

M2REG-DEFINT-2425-ASM-SET 1-MATH

建議題解

$$\begin{aligned}
 1. \quad \int_2^4 \left(x - \frac{1}{x}\right)^3 dx &= \int_2^4 \left[x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}\right] dx && 1M \\
 &= \left[\frac{x^4}{4} - \frac{3x^2}{2} + 3 \ln|x| - \frac{1}{2x^2}\right]_2^4 \\
 &= \frac{1341}{32} + 3 \ln 2 && 1A
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (a) \quad \frac{A}{(x-2)^2} + \frac{B}{x^2} &= \frac{Ax^2 + B(x-2)^2}{x^2(x-2)^2} \\
 &= \frac{(A+B)x^2 - 4Bx + 4B}{x^2(x-2)^2} \\
 \text{可得 } A+B &= 0, -4B = 1 \text{ 及 } 4B = -1. && 1M \\
 \text{求解後, 可得 } A &= \frac{5}{4} \text{ 及 } B = -\frac{1}{4}. && 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \int_4^6 \frac{x-1}{x^2(x-2)^2} dx &= \int_4^6 \left(\frac{1}{4(x-2)^2} - \frac{1}{4x^2}\right) dx && 1M \\
 &= \left[-\frac{1}{4(x-2)} + \frac{1}{4x}\right]_4^6 \\
 &= \frac{1}{24} && 1A
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \int_0^2 \frac{1}{\sqrt{2x+1} - \sqrt{2x}} dx &= \int_0^2 \left(\frac{1}{\sqrt{2x+1} - \sqrt{2x}} \times \frac{\sqrt{2x+1} + \sqrt{2x}}{\sqrt{2x+1} + \sqrt{2x}}\right) dx && 1M \\
 &= \int_0^2 (\sqrt{2x+1} + \sqrt{2x}) dx \\
 &= \left[\frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(2x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 && 1A \\
 &= \frac{5\sqrt{5} + 7}{3} && 1A
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \int_1^9 \frac{x-16}{\sqrt{x}-4} dx &= \int_1^9 \frac{(\sqrt{x}+4)(\sqrt{x}-4)}{\sqrt{x}-4} dx && 1M \\
 &= \int_1^9 (\sqrt{x}+4) dx \\
 &= \left[\frac{2}{3}x^{\frac{3}{2}} + 4x\right]_1^9 \\
 &= \frac{148}{3} && 1A
 \end{aligned}$$

$$\begin{aligned}
5. \int_{-\pi}^{\frac{\pi}{2}} \frac{e^{2x} - \pi e^x - 2\pi^2}{e^x + \pi} dx &= \int_{-\pi}^{\frac{\pi}{2}} \frac{e^x(e^x + \pi) - 2\pi(e^x + \pi)}{e^x + \pi} dx \\
&= \int_{-\pi}^{\frac{\pi}{2}} (e^x - 2\pi) dx && 1M \\
&= \left[e^x - 2\pi x \right]_{-\pi}^{\frac{\pi}{2}} \\
&= e^{\frac{\pi}{2}} - e^{-\pi} - 3\pi^2 && 1A
\end{aligned}$$

$$\begin{aligned}
6. \int_0^2 (3 - e^{-2x})^2 dx &= \int_0^2 (9 - 6e^{-2x} + e^{-4x}) dx && 1M \\
&= \left[9x + 3e^{-2x} - \frac{1}{4}e^{-4x} \right]_0^2 \\
&= \frac{61}{4} + 3e^{-4} - \frac{1}{4}e^{-8} && 1A
\end{aligned}$$

$$\begin{aligned}
7. \int_0^{\frac{\pi}{4}} \sin^4 x dx &= \frac{1}{4} \int_0^{\frac{\pi}{4}} (1 - \cos 2x)^2 dx && 1M \\
&= \frac{1}{4} \int_0^{\frac{\pi}{4}} (1 - 2\cos 2x + \cos^2 2x) dx \\
&= \frac{1}{4} \int_0^{\frac{\pi}{4}} \left[1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x) \right] dx && 1M \\
&= \frac{1}{4} \left[\frac{3x}{2} - \sin 2x + \frac{\sin 4x}{8} \right]_0^{\frac{\pi}{4}} \\
&= \frac{1}{4} \left(\frac{3\pi}{8} - 1 + 0 \right) - \frac{1}{4}(0) \\
&= \frac{3\pi}{32} - \frac{1}{4} && 1A
\end{aligned}$$

$$\begin{aligned}
8. \int_0^{\frac{\pi}{6}} \cos 6x \cos 2x dx &= \frac{1}{2} \int_0^{\frac{\pi}{6}} (\cos 4x + \cos 8x) dx && 1M \\
&= \frac{1}{2} \left[\frac{\sin 4x}{4} + \frac{\sin 8x}{8} \right]_0^{\frac{\pi}{6}} \\
&= \frac{\sqrt{3}}{32} && 1A
\end{aligned}$$

$$\begin{aligned}
9. \int_0^{\frac{\pi}{8}} \sin 5x \sin 3x dx &= \frac{1}{2} \int_0^{\frac{\pi}{8}} (\cos 2x - \cos 8x) dx && 1M \\
&= \frac{1}{2} \left[\frac{\sin 2x}{2} - \frac{\sin 8x}{8} \right]_0^{\frac{\pi}{8}} \\
&= \frac{\sqrt{2}}{8} && 1A
\end{aligned}$$

$$\begin{aligned}
10. \quad \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \cos 2x} \, dx &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + (2 \cos^2 x - 1)} \, dx && 1M \\
&= \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \, dx && 1A \\
&= \frac{1}{2} \left[\tan x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\
&= \frac{\sqrt{3}}{3} && 1A
\end{aligned}$$

$$\begin{aligned}
11. \quad \int_0^{\frac{\pi}{4}} \frac{1 - \cos 2x}{1 + \cos 2x} \, dx &= \int_0^{\frac{\pi}{4}} \frac{1 - (1 - 2 \sin^2 x)}{1 + (2 \cos^2 x - 1)} \, dx && 1M \\
&= \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \\
&= \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \, dx && 1M \\
&= \left[\tan x - x \right]_0^{\frac{\pi}{4}} \\
&= 1 - \frac{\pi}{4} && 1A
\end{aligned}$$

$$\begin{aligned}
12. \quad \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{2(1 + \sin x)} \, dx + \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos 2x}{2(1 + \sin x)} \, dx \\
&= \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1 + (1 - 2 \sin^2 x)}{2(1 + \sin x)} \, dx && 1M \\
&= \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(1 + \sin x)(1 - \sin x)}{1 + \sin x} \, dx \\
&= \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (1 - \sin x) \, dx && 1A \\
&= \left[x + \cos x \right]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \\
&= \frac{2\pi}{3} - \frac{\sqrt{3}}{2} && 1A
\end{aligned}$$

$$\begin{aligned}
13. \quad \int_1^4 \frac{3x}{1 + e^x} \, dx - 3 \int_4^1 \frac{x e^x}{1 + e^x} \, dx \\
&= \int_1^4 \frac{3x}{1 + e^x} + 3 \int_1^4 \frac{x e^x}{1 + e^x} \, dx && 1M \\
&= \int_1^4 \frac{3x(1 + e^x)}{1 + e^x} \, dx \\
&= \int_1^4 3x \, dx \\
&= \left[\frac{3x^2}{2} \right]_1^4 \\
&= \frac{45}{2} && 1A
\end{aligned}$$

$$\begin{aligned}
 14. \quad \int_0^4 f(x) \, dx + \int_5^2 f(x) \, dx &= \int_0^2 f(x) \, dx + \int_2^4 f(x) \, dx - \int_2^5 f(x) \, dx & 1M \\
 &= \int_0^2 f(x) \, dx - \left(\int_4^2 f(x) \, dx + \int_2^5 f(x) \, dx \right) \\
 &= \int_0^2 f(x) \, dx - \int_4^5 f(x) \, dx & 1M \\
 &= k & 1A
 \end{aligned}$$

$$\begin{aligned}
 15. \quad (a) \quad \int_{-1}^1 p(x) \, dx &= \int_{-1}^4 p(x) \, dx + \int_4^1 p(x) \, dx \\
 &= \int_{-1}^4 p(x) \, dx - \int_1^4 p(x) \, dx & 1M \\
 &= 2 & 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \int_{-1}^1 5[p(x) - 3q(x)] \, dx &= 5 \int_{-1}^1 p(x) \, dx - 3 \int_{-1}^1 q(x) \, dx & 1M \\
 &= 5(2) - 15(-5) \\
 &= 85 & 1A
 \end{aligned}$$

$$\begin{aligned}
 16. \quad (a) \quad \frac{1}{4} \int_{-3}^2 f(x) \, dx &= \frac{1}{4} \left[\int_{-3}^{-1.5} f(x) \, dx + \int_{-1.5}^2 f(x) \, dx \right] & 1M \\
 &= \frac{1}{4}(2 + 4) \\
 &= \frac{3}{2} & 1A
 \end{aligned}$$

$$(b) \quad \int_{-1}^{-1} f(x) \, dx = 0 \quad 1A$$

$$\begin{aligned}
 (c) \quad \int_{-3}^{-1.5} \left[f(t) + \frac{1}{t} \right] dt &= \int_{-3}^{-1.5} f(x) \, dx + \left[\ln |t| \right]_{-3}^{-1.5} & 1A \\
 &= 2 - \ln 2 & 1A
 \end{aligned}$$

$$\begin{aligned}
 17. \quad (a) \quad \int_4^2 \frac{2f(x)}{f(x) - 2g(x)} \, dx &= -2 \int_2^4 \frac{f(x)}{f(x) - 2g(x)} \, dx \\
 &= -2(10) \\
 &= -20 & 1A
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \int_2^4 \frac{g(t)}{f(t) - 2g(t)} \, dt &= \int_2^4 \frac{g(x)}{f(x) - 2g(x)} \, dx \\
 &= -\frac{1}{2} \int_2^4 \left[1 - \frac{f(x)}{f(x) - 2g(x)} \right] dx & 1M \\
 &= -\frac{1}{2} \left[x \right]_2^4 + \frac{1}{2} \int_2^4 \frac{f(x)}{f(x) - 2g(x)} \, dx \\
 &= -\frac{1}{2}(4 - 2) + \frac{1}{2}(10) \\
 &= 4 & 1A
 \end{aligned}$$