**REG-LOCUS-2425-ASM-SET 2-MATH**
**Suggested solutions**

**Conventional Questions**

1. (a) The locus of $P$ is parallel to $L$. 1A

   (b) Let $(x,\ y)$ be the coordinates of $P$.
   Distance of $P$ from $L$ is 2.

$$y-(-1)=2 \quad \text{1M}$$
$$y-1=0 \quad \text{1A}$$

   The equation of the locus of $P$ is $y-1=0$.

2. Let $(x,\ y)$ be the coordinates of $P$.

$$AP=\sqrt{7}$$
$$\sqrt{[x-(-3)]^2+(y-2)^2}=\sqrt{7} \quad \text{1M}$$
$$(x+3)^2+(y-2)^2=7$$
$$x^2+y^2+6x-4y+6=0 \quad \text{1A}$$

   The equation of the locus of $P$ is $x^2+y^2+6x-4y+6=0$.

3. Let $(x,\ y)$ be the coordinates of $P$.

$$PB=2PA$$
$$\sqrt{(x-4)^2+(y+6)^2}=2\sqrt{(x-3)^2+(y-0)^2} \quad \text{1M}$$
$$(x-4)^2+(y+6)^2=4(x-3)^2+4y^2$$
$$3x^2+3y^2-16x-12y-16=0 \quad \text{1A}$$

   The equation of the locus of $P$ is $3x^2+3y^2-16x-12y-16=0$.

4. (a) The locus of $P$ is the perpendicular bisector of the line segment $AB$. 1A

   (b) Let $(x,\ y)$ be the coordinates of $P$.

$$PA=PB$$
$$\sqrt{(x-9)^2+[y-(-7)]^2}=\sqrt{[x-(-4)]^2+(y-6)^2} \quad \text{1M}$$
$$(x-9)^2+(y+7)^2=(x+4)^2+(y-6)^2$$
$$26x-26y-78=0$$
$$x-y-3=0 \quad \text{1A}$$

   The equation of the locus of $P$ is $x-y-3=0$.

5. Let $(x,\ y)$ be the coordinates of $P$.

$$2PA = 3PB$$

$$2\sqrt{(x-9)^2+(y-3)^2} = 3\sqrt{(x-4)^2+\left(y-\frac{4}{3}\right)^2} \quad \text{1M}$$

$$4[(x-9)^2+(y-3)^2] = 9\left[(x-4)^2+\left(y-\frac{4}{3}\right)^2\right]$$

$$5x^2+5y^2-200=0$$

$$x^2+y^2-40=0 \quad \text{1A}$$

The equation of the locus of $P$ is $x^2+y^2-40=0$.

6. Let $(x,\ y)$ be the coordinates of $P$.

Slope of $AP = \dfrac{y-(-2)}{x-6} = \dfrac{y+2}{x-6}$ 1M

Slope of $PB = \dfrac{y-4}{x-2}$

$$\frac{y+2}{x-6}\cdot\frac{y-4}{x-2} = -1 \quad \text{1M}$$

$$(y+2)(y-4) = -(x-6)(x-2)$$

$$x^2+y^2-8x-2y+4=0 \quad \text{1A}$$

The equation of the locus of $P$ is $x^2+y^2-8x-2y+4=0$.

7. The locus of $P$ is two circles with centre $O$ and radii 7 and 11. 1M

When $P$ maintains a fixed distance of 7 from $O$,
the equation of locus is $x^2+y^2=49$. 1A

When $P$ maintains a fixed distance of 11 from $O$,
the equation of locus is $x^2+y^2=121$. 1A

8. (a) The locus of $P$ is a pair of angle bisectors of the angles formed by $L$ and the $x$-axis. 1A

(b) Let $\theta$ be the inclination of $L$.

$$\tan\theta = -\sqrt{3} \quad \text{1M}$$

$$\theta = 60^\circ$$

The inclinations of the two angle bisectors are $30^\circ$ and $120^\circ$ 1A

$x$-intercept of $L = \dfrac{2}{\sqrt{3}}$

The equations of the locus of $P$ are

$$y-0=(\tan 30^\circ)\left(x-\frac{2}{\sqrt{3}}\right) \quad \text{and} \quad y-0=(\tan 120^\circ)\left(x-\frac{2}{\sqrt{3}}\right)$$

$$\sqrt{3}x-3y-2=0 \qquad \sqrt{3}x+y-2=0 \quad \text{1A+1A}$$

9. (a) The coordinates of $B$ are $(4,\ -3)$. 1A

The coordinates of $C$ are $(-8,\ 9)$. 1A

(b) (i) Let $(x,\ y)$ be the coordinates of $P$.

$$BP = CP$$

$$\sqrt{(x-4)^2 + [y-(-3)]^2} = \sqrt{[x-(-8)]^2 + (y-9)^2} \quad \text{1M}$$

$$(x-4)^2 + (y+3)^2 = (x+8)^2 + (y-9)^2$$

$$-24x + 24y - 120 = 0$$

$$x - y + 5 = 0 \quad \text{1A}$$

The equation of the locus of $P$ is $x - y + 5 = 0$.

(ii) Substitute $(-2,\ 3)$ into the equation of the locus of $P$,

$$\text{L.H.S.} = -2 - 3 + 5 \quad \text{1M}$$

$$= 0$$

$$= \text{R.H.S.}$$

The locus of $P$ passes through $D\ (-2,\ 3)$. 1A

10. Let $(x,\ y)$ be the coordinates of $P$.

Coordinates of $A'$ are $(10,\ -24)$. 1A

$$AP = A'P$$

$$\sqrt{[x-(-10)]^2 + (y-24)^2} = \sqrt{(x-10)^2 + [y-(-24)]^2} \quad \text{1M}$$

$$(x+10)^2 + (y-24)^2 = (x-10)^2 + (y+24)^2$$

$$40x - 96y = 0$$

$$5x - 12y = 0 \quad \text{1A}$$

The equation of the locus of $P$ is $5x - 12y = 0$.

11. Let $h$ be the perpendicular distance from $P$ to $AB$.

$$\frac{1}{2} \times (3+2) \times h = 25$$

$$h = 10$$

The locus of $P$ is a pair of straight lines parallel to and 10 apart from $AB$. 1A

Let $(x,\ y)$ be the coordinates of $P$.

$$x + 2 = 10 \quad \text{or} \quad -2 - x = 10 \quad \text{1M}$$

$$x - 8 = 0 \qquad x + 12 = 0 \quad \text{1A}$$

The equations of the locus of $P$ are $x - 8 = 0$ and $x + 12 = 0$.

12. (a) Let $(x,\ y)$ be the coordinates of $P$.

$$\sqrt{(x-1)^2+[y-(-2)]^2} = \sqrt{(y-5)^2} \qquad \text{1M}$$

$$(x-1)^2+(y+2)^2 = (5-y)^2$$

$$x^2-2x+14y-20=0 \qquad \text{1A}$$

The equation of the locus of $P$ is $x^2-2x+14y-20=0$.

(b) Let $(x,\ y)$ be the coordinates of $Q$.

$$QA = QB$$

$$\sqrt{(x-2)^2+(y-5)^2} = \sqrt{(x-1)^2+[y-(-2)]^2} \qquad \text{1M}$$

$$(x-2)^2+(y-5)^2 = (x-1)^2+(y+2)^2$$

$$-2x-14y+28=0$$

$$x+7y-12=0 \qquad \text{1A}$$

The equation of the locus of $Q$ is $x+7y-12=0$.

(c) Substitute $y=-\dfrac{x^2}{14}+\dfrac{x}{7}+\dfrac{10}{7}$ into $x+7y-12=0$,

$$x+7\left(-\frac{x^2}{14}+\frac{x}{7}+\frac{10}{7}\right)-12=0 \qquad \text{1M}$$

$$-\frac{x^2}{2}+2x-2=0$$

$$x=2$$

Substitute $x=2$ into $y=-\dfrac{x^2}{14}+\dfrac{x}{7}+\dfrac{10}{7}$,

$$y=-\frac{2^2}{14}+\frac{2}{7}+\frac{10}{7}$$

$$=\frac{10}{7}$$

The coordinates of $D$ are $\left(2,\ \dfrac{10}{7}\right)$. 1A

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