

REG-LOCUS-2425-ASM-SET 3-MATH

Suggested solutions

Conventional Questions

1. (a) (i) Slope of $L = \frac{-2}{3}$.
The equation of ℓ is

$$y - 0 = \frac{3}{2}(x - 14) \quad 1M$$

$$y = \frac{3x}{2} - 21 \quad 1A$$
 - (ii) Required distance = $\left(\frac{-2k}{3} + 5\right) - \left(\frac{3k}{2} - 21\right) \quad 1M$

$$= \frac{-13k}{6} + 26 \quad 1A$$
 - (b) (i) AB is the radius of Γ . 1A
 - (ii) $\frac{-13k}{6} + 26 = 39 \quad 1M$

$$k = -6$$

Coordinates of A are $(-6, 9)$.
 Denote the intersection of L and ℓ by E .
 Solving L and ℓ , the coordinate of E are $(12, -3)$. 1M

$$AE = \sqrt{(12+6)^2 + (9+3)^2} = \sqrt{468}$$

$$\frac{r}{1} = \frac{\sqrt{468}}{39 - \sqrt{468}} \quad 1M$$

$$r \approx 1.25 \quad 1A$$
2. (a) (i) Let $P(x, y)$.

$$\frac{y+1}{x-5} \times \frac{y-5}{x+3} = -1 \quad 1M+1M$$

$$(x+3)(x-5) + (y+1)(y-5) = 0$$

$$x^2 + y^2 - 2x - 4y - 20 = 0 \quad 1A$$

The equation of locus of P is $x^2 + y^2 - 2x - 4y - 20 = 0$.
 - (ii) The locus of P is a circle with AB as diameter, excluding points A and B . 1A
 - (b) Centre of C is at $(9, 8)$.
 Distance between centres = $\sqrt{(9-1)^2 + (8-2)^2} = 10$. 1M
 Sum of radii = $\sqrt{25} + 5 = 10 =$ distance between centres 1M
 The circles touch each other externally.
 The claim is disagreed. 1A
3. (a) $(0, 7)$ 1A

- (b) (i) Let $P(x, y)$.

$$\sqrt{x^2 + (y - 7)^2} = \sqrt{(x - 8)^2 + (y - 1)^2} \quad 1M$$

$$x^2 + y^2 - 14y + 49 = x^2 + y^2 - 16x - 2y + 65$$

$$4x - 3y - 4 = 0 \quad 1A$$

The equation of locus of P is $4x - 3y - 4 = 0$.

- (ii) Let the mid-point of AE be F . Then $F(4, 4)$.

$$AF = \sqrt{4^2 + 3^2} = 5 \text{ and radius of } C = \sqrt{7^2 - 40} = 3 < 5. \quad 1M$$

Thus, F lies outside the circle and C does not have any intersection with Γ . 1A

- (c) Required ratio = $AH : AE$

$$= 3 : 2 \times 5$$

$$= 3 : 10 \quad 1A$$

4. (a) mid-point of PQ is $(0, 2)$.

Equation of C is

$$x^2 + (y - 2)^2 = (8)^2 + (8 - 2)^2 \quad 1M$$

$$x^2 + (y - 2)^2 = 100 \quad 1A$$

- (b) $GR = \sqrt{(4 - 0)^2 + (5 - 2)^2} = 5 < 10$ 1A

Thus, R lies inside C . 1

- (c) $RU = \sqrt{10^2 - 5^2} = \sqrt{75}$ 1M

$TR \perp UV$ and $\triangle TUV$ is an isosceles triangle.

$$TU = \sqrt{(\sqrt{75})^2 + (10 + 5)^2} = \sqrt{300}$$

$$\text{Perimeter of } \triangle TUV = 2\sqrt{300} + 2\sqrt{75} \quad 1M$$

$$\approx 52.0 > 50 \quad 1A$$

The claim is agreed. 1A

5. (a) (i) $\sqrt{(x - 7)^2 + (y + 3)^2} = \sqrt{(x - 1)^2 + (y + 1)^2}$ 1M+1A

$$-12x + 4y + 56 = 0$$

$$3x - y - 14 = 0 \quad 1A$$

The equation of Γ is $3x - y - 14 = 0$.

- (ii) Γ is the perpendicular bisector of AB . 1A

- (b) (i) 3 1A

- (ii) When $\triangle AQB$ is an acute-angled triangle,

$$\angle AQB = \frac{1}{2} \angle AMB = 30^\circ. \quad 1M+1A$$

When $\triangle AQB$ is an obtuse-angled triangle,

$$\angle AQB = 180^\circ - 30^\circ = 150^\circ. \quad 1A$$

6. (a) (i) Slope of $L = \frac{5-4}{5-2} = \frac{1}{3}$
Equation of L is

$$y - 5 = \frac{1}{3}(x - 5) \quad 1M$$

$$x - 3y + 10 = 0 \quad 1A$$
- (ii) $0 - 3y + 10 = 0$

$$y = \frac{10}{3}$$

The coordinates of C are $\left(0, \frac{10}{3}\right)$. 1A
- (b) (i) $\sqrt{(x-2)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-5)^2}$ 1M

$$x^2 + y^2 - 4x - 8y + 20 = x^2 + y^2 - 10x - 10y + 50$$

$$3x + y - 15 = 0 \quad 1A$$

Required equation is $3x + y - 15 = 0$.
- (ii) $3x - 0 - 15 = 0$

$$x = 5$$

The coordinates of D are $(5, 0)$. 1A
- (c) Solve $\begin{cases} x - 3y - 10 = 0 \\ 3x + y - 15 = 0 \end{cases}$, we have $(x, y) = \left(\frac{7}{2}, \frac{9}{2}\right)$. 1A
Area of $OCED = \frac{1}{2} \left(\frac{10}{3}\right) \left(\frac{7}{2}\right) + \frac{1}{2}(5) \left(\frac{9}{2}\right)$ 1M

$$= \frac{205}{12} < 20$$

The claim is agreed. 1A
7. (a) AD is the perpendicular bisector of BC . The coordinates of D are $(-8, -6)$. 1A
Since D is the mid-point of BC , the coordinates of C are $(-8, -16)$. 1A
mid-point of $AC = (1, -11)$ and slope of $AC = \frac{-6+16}{10+8} = \frac{5}{9}$ 1A
Required equation is

$$y + 11 = -\frac{9}{5}(x - 1) \quad 1M$$

$$9x + 5y + 46 = 0 \quad 1A$$
- (b) Since AD is perpendicular bisector of BC , y -coordinate of circumcentre $= -6$. 1M
Put $y = -6$ into $9x + 5y + 46 = 0$, we have $x = -\frac{16}{9}$.
Required coordinates are $\left(-\frac{16}{9}, -6\right)$. 1A
- (c) Required equation is

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \left(10 + \frac{16}{9}\right)^2 + (-6 + 6)^2 \quad 1M$$

$$\left(x + \frac{16}{9}\right)^2 + (y + 6)^2 = \frac{11\,236}{81} \quad 1A$$

(d) (i) Locus of P is a parabola (opens rightwards). 1A

(ii) Let the coordinates of P be (x, y) .

$$\sqrt{(x - 10)^2 + (y + 6)^2} = \sqrt{(x + 8)^2} \quad 1M$$

$$y^2 - 36x + 12y + 72 = 0$$

Required equation is $y^2 - 36x + 12y + 72 = 0$. 1A