

REG-LOCUS-2425-ASM-SET 3-MATH

Suggested solutions

Conventional Questions

1. (a) (i) Slope of $L = \frac{-2}{3}$.

The equation of ℓ is

$$y - 0 = \frac{3}{2}(x - 14)$$

1M

$$y = \frac{3x}{2} - 21$$

1A

(ii) Required distance $= \left(\frac{-2k}{3} + 5\right) - \left(\frac{3k}{2} - 21\right)$

$$= \frac{-13k}{6} + 26$$

1M

1A

(b) (i) AB is the radius of Γ .

1A

(ii) $\frac{-13k}{6} + 26 = 39$

1M

$$k = -6$$

Coordinates of A are $(-6, 9)$.

Denote the intersection of L and ℓ by E .

Solving L and ℓ , the coordinate of E are $(12, -3)$.

1M

$$AE = \sqrt{(12 + 6)^2 + (9 + 3)^2} = \sqrt{468}$$

$$\frac{r}{1} = \frac{\sqrt{468}}{39 - \sqrt{468}}$$

1M

$$r \approx 1.25$$

1A

2. (a) (i) Let $P(x, y)$.

$$\frac{y+1}{x-5} \times \frac{y-5}{x+3} = -1$$

1M+1M

$$(x+3)(x-5) + (y+1)(y-5) = 0$$

$$x^2 + y^2 - 2x - 4y - 20 = 0$$

1A

The equation of locus of P is $x^2 + y^2 - 2x - 4y - 20 = 0$.

(ii) The locus of P is a circle with AB as diameter, excluding points A and B .

1A

(b) Centre of C is at $(9, 8)$.

Distance between centres $= \sqrt{(9-1)^2 + (8-2)^2} = 10$.

1M

Sum of radii $= \sqrt{25} + 5 = 10 =$ distance between centres

1M

The circles touch each other externally.

The claim is disagreed.

1A

3. (a) $(0, 7)$

1A

(b) (i) Let $P(x, y)$.

$$\begin{aligned}\sqrt{x^2 + (y-7)^2} &= \sqrt{(x-8)^2 + (y-1)^2} \\ x^2 + y^2 - 14y + 49 &= x^2 + y^2 - 16x - 2y + 65 \\ 4x - 3y - 4 &= 0\end{aligned}$$

1M

1A

The equation of locus of P is $4x - 3y - 4 = 0$.

(ii) Let the mid-point of AE be F . Then $F(4, 4)$.

$$AF = \sqrt{4^2 + 3^2} = 5 \text{ and radius of } C = \sqrt{7^2 - 40} = 3 < 5.$$

1M

Thus, F lies outside the circle and C does not have any intersection with Γ .

1A

(c) Required ratio = $AH : AE$

$$\begin{aligned}&= 3 : 2 \times 5 \\ &= 3 : 10\end{aligned}$$

1A

4. (a) mid-point of PQ is $(0, 2)$.

Equation of C is

$$\begin{aligned}x^2 + (y-2)^2 &= (8)^2 + (8-2)^2 \\ x^2 + (y-2)^2 &= 100\end{aligned}$$

1M

1A

(b) $GR = \sqrt{(4-0)^2 + (5-2)^2} = 5 < 10$

1A

Thus, R lies inside C .

1

(c) $RU = \sqrt{10^2 - 5^2} = \sqrt{75}$

1M

$TR \perp UV$ and $\triangle TUV$ is an isosceles triangle.

$$TU = \sqrt{(\sqrt{75})^2 + (10+5)^2} = \sqrt{300}$$

1M

$$\text{Perimeter of } \triangle TUV = 2\sqrt{300} + 2\sqrt{75}$$

1M

$$\approx 52.0 > 50$$

1A

The claim is agreed.

1A

5. (a) (i) $\sqrt{(x-7)^2 + (y+3)^2} = \sqrt{(x-1)^2 + (y+1)^2}$

1M+1A

$$-12x + 4y + 56 = 0$$

1A

$$3x - y - 14 = 0$$

The equation of Γ is $3x - y - 14 = 0$.

1A

(ii) Γ is the perpendicular bisector of AB .

1A

(b) (i) 3

1A

(ii) When $\triangle AQB$ is an acute-angled triangle,

$$\angle AQB = \frac{1}{2} \angle AMB = 30^\circ.$$

1M+1A

When $\triangle AQB$ is an obtuse-angled triangle,

$$\angle AQB = 180^\circ - 30^\circ = 150^\circ.$$

1A

6. (a) (i) Slope of $L = \frac{5-4}{5-2} = \frac{1}{3}$
 Equation of L is

$$y - 5 = \frac{1}{3}(x - 5)$$

$$x - 3y + 10 = 0$$

1M

1A

$$(ii) 0 - 3y + 10 = 0$$

$$y = \frac{10}{3}$$

The coordinates of C are $\left(0, \frac{10}{3}\right)$.

1A

$$(b) (i) \sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-5)^2}$$

$$x^2 + y^2 - 4x - 8y + 20 = x^2 + y^2 - 10x - 10y + 50$$

$$3x + y - 15 = 0$$

Required equation is $3x + y - 15 = 0$.

1A

$$(ii) 3x - 0 - 15 = 0$$

$$x = 5$$

The coordinates of D are $(5, 0)$.

1A

$$(c) \text{ Solve } \begin{cases} x - 3y - 10 = 0 \\ 3x + y - 15 = 0 \end{cases}, \text{ we have } (x, y) = \left(\frac{7}{2}, \frac{9}{2}\right).$$

1A

$$\begin{aligned} \text{Area of } OCED &= \frac{1}{2} \left(\frac{10}{3}\right) \left(\frac{7}{2}\right) + \frac{1}{2}(5) \left(\frac{9}{2}\right) \\ &= \frac{205}{12} < 20 \end{aligned}$$

1M

The claim is agreed.

1A

7. (a) AD is the perpendicular bisector of BC . The coordinates of D are $(-8, -6)$.

1A

Since D is the mid-point of BC , the coordinates of C are $(-8, -16)$.

1A

$$\text{mid-point of } AC = (1, -11) \text{ and slope of } AC = \frac{-6 + 16}{10 + 8} = \frac{5}{9}$$

1A

Required equation is

$$y + 11 = -\frac{9}{5}(x - 1)$$

1M

$$9x + 5y + 46 = 0$$

1A

(b) Since AD is perpendicular bisector of BC , y -coordinate of circumcentre = -6 .

1M

$$\text{Put } y = -6 \text{ into } 9x + 5y + 46 = 0, \text{ we have } x = -\frac{16}{9}.$$

16
9

$$\text{Required coordinates are } \left(-\frac{16}{9}, -6\right).$$

1A

(c) Required equation is

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \left(10 + \frac{16}{9}\right)^2 + (-6 + 6)^2$$

1M

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \frac{11236}{81}$$

1A

(d) (i) Locus of P is a parabola (opens rightwards).

1A

(ii) Let the coordinates of P be (x, y) .

$$\sqrt{(x - 10)^2 + (y + 6)^2} = \sqrt{(x + 8)^2}$$

$$y^2 - 36x + 12y + 72 = 0$$

Required equation is $y^2 - 36x + 12y + 72 = 0$.

1A