

M2REG-DIFF-2425-ASM-SET 3-MATH

建議題解

$$\begin{aligned}
 1. \quad y &= \sqrt{(e^x)^3} \\
 &= e^{\frac{3x}{2}} \\
 \frac{dy}{dx} &= e^{\frac{3x}{2}} \left(\frac{3}{2} \right) && 1M \\
 &= \frac{3}{2} e^{\frac{3x}{2}} && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad y &= \ln(4x^3 + 5x^2) \\
 \frac{dy}{dx} &= \frac{12x^2 + 10x}{4x^3 + 5x^2} && 1M \\
 &= \frac{12x + 10}{4x^2 + 5x} && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad y &= \log_2(2 + e^{2x}) \\
 &= \frac{\ln(2 + e^{2x})}{\ln 2} \\
 \frac{dy}{dx} &= \frac{1}{\ln 2} \cdot \frac{2e^{2x}}{2 + e^{2x}} && 1M \\
 &= \frac{2e^{2x}}{(2 + e^{2x}) \ln 2} && 1A
 \end{aligned}$$

$$\begin{aligned}
 4. \quad y &= \sqrt{x}e^{-3x} \\
 &= x^{\frac{1}{2}}e^{-3x} \\
 \frac{dy}{dx} &= \frac{1}{2}x^{-\frac{1}{2}}e^{-3x} + x^{\frac{1}{2}}e^{-3x}(-3) && 1M \\
 &= e^{-3x} \left(\frac{1}{2\sqrt{x}} - 3\sqrt{x} \right) && 1A
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \ln x + \ln y &= 2x^2 + 3x(x + y) \\
 \ln x + \ln y &= 5x^2 + 3xy \\
 \frac{1}{x} + \frac{1}{y} \times \frac{dy}{dx} &= 10x + 3y + 3x \frac{dy}{dx} && 1M \\
 \left(\frac{1}{y} - 3x \right) \frac{dy}{dx} &= 10x + 3y - \frac{1}{x} \\
 \frac{1 - 3xy}{y} \times \frac{dy}{dx} &= \frac{10x^2 + 3xy - 1}{x} \\
 \frac{dy}{dx} &= \frac{y(10x^2 + 3xy - 1)}{x(1 - 3xy)} && 1A
 \end{aligned}$$

6. $\ln(x^2y) = 4x^3 - 10y$
- $$\frac{1}{x^2y} \left(2xy + x^2 \frac{dy}{dx} \right) = 12x^2 - 10 \frac{dy}{dx} \quad 1M$$
- $$\left(\frac{1}{y} + 10 \right) \frac{dy}{dx} = 12x^2 - \frac{2}{x}$$
- $$\left(\frac{1+10y}{y} \right) \frac{dy}{dx} = \frac{12x^3 - 2}{x}$$
- $$\frac{dy}{dx} = \frac{2y(6x^3 - 1)}{x(1+10y)} \quad 1A$$
7. $\ln y = 2 \ln x + 3 \ln(3x - 1) - (-x) - \frac{1}{3} \ln(9 + 4x^2)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x} + \frac{3(3)}{3x-1} + 1 - \frac{1}{3} \left(\frac{8x}{9+4x^2} \right) \quad 1M$$
- $$\frac{dy}{dx} = \frac{x^2 e^x (3x-1)^3}{\sqrt[3]{9+4x^2}} \left[\frac{2}{x} + 1 + \frac{9}{3x-1} - \frac{8x}{3(9+4x^2)} \right] \quad 1A$$
8. $\ln y = \frac{2}{3} \ln(3x+4) + \frac{4}{3} \ln(5+2x) - \frac{1}{3} \ln(2e^{3x} + 3x)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{3} \left(\frac{3}{3x+4} \right) + \frac{4}{3} \left(\frac{2}{5+2x} \right) - \frac{1}{3} \left(\frac{6e^{3x} + 3}{2e^{3x} + 3x} \right) \quad 1M$$
- $$\frac{dy}{dx} = \sqrt[3]{\frac{(3x+4)^2(5+2x)^4}{2e^{3x} + 3x}} \left[\frac{2}{3x+4} + \frac{8}{3(5+2x)} - \frac{2e^{3x} + 1}{2e^{3x} + 3x} \right] \quad 1A$$
9. $\ln y = \ln x \ln(x \tan x + e^x)$ 1M
- $$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{x} \ln(x \tan x + e^x) + \ln x \times \frac{\tan x + x \sec^2 x + e^x}{x \tan x + e^x} \quad 1M$$
- $$\frac{dy}{dx} = (x \tan x + e^x)^{\ln x} \left[\frac{(x \sec^2 x + \tan x + e^x) \ln x}{x \tan x + e^x} + \frac{\ln(x \tan x + e^x)}{x} \right] \quad 1A$$
10. $\sin xy = \ln(x^2 + 2)$
- $$\cos xy \left(y + x \frac{dy}{dx} \right) = \frac{2x}{x^2 + 2} \quad 1M$$
- $$x \cos xy \frac{dy}{dx} + y \cos xy = \frac{2x}{x^2 + 2}$$
- $$x \cos xy \frac{dy}{dx} = \frac{2x - (x^2 + 2)y \cos xy}{x^2 + 2}$$
- $$\frac{dy}{dx} = \frac{2x - (x^2 + 2)y \cos xy}{(x^2 + 2)x \cos xy} \quad 1A$$
11. $\ln(\sin y) = \cos xy + 3$
- $$\frac{\cos y}{\sin y} \times \frac{dy}{dx} = -\sin xy \left(y + x \frac{dy}{dx} \right) \quad 1M$$
- $$(\cot y + x \sin xy) \frac{dy}{dx} = -y \sin xy$$
- $$\frac{dy}{dx} = -\frac{y \sin xy}{\cot y + x \sin xy} \quad 1A$$

12. $y^3 = 2 \cos(x + y)$

$$3y^2 \frac{dy}{dx} = -2 \sin(x + y) \left(1 + \frac{dy}{dx} \right) \quad 1M$$

$$[3y^2 + 2 \sin(x + y)] \frac{dy}{dx} = -2 \sin(x + y)$$

$$\frac{dy}{dx} = -\frac{2 \sin(x + y)}{3y^2 + 2 \sin(x + y)} \quad 1A$$

13. $\sin x^2 y - \cos y = 3x^2$

$$\cos x^2 y \left(2xy + x^2 \frac{dy}{dx} \right) + \sin y \frac{dy}{dx} = 6x \quad 1M$$

$$(x^2 \cos x^2 y + \sin y) \frac{dy}{dx} = 6x - 2xy \cos x^2 y$$

$$\frac{dy}{dx} = \frac{6x - 2xy \cos x^2 y}{x^2 \cos x^2 y + \sin y} \quad 1A$$