REG-EOC-2425-ASM-SET 7-MATH
Suggested solutions

Conventional Questions

1. $x^2 + (-3x - k)^2 + 4x - 2(-3x - k) - 5 = 0$ 1M

$(1 + 9)x^2 + (6k + 4 + 6)x + (k^2 + 2k - 5) = 0$

$10x^2 + (6k + 10)x + (k^2 + 2k - 5) = 0$

$\Delta = (6k + 10)^2 - 4(10)(k^2 + 2k - 5) = 0$ 1M

$-4k^2 + 40k + 300 = 0$

$k = -5 \quad \text{or} \quad 15$ 1A+1A

2. $x^2 + (3x + 8)^2 - 5x - 4(3x + 8) + k = 0$ 1M

$(1 + 9)x^2 + (48 - 5 - 12)x + (64 - 32 + k) = 0$

$10x^2 + 31x + 32 + k = 0$ 1A

$\Delta = 31^2 - 4(10)(32 + k) < 0$ 1M

$-319 - 40k < 0$

$k > -\dfrac{319}{40}$

Required value is -7. 1A

3. (a) $(-3y + 10)^2 + y^2 - 8(-3y + 10) - 4y - k^2 = 0$ 1M

$(9 + 1)y^2 + (-60 + 24 - 4)y + (100 - 80 - k^2) = 0$

$10y^2 - 40y + (20 - k^2) = 0$

$\Delta = 40^2 - 4(10)(20 - k^2)$ 1M

$= 40k^2 + 800$

> 0

Thus, C and L intersect at two points. 1

(b) y-coordinate of the mid-point

$= \dfrac{1}{2}\left(-\dfrac{-40}{10}\right)$ 1M

$= 2$

$x + 3(2) - 10 = 0$ 1M

$x = 4$

Required coordinates are $(4,\ 2)$. 1A

4. (a) Centre $(-4,\ 3)$ and radius $= \sqrt{4^2 + 3^2 - 5} = 2\sqrt{5}$ 1A

(b) $(2y - k)^2 + y^2 + 8(2y - k) - 6y + 5 = 0$ 1M

$(4 + 1)y^2 + (-4k + 16 - 6)y + (k^2 - 8k + 5) = 0$

$5y^2 + (-4k + 10)y + (k^2 - 8k + 5) = 0$

y-coordinate of $M = \frac{1}{2}\left(-\frac{-4k+10}{5}\right) = \frac{2k-5}{5}$ 1M

When $y = \frac{2k-5}{5}$, $x = 2\left(\frac{2k-5}{5}\right) - k = \frac{-k-10}{5}$.

Required coordinates are $\left(\frac{-k-10}{5}, \frac{2k-5}{5}\right)$. 1A

(c) If M lies above the x-axis, then $2k - 5 > 0$. So, $k > \frac{5}{2}$. 1A

Consider the equation $5y^2 + (-4k+10)y + (k^2 - 8k + 5) = 0$.

If L cuts C at two distinct points,

$$\Delta = (-4k+10)^2 - 4(5)(k^2 - 8k + 5) > 0 \quad \text{1M}$$

$$-4k^2 + 80k > 0$$

$$0 < k < 20 \quad \text{1A}$$

Combine $k > \frac{5}{2}$ and $0 < k < 20$, we have $\frac{5}{2} < k < 20$.

Thus, it is possible. 1A

5. (a) $CE \perp AB$ *(property of orthocentre)*

$BD \perp AC$ *(property of orthocentre)*

$\angle BEC = \angle BDC = 90°$

Thus, $BCDE$ is a cyclic quadrilateral. *(converse of $\angle$s in the same segment)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i) Coordinates of centre $= \left(\frac{-6+14}{2}, \frac{-6-6}{2}\right) = (4, -6)$ 1A

The equation of the circle is

$$(x-4)^2 + (y+6)^2 = (0-4)^2 + (8+6)^2 \quad \text{1M}$$

$$(x-4)^2 + (y+6)^2 = 100 \quad \text{1A}$$

(ii) Distance between A and centre $= \sqrt{4^2 + (-6-8)^2} = \sqrt{212}$

Radius of circle = 10

Angle between two tangents $= 2 \times \sin^{-1}\frac{10}{\sqrt{212}} \approx 86.8° \neq 90°$ 1M+1A

The claim is not agreed. 1A

6. (a) $AB^2 = 9^2 + 18^2 = 405$

$AC^2 = 13^2 + 16^2 = 425$

$BC^2 = 4^2 + 2^2 = 20$

Therefore, $AB^2 + BC^2 = AC^2$. 1M

Thus, $\angle ABC = 90°$.

$\triangle ABC$ is a right-angled triangle. 1A

(b) The coordinates of the centre $= \left(\frac{-9+4}{2},\ \frac{-8+8}{2}\right) = \left(-\frac{5}{2},\ 0\right)$ 1A

The equation of Ω is

$$\left(x+\frac{5}{2}\right)^2 + y^2 = \left(0+\frac{5}{2}\right)^2 + 10^2 \qquad \text{1M}$$

$$x^2 + y^2 + 5x - 100 = 0 \qquad \text{1A}$$

(c) (i) The coordinates of D are $(10,\ 0)$. 1A

$$(10)^2 + 0^2 + 5(10) - 100 = 50 \neq 0$$

Therefore, D does not lie on the circle passing through A, B and C. 1M

Thus, $ABCD$ is not a cyclic quadrilateral. 1A

(ii) Let the slope of L be m.

The equation of L is $y = m(x-10)$. 1M

Substitute L into Ω,

$$x^2 + (mx - 10m)^2 + 5x - 100 = 0 \qquad \text{1M}$$

$$(1+m^2)x^2 + (-20m^2+5)x + (100m^2 - 100) = 0$$

Since L is tangent, $\Delta = 0$.

$$(20m^2 - 5)^2 - 4(1+m^2)(100m^2 - 100) = 0 \qquad \text{1M}$$

$$-200m^2 + 425 = 0$$

$$m = \pm\frac{\sqrt{34}}{4} \qquad \text{1A}$$

The equations of L are $y = \frac{\sqrt{34}}{4}(x-10)$ and $y = -\frac{\sqrt{34}}{4}(x-10)$.

END OF PAPER