REG-EOC-2425-ASM-SET 5-MATH
Suggested solutions

Conventional Questions

1. The equation of C is $x^2 + y^2 - 14x + 8y + \frac{13}{2} = 0$.
   Let G be the centre of C.
   The coordinates of G are $(7, -4)$. 1A
   Slope of $GP = \dfrac{-4 + \frac{11}{2}}{7 + \frac{1}{2}} = \dfrac{1}{5}$
   Slope of tangent $= -5$ 1M
   Required equation is
   $$y + \frac{11}{2} = -5\left(x + \frac{1}{2}\right) \quad \text{1M}$$
   $$5x + y + 8 = 0 \quad \text{1A}$$

2. Let G be the centre of C.
   The coordinates of G are $(4, -15)$. 1M
   Slope of $GP = \dfrac{-15 + 3}{4 - 5} = 12$
   Slope of tangent $= -\dfrac{1}{12}$ 1M
   Required equation is
   $$y + 3 = -\frac{1}{12}(x - 5) \quad \text{1M}$$
   $$x + 12y + 31 = 0 \quad \text{1A}$$

3. $4x^2 + 4\left(-\frac{x}{2} + \frac{5}{4}\right)^2 - 16x + 8\left(-\frac{x}{2} + \frac{5}{4}\right) - 5 = 0$ 1M
   $$(4 + 1)x^2 + (-5 - 16 - 4)x + \left(\frac{25}{4} + 10 - 5\right) = 0$$
   $$5x^2 - 25x + \frac{45}{4} = 0$$
   $$x = \frac{9}{2} \quad \text{or} \quad \frac{1}{2}$$
   Required coordinates are $\left(\frac{9}{2}, -1\right)$ and $\left(\frac{1}{2}, 1\right)$. 1A+1A

4. $$(x - 3)^2 + (-2x - 4)^2 = 65 \quad \text{1M}$$
   $$(1 + 4)x^2 + (-6 + 16)x + (9 + 16 - 65) = 0$$
   $$5x^2 + 10x - 40 = 0$$
   $$x = -4 \quad \text{or} \quad 2$$

Required coordinates are $(-4,\ 8)$ and $(2,\ -4)$. 1A+1A

5. $(-3y+5)^2+y^2+2(-3y+5)-14y+5=0$ 1M

$$(9+1)y^2+(-30-6-14)y+(25+10+5)=0$$
$$10y^2-50y+40=0$$
$$y=4 \quad \text{or} \quad 1$$

Required coordinates are $(-7,\ 4)$ and $(2,\ 1)$. 1A+1A

6. $4x^2+4\left(\dfrac{x}{2}+\dfrac{3}{4}\right)^2-16\left(\dfrac{x}{2}+\dfrac{3}{4}\right)-69=0$ 1M

$$(4+1)x^2+(3-8)x+\left(\frac{9}{4}-12-69\right)=0$$
$$5x^2-5x-\frac{315}{4}=0$$
$$x=\frac{9}{2} \quad \text{or} \quad -\frac{7}{2}$$

Required coordinates are $\left(\dfrac{9}{2},\ 3\right)$ and $\left(-\dfrac{7}{2},\ -1\right)$. 1A+1A

7. (a) $(y+4)^2+y^2-8(y+4)+4y-14=0$ 1M

$$(1+1)y^2+(8-8+4)y+(16-32-14)=0$$
$$2y^2+4y-30=0$$
$$y=3 \quad \text{or} \quad -5$$

The coordinates of A and B are $(7,\ 3)$ and $(-1,\ -5)$ respectively. 1A+1A

(b) $AB=\sqrt{(7+1)^2+(3+5)^2}$ 1M

$=8\sqrt{2}$ 1A

8. $x^2+(-x+4)^2-6x+2(-x+4)=0$ 1M

$$(1+1)x^2+(-8-6-2)x+(16+8)=0$$
$$2x^2-16x+24=0$$
$$x=6 \quad \text{or} \quad 2$$

Required coordinates are $(6,\ -2)$ and $(2,\ 2)$. 1A+1A

9. (a) $(-4y)^2+y^2-3(-4y)+5y-34=0$ 1M

$$(16+1)y^2+(12+5)y-34=0$$
$$17y^2+17y-34=0$$
$$y=1 \quad \text{or} \quad -2$$

Required coordinates are $(-4,\ 1)$ and $(8,\ -2)$. 1A+1A

(b) $x^2 + \left(-\frac{4x}{3} - \frac{1}{3}\right)^2 - 6x - 8\left(-\frac{4x}{3} - \frac{1}{3}\right) = 0$ 1M

$$\left(1 + \frac{16}{9}\right)x^2 + \left(\frac{8}{9} - 6 + \frac{32}{3}\right)x + \left(\frac{1}{9} + \frac{8}{3}\right) = 0$$

$$\frac{25x^2}{9} + \frac{50x}{9} + \frac{25}{9} = 0$$

$$x = -1$$ 1A

Required coordinates are $(-1,\ 1)$. 1A

10. (a) Let $f(x) = ax + bx^2$, where a and b are non-zero constants. 1A

$$\begin{cases} -160 = -5a + 25b \\ -16 = 4a + 16b \end{cases}$$ 1M

Solving, we have $a = 12$ and $b = -4$.

$f(3) = 12(3) - 4(3)^2 = 0$ 1A

(b) $p = f(3) = 0$

$12q - 4q^2 = 8$ 1M

$-4q^2 + 12q - 8 = 0$

$q = 1 \quad \text{or} \quad 2$

Note that $\angle QRP = 90°$.

PQ is a diameter of C. 1M

When $q = 1$, area of $C = \pi\left(\frac{\sqrt{2^2 + 8^2}}{2}\right)^2 = 17\pi$. 1M

When $q = 2$, area of $C = \pi\left(\frac{\sqrt{1^2 + 8^2}}{2}\right)^2 = \frac{65\pi}{4} < 17\pi$.

Thus, it is not possible. 1A

11. (a) The coordinates of G are $(4,\ -5)$. 1M

Radius $= \sqrt{4^2 + 5^2 + 248} = 17$ 1M

$GH = \sqrt{(4 + 14)^2 + (19 + 5)^2} = 30 > 17$ 1M

H lies outside C. 1

(b) (i) H, G and P are collinear. 1A

(ii) $AB = 2\sqrt{17^2 - \left(\frac{30}{2}\right)^2}$ 1M

$= 16$

Area of $\triangle ABP$

$= \frac{1}{2} \times 16 \times \left(\frac{30}{2} + 17\right)$

$= 256$ 1A

END OF PAPER