REG-EOC-2425-ASM-SET 8-MATH
Suggested solutions

Conventional Questions

1. (a) (i)

$CE = EB$ *(given)*

$OA = OB$ *(radii)*

$OE // AC$ *(mid-pt. theorem)*

$\angle BOE = \angle OAD$ *(corr. $\angle$s, $AC // OE$)*

$\angle DOE + \angle BOE = 2\angle OAD$ *($\angle$ at centre twice $\angle$ at $\odot^{ce}$)*

$\angle DOE + \angle BOE = 2\angle BOE$

$\angle DOE = \angle BOE$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(ii) $\angle DOE = \angle BOE$ *(proved)*

$OE = OE$ *(common side)*

$OD = OB$ *(radii)*

$\triangle DOE \cong \triangle BOE$ *(SAS)*

$\angle OBE = 90^\circ$ *(tangent $\perp$ radius)*

$\angle ODE = \angle OBE$ *(corr. $\angle$s, $\cong$ $\triangle$s)*

$= 90^\circ$

Thus, DE is the tangent to the circle at D. *(converse of tangent $\perp$ radius)*.

Marking Scheme		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step with reason.	1

(b) $\angle ODE + \angle OBE = 90^\circ + 90^\circ = 180^\circ$ 1M

Thus, O, B, E, D are concyclic.

Since $\angle OBE = 90^\circ$, OE is a diameter of the circle $OBED$.

The coordinates of O are $(0,\ 8)$. 1M

Slope of $OE = \dfrac{8-0}{0+6} = \dfrac{4}{3}$

Slope of required tangent $= -\dfrac{3}{4}$ 1M

Required equation is

$$y - 0 = -\frac{3}{4}(x+6)$$ 1M

$$3x + 4y + 18 = 0$$ 1A

2. (a) $\angle CAB = \angle BAD$ *(common ∠)*

$\angle ABC = \angle ADB$ *(∠ in alt. segment)*

$\triangle ABC \sim \triangle ADB$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $\dfrac{AC}{AB} = \dfrac{AB}{AD}$ 1M

$$\frac{\sqrt{9^2 + 12^2}}{36 + 9} = \frac{36 + 9}{\sqrt{9^2 + 12^2} + CD}$$

$CD = 120$

Radius of Γ is 60.

Note that $\angle EBA = 90°$.

The coordinates of E are $(60,\ 36)$. 1M

Required equation is

$$(x - 60)^2 + (y - 36)^2 = 60^2$$

$$(x - 60)^2 + (y - 36)^2 = 3600$$ 1A

(ii) E is the mid-point of CD.

The coordinates of D are $(108,\ 72)$. 1A

Let $x^2 + y^2 + dx + ey + f = 0$ be the equation of the circumcircle of $\triangle BED$.

$$\begin{cases} 0^2 + 36^2 + 0 + 36e + f = 0 \\ 60^2 + 36^2 + 60d + 36e + f = 0 \\ 108^2 + 72^2 + 108d + 72e + f = 0 \end{cases}$$ 1M

Solving, we have $d = -60$, $e = -252$ and $f = 7776$.

Required equation is $x^2 + y^2 - 60x - 252y + 7776 = 0$. 1A

> E is the mid-point of CD.
>
> The coordinates of D are $(108,\ 72)$. 1A
>
> Note that the centre of the circumcircle lies on the perpendicular bisector of BE, which is $x = 30$.
>
> Let the coordinates of the centre of the circumcentre of $\triangle BED$ be $(30,\ k)$.
>
> $$\sqrt{(30 - 0)^2 + (k - 36)^2} = \sqrt{(30 - 108)^2 + (k - 72)^2}$$ 1M
>
> $$k^2 - 72k + 2196 = k^2 - 144k + 11\,268$$
>
> $$k = 126$$
>
> Required equation is
>
> $$(x - 30)^2 + (y - 126)^2 = (0 - 30)^2 + (36 - 126)^2$$
>
> $$(x - 30)^2 + (y - 126)^2 = 9000$$ 1A

(iii) Area of the circumcircle of $\triangle BED$

$= (\sqrt{30^2 + 126^2 - 7776})^2\pi$ 1M

$\approx 28\,300$

Let r be the radius of the inscribed circle of $\triangle AED$.

Consider the area of $\triangle AED$.

$$\frac{(36+9)(108)}{2} = \frac{(AB)(r)}{2} + \frac{(BD)(r)}{2} + \frac{(AD)(r)}{2}$$ 1M

$$r \approx 16.5$$

(Area of the inscribed circle of $\triangle BED$) $\times 30$

$= \pi r^2 \times 30$

$\approx 25\,800$

$<$ area of the circumcircle of $\triangle BED$

The claim is agreed. 1

3. (a) The coordinates of G are $(4,\ 10)$. 1M

Required equation is

$$(x-4)^2 + (y-10)^2 = (14-4)^2 + (20-10)^2$$ 1M

$$(x-4)^2 + (y-10)^2 = 200$$ 1A

(b) The equation of L_1 is

$$\frac{y-0}{x+6} = \frac{20-0}{14+6}$$ 1M

$$y = x + 6$$

The coordinates of the three vertices of the bounded region are $(0,\ 6)$, $(0,\ k)$ and $(k-6,\ k)$. 1M

$$\frac{(k-6)(k-6-0)}{2} = 200$$

$$k - 6 = \pm 20$$

$$k = 26 \quad \text{or} \quad -14 \text{ (rejected)}$$ 1A

(c) $\sqrt{(x-4)^2 + (y-10)^2} = \sqrt{(y-26)^2}$ 1M+1M

$x^2 + y^2 - 8x - 20y + 116 = y^2 - 52y + 676$ 1M

$x^2 - 8x + 32y - 560 = 0$ 1A

4. (a) Let the coordinates of G be $(h,\ 26)$.

Note that G lies on the perpendicular bisector of AB.

$$h = \frac{5+13}{2}$$ 1M

$= 9$

The equation of C is

$$(x-9)^2+(y-26)^2=(5-9)^2+(23-26)^2 \quad \text{1M}$$

$$(x-9)^2+(y-26)^2=25 \quad \text{1A}$$

(b) $\sqrt{(k-9)^2+(38-26)^2}=15$ 1M

$$k^2-18k=0$$

$$k=18 \quad \text{or} \quad 0 \text{ (rejected)} \quad \text{1A}$$

(c) (i) T, P and G are collinear. 1A

(ii) Radius of C is 5.

Required ratio $= GP : PT$ 1M

$= 5 : (15-5)$

$= 1 : 2$ 1A

5. (a) (i) $x^2+(mx)^2-400x-300mx+40\,000=0$ 1M

$(1+m^2)x^2-(300m+400)x+40\,000=0$

$\Delta=(300m+400)^2-4(1+m^2)(40\,000)>0$ 1M

$$10\,000(-7m^2+24m)>0$$

$$0<m<\frac{24}{7} \quad \text{1A}$$

(ii) x-coordinate of $M = \frac{1}{2}\left[\frac{300m+400}{1+m^2}\right]$

$= \frac{50(3m+4)}{1+m^2}$

y-coordinate of $M = m\cdot\frac{50(3m+4)}{1+m^2}$

$= \frac{50m(3m+4)}{1+m^2}$ 1

(b) (i) Perpendicular bisector of AB is the line passing through O and the centre of C. 1M

The coordinates of the centre of C are $(200,\ 150)$.

Required equation is

$$y-0=\frac{150-0}{200-0}(x-0)$$

$$3x-4y=0 \quad \text{1A}$$

(ii) Note that L touches C when $m=0$ or $\frac{24}{7}$.

When $m=0$, the coordinates of the intersection of L and C are $(200,\ 0)$.

The coordinates of B are $(200,\ 0)$. 1A

When $m=\frac{24}{7}$, we have the coordinates of A are $(56,\ 192)$.

Denote the centre of C by G $(200,\ 150)$.

Suppose G and M are distinct points.

Note that $\angle GMO=90^\circ$ and $\angle OBG=90^\circ$.

We have O, B, G, M are concyclic. 1M

Since $\angle OAG = 90^\circ$ and $\angle GMO = 90^\circ$, we have O, A, G, M are concyclic.
Thus, O, A, M, G and B are concyclic.
If G and M coincides, O, A, G, B are also concyclic.
Note that OG is a diameter of the circle.
The coordinates of the centre of the required circle are $(100,\ 75)$. 1M
Required equation is

$$(x-100)^2 + (y-75)^2 = (0-100)^2 + (0-75)^2$$
$$(x-100)^2 + (y-75)^2 = 15\,625$$ 1A

(iii) Denote the centre of the circle AMB by D.

$$\tan\angle BOG = \frac{150}{200}$$
$$\angle BOG \approx 36.9^\circ$$

$\angle ADB = 2\angle AOB = 2(2\angle BOG) \approx 147^\circ$ 1M

Length of $\Gamma \le 2\pi(125) \times \dfrac{\angle ADB}{360^\circ}$ 1M

$\approx 322 < 330$

The claim is disagreed. 1A

6. (a) (i) $\angle BEC = \angle BDC$ *($\angle$s in the same segment)*

$\angle ECG = \angle BEC$ *(alt. $\angle$s, $BE // CG$)*

$= \angle BDC$

$\angle BCE = \angle BDE$ *($\angle$s in the same segment)*

$\angle BCG = \angle BCE + \angle ECG$

$= \angle BDE + \angle BDC$ *(proved)*

$= \angle CDG$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(ii) $\angle CBD = \angle DBE$ *(equal arcs, equal $\angle$s)*

$\angle BHC = \angle DBE$ *(alt. $\angle$s, $BE // CG$)*

$= \angle CBD$

$BC = HC$ *(sides opp. equal $\angle$s)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) $BH : DH = \frac{144}{25} : 8$ 1M

$= 18 : 7$ 1A

x-coordinate of $H = \frac{18}{25} \times (6) = \frac{108}{25}$ 1M

The coordinates of H are $\left(\frac{108}{25},\ -\frac{144}{25}\right)$. 1A

(ii) Slope of $BD = \frac{-8}{6} = -\frac{4}{3}$

Slope of $CE = \frac{0 + \frac{144}{25}}{6 + \frac{42}{25}} = \frac{3}{4}$ 1M

Since $\frac{-4}{3} \times \frac{3}{4} = -1$, $BD \perp CE$ and so $\angle CFD = 90^\circ = \angle CGD$. 1A

C, D, G and F are concyclic. *(converse of ∠s in the same segment)* 1

(iii) CD is a diameter of the circle $CDGF$.

Slope of $CD = \frac{-8 + \frac{144}{25}}{6 + \frac{42}{25}} = -\frac{7}{24}$

Slope of tangent required $= \frac{24}{7}$ 1M

Required equation is

$$y + 8 = \frac{24}{7}(x - 6)$$

$24x - 7y - 200 = 0$ 1A

7. (a) $(x - 2)^2 + (y - 6)^2 = r^2$ 1A

(b) (i) Let G be the centre of C'. Then $G\ (-2,\ 6 - c)$. 1A

Since $AG \perp PQ$,

$$\frac{6 - c - 6}{-2 - 2} \times \left(-\frac{1}{2}\right) = -1$$ 1M

$$c = 8$$ 1A

(ii) mid-point of AG lies on PQ, i.e., $(0,\ 2)$ lies on PQ. 1M

The equation of PQ is $y = -\frac{x}{2} + 2$. 1A

(iii) $(x - 2)^2 + \left(-\frac{x}{2} + 2 - 6\right)^2 = r^2$ 1M

$$\frac{5}{4}x^2 + 20 - r^2 = 0$$

a and d are roots of the equation.

So, $a + d = 0$ and $ad = \frac{4(20 - r^2)}{5}$. 1M

$(a - d)^2 = (a + d)^2 - 4ad$

$= \frac{16(r^2 - 20)}{5}$ 1A

(c) $PQ^2 = (a - d)^2 + \left[\left(-\frac{a}{2} + 2\right) - \left(-\frac{d}{2} + 2\right)\right]^2$

$80 = \frac{5}{4}(a - d)^2$

$= 4(r^2 - 20)$ 1M

$r^2 = 40$

$r = 2\sqrt{10}$ or $-2\sqrt{10}$ (rejected)

$AB = \sqrt{(2 + 1)^2 + (6 - 1)^2} = \sqrt{34} < r$

So, B lies inside C.

The claim is agreed. 1A

 END OF PAPER