad 1A$$

$$(x^2-1) \frac{d^2y}{dx^2} + 2(x+k) \frac{dy}{dx}$$

$$= (x^2-1) \left[\frac{2k}{1-x^2} \frac{dy}{dx} + \frac{4kxy}{(1-x^2)^2} \right] + 2(x+k) \left[\frac{2ky}{1-x^2} \right]$$

$$= -2k \left(\frac{2ky}{1-x^2} \right) - \frac{4kxy}{1-x^2} + \frac{4kxy}{1-x^2} + \frac{4k^2y}{1-x^2}$$

$$= 0 \quad 1$$

$$(b) \quad y = \left(\frac{0+1}{0-1} \right)^k$$

$$= (-1)^k$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{2k(-1)^k}{1-0}$$

$$= 2k(-1)^k$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=0} = \frac{2k}{1} [2k(-1)^k] + 0$$

$$16 = 4k^2(-1)^k \quad 1A$$

可得 k 為正偶數，且 $4k^2 = 16$ 。 $1M$

因此， $k = 2$ 。 $1A$