Solution	Marks
REG-2425-MOCK-SET 9-MATH-EP(M2)	
Suggested solutions	
1. (a) $(5x-1)^2(1+3x)^n = (1-10x+25x^2)[1+C_1^n(3x)+C_2^n(3x)^2+\ldots]$	1M
$= (1-10x+25x^2)\left[1+3nx+\dfrac{9n(n-1)}{2}x^2+\ldots\right]$	
The coefficient of x is 8.	
$-10+3n = 8$	
$n = 6$	1A
(b) $(5x-1)^2(1+3x)^6 = (1-10x+25x^2)(1+18x+135x^2+\ldots)$	
Coefficient of $x^2 = 25(1)+(-10)(18)+(1)(135)$	1M
$= -20$	1A
2. $f'(2) = \lim\limits_{h\to 0}\dfrac{\sqrt{(2+h)^2-1}-\sqrt{2^2-1}}{h}$	1M
$= \lim\limits_{h\to 0}\dfrac{(h^2+4h+1)-3}{h(\sqrt{h^2+4h+3}+\sqrt{3})}$	1M
$= \lim\limits_{h\to 0}\dfrac{h+4}{\sqrt{h^2+4h+3}+\sqrt{3}}$	1M
$= \dfrac{2}{\sqrt{3}}$	1A
3. (a) $\cot\theta - \cot 2\theta$	
$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\sin 2\theta}$	
$= \dfrac{\cos\theta\sin 2\theta - \cos 2\theta\sin\theta}{\sin\theta\sin 2\theta}$	1M
$= \dfrac{\sin(2\theta-\theta)}{\sin\sin\theta\sin 2\theta}$	
$= \dfrac{1}{\sin 2\theta}$	
$= \csc 2\theta$	1
(b) (i) $g(x) = \dfrac{7+8\cot 4x}{(\cot 4x-\cot 8x)+\cot 8x}$	1M
$= \dfrac{7+8\cot 4x}{\cos 4x}$	
$= 7\tan 4x + 8$	1A
(ii) $7\tan 4x + 8 = 15$	
$\tan 4x = 1$	1M
$4x = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$	
$x = \dfrac{\pi}{16}$ or $\dfrac{5\pi}{16}$	1A

Solution	Marks
4. (a) $\int \sin^2\theta\cos^2\theta\,\mathrm{d}\theta$	
$= \dfrac{1}{4}\int \sin^2 2\theta\,\mathrm{d}\theta$	1M
$= \dfrac{1}{8}\int (1-\cos 4\theta)\,\mathrm{d}\theta$	
$= \dfrac{\theta}{8} - \dfrac{\sin 4\theta}{32} + \text{constant}$	1A
(b) Volume	
$= \pi\int_0^1 \left[6x(1-x^2)^{\frac{1}{4}}\right]^2 \mathrm{d}x$	1M
$= 36\pi\int_0^1 x^2(1-x^2)^{\frac{1}{2}}\,\mathrm{d}x$	
Let $x = \sin\theta$. Then $\mathrm{d}x = \cos\theta\,\mathrm{d}\theta$.	1M
Volume	
$= 36\pi\int_0^{\frac{\pi}{2}} \sin^2\theta(1-\sin^2\theta)^{\frac{1}{2}}\cos\theta\,\mathrm{d}\theta$	
$= 36\pi\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\,\mathrm{d}\theta$	
$= 36\pi\left[\dfrac{\theta}{8} - \dfrac{\sin 4\theta}{32}\right]_0^{\frac{\pi}{2}}$	1M
$= \dfrac{9\pi^2}{4}$	1A

Solution	Marks
5. (a) When $n = 1$,	
L.H.S. $= 1(3^{-1}) = \frac{1}{3}$	
R.H.S. $= \frac{3}{4} - \frac{2+3}{4}(3^{-1}) = \frac{1}{3} =$ L.H.S.	
It is true for $n = 1$.	1
Assume $\sum_{r=1}^{k} r(3^{-r}) = \frac{3}{4} - \left(\frac{2k+3}{4}\right)(3^{-k})$, where $k \in \mathbb{Z}^+$.	1
$\sum_{r=1}^{k+1} r(3^{-r})$	
$= \sum_{r=1}^{k} r(3^{-r}) + (k+1)(3^{-k-1})$	
$= \frac{3}{4} - \left(\frac{2k+3}{4}\right)(3^{-k}) + (k+1)(3^{-k-1})$	1M
$= \frac{3}{4} - \left[\frac{3(2k+3) - 4(k+1)}{4}\right](3^{-k-1})$	
$= \frac{3}{4} - \left(\frac{2(k+1)+3}{4}\right)(3^{-k-1})$	
It is true for $n = k + 1$.	
By M.I., it is true $\forall n \in \mathbb{Z}^+$.	1
(b) (i) $\sum_{r=100}^{199} r(3^{-r}) = \sum_{r=1}^{199} r(3^{-r}) - \sum_{r=1}^{99} r(3^{-r})$	
$= \left[\frac{3}{4} - \frac{2(199)+3}{4}(3^{-199})\right] - \left[\frac{3}{4} - \frac{2(99)+3}{4}(3^{-99})\right]$	
$= \frac{201}{4(3^{99})} - \frac{401}{4(3^{199})}$	1A
(ii) Let $u = 200 - r$.	
$\sum_{r=1}^{100} (200-r)3^r = \sum_{u=100}^{199} u(3^{200-u})$	1M
$= 3^{200} \sum_{u=100} u(3^{-u})$	
$= 3^{200}\left[\frac{201}{4(3^{99})} - \frac{401}{4(3^{199})}\right]$	
$= \frac{201(3^{101})}{4} - \frac{1203}{4}$	1A

Solution	Marks
6. (a) Let r cm be the radius of the water surface in the vessel.	
We have $r = h\tan\frac{\pi}{3} = \sqrt{3}h$.	1M
$A = \pi(\sqrt{3}h)\sqrt{h^2 + (\sqrt{3}h)^2}$	1M
$= 2\sqrt{3}\pi h^2$	1
(b) When the volume of water in the vessel is 216π cm^3.	
$\frac{1}{3}\pi(\sqrt{3}h)^2 h = 216\pi$	1M
$h = 6$	1A
From (a), we have $A = 2\sqrt{3}\pi h^2$.	
$\frac{\mathrm{d}A}{\mathrm{d}t} = 4\sqrt{3}\pi h\frac{\mathrm{d}h}{\mathrm{d}t}$	1M
Put $\frac{\mathrm{d}A}{\mathrm{d}t} = -12\pi$ and $h = 6$.	
$-12\pi = 4\sqrt{3}\pi(6)\frac{\mathrm{d}h}{\mathrm{d}t}$	
$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{\sqrt{3}}{6}$	1A
Required rate is $-\frac{1}{2\sqrt{3}}$ cm/s.	

Solution	Marks
7. (a) $f(x) = \int (2x-4)\,\mathrm{d}x$	
$= x^2 - 4x + C$	1M
Γ passes through $(5,\ 15)$.	
$15 = 5^2 - 4(5) + C$	1M
$C = 10$	
Required equation is $y = x^2 - 4x + 10$.	1A
(b) (i) Let $(a,\ b)$ be the coordinates of P.	
$\dfrac{b+1}{a+1} = 2a - 4$	1M
$\dfrac{(a^2 - 4a + 10) + 1}{a+1} = 2a - 4$	
$a^2 - 4a + 11 = 2a^2 - 2a - 4$	
$0 = a^2 + 2a - 15$	
$a = 3 \quad \text{or} \quad -5$	
Note that the slope of Γ is positive, we have $a = 3$ only.	
The coordinates of P are $(3,\ 7)$.	1A
(ii) Slope of $L = f'(3) = 2$	
Slope of required straight line $= -\dfrac{1}{2}$	1M
Required equation is	
$y - 7 = -\dfrac{1}{2}(x-3)$	1M
$x + 2y - 17 = 0$	1A

Solution	Marks
8. (a) $MX = X^T M^T$	
$\begin{pmatrix} 1 & 4 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} a & a \\ 3b & b \end{pmatrix} = \begin{pmatrix} a & 3b \\ a & b \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 4 & 1 \end{pmatrix}$	
$\begin{pmatrix} a+12b & a+4b \\ -3a+3b & -3a+b \end{pmatrix} = \begin{pmatrix} a+12b & -3a+3b \\ a+4b & -3a+b \end{pmatrix}$	1M
Compare the elements of two matrices.	
$a+4b = -3a+3b$	1M
$b = -4a$	1A
(b) $X = \begin{pmatrix} a & a \\ -12a & -4a \end{pmatrix}$	
$\det X = -4a^2 + 12a^2 = 8a^2$	
Since X is a non-zero matrix, we have $\det X = 8a^2 \neq 0$.	
X is a non-singular matrix.	
The claim is agreed.	1A
(c) $(X^T)^{-1} = \begin{pmatrix} a & -12a \\ a & -4a \end{pmatrix}^{-1}$	1M
$= \dfrac{1}{8a^2}\begin{pmatrix} -4a & 12a \\ -a & a \end{pmatrix}$	
$= \dfrac{1}{8a}\begin{pmatrix} -4 & 12 \\ -1 & 1 \end{pmatrix}$	1A

Solution	Marks
9. (a) Vertical asymptote is $x = 1$.	1A
$f(x) = x + 4 + \dfrac{7}{x-1} + \dfrac{3}{(x-1)^2}$	1M
Oblique asymptote is $y = x + 4$.	1A
(b) $f'(x) = 1 - \dfrac{7}{(x-1)^2} - \dfrac{6}{(x-1)^3}$	1M
$= \dfrac{x(x-4)(x+1)}{(x-1)^3}$	1A
$f''(x) = \dfrac{14}{(x-1)^3} + \dfrac{18}{(x-1)^4}$	
$= \dfrac{14x+4}{(x-1)^4}$	1A
(c) When $f'(x) = 0$, $x = -1$ or 0 or 4.	1M
(table below)	1M
Maximum point is $\left(-1,\ \dfrac{1}{4}\right)$.	1A
Minimum points are $(0,\ 0)$ and $\left(4,\ \dfrac{32}{3}\right)$.	1A
(d) Required area	
$= \displaystyle\int_{-2}^{0} f(x)\,\mathrm{d}x$	
$= \displaystyle\int_{-2}^{0} \left(x + 4 + \dfrac{7}{x-1} + \dfrac{3}{(x-1)^2}\right)\mathrm{d}x$	
$= \left[\dfrac{x^2}{2} + 4x + 7\ln\lvert x-1\rvert - \dfrac{3}{x-1}\right]_{-2}^{0}$	1M
$= 8 - 7\ln 3$	1A

x	$x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x < 4$	$x > 4$
$f'(x)$	$+$	$-$	$+$	$-$	$+$

Solution	Marks
10. (a) $\displaystyle\int_0^{\pi} x^2\cos^2 x\,\mathrm{d}x = \frac{1}{2}\int_0^{\pi} x^2(1+\cos 2x)\,\mathrm{d}x$	1M
$\displaystyle = \left[\frac{x^3}{6}\right]_0^{\pi} + \frac{1}{4}\left[x^2\sin 2x\right]_0^{\pi} - \frac{1}{2}\int_0^{\pi} x\sin 2x\,\mathrm{d}x$	1M
$\displaystyle = \frac{\pi^3}{6} + 0 + \frac{1}{4}\left[x\cos 2x\right]_0^{\pi} - \frac{1}{4}\int_0^{\pi}\cos 2x\,\mathrm{d}x$	1M
$\displaystyle = \frac{\pi^3}{6} + \frac{\pi}{4} - \frac{1}{8}\left[\sin 2x\right]_0^{\pi}$	
$\displaystyle = \frac{\pi^3}{6} + \frac{\pi}{4}$	1A
(b) (i) Let $u = -x$. Then $\mathrm{d}u = -\,\mathrm{d}x$.	1M
$\displaystyle\int_{-a}^{a} f(x)\,\mathrm{d}x = \int_{-a}^{0} f(x)\,\mathrm{d}x + \int_0^{a} f(x)\,\mathrm{d}x$	
$\displaystyle = -\int_a^{0} f(-u)\,\mathrm{d}u + \int_0^{a} f(x)\,\mathrm{d}x$	
$\displaystyle = \int_0^{a} f(-u)\,\mathrm{d}u + \int_0^{a} f(x)\,\mathrm{d}x$	
$\displaystyle = \int_0^{a} [f(x)+f(-x)]\,\mathrm{d}x$	1
(ii) Let $u = a - x$. Then $\mathrm{d}u = -\,\mathrm{d}x$.	1M
$\displaystyle\int_0^{a} xg(x)\,\mathrm{d}x = -\int_a^{0}(a-u)g(a-u)\,\mathrm{d}u$	
$\displaystyle = \int_0^{a}(a-u)g(u)\,\mathrm{d}u$	
$\displaystyle = a\int_0^{a} g(x)\,\mathrm{d}x - \int_0^{a} xg(x)\,\mathrm{d}x$	
$\displaystyle = \frac{a}{2}\int_0^{a} g(x)\,\mathrm{d}x$	1
(c) Put $f(x) = \ln(1+e^x)(\sin^3 x + x)\cos^2 x$.	
$f(x)+f(-x) = \ln(1+e^x)(\sin^3 x + x)\cos^2 x + \ln(1+e^{-x})(\sin^3(-x) - x)\cos^2(-x)$	
$\displaystyle = \ln(1+e^x)(\sin^3 x + x)\cos^2 x - \ln\left(\frac{e^x+1}{e^x}\right)(\sin^3 x + x)\cos^2 x$	
$= \ln(e^x)(\sin^3 x + x)\cos^2 x$	
$= x(\sin^3 x + x)\cos^2 x$	
Use the result of (b)(i).	
$\displaystyle\int_{-\pi}^{\pi}\ln(1+e^x)(\sin^3 x + x)\cos^2 x\,\mathrm{d}x = \int_0^{\pi} x(\sin^3 x + x)\cos^2 x\,\mathrm{d}x$	1M
$\displaystyle = \int_0^{\pi} x\sin^3 x\cos^2 x\,\mathrm{d}x + \int_0^{\pi} x^2\cos^2 x\,\mathrm{d}x$	
$\displaystyle = \int_0^{\pi} x\sin^3 x\cos^2 x\,\mathrm{d}x + \frac{\pi^3}{6} + \frac{\pi}{4}$	
Put $g(x) = \sin^3 x\cos^2 x$. Then $g(\pi - x) = \sin^3(\pi - x)\cos^2(\pi - x) = \sin^3 x\cos^2 x = g(x)$.	

Solution	Marks
Let $u = \cos x$. Then $\mathrm{d}u = -\sin x \,\mathrm{d}x$.	1M
$\displaystyle\int_0^{\pi} x\sin^3 x\cos^2 x\,\mathrm{d}x = \frac{\pi}{2}\int_0^{\pi}\sin^3 x\cos^2 x\,\mathrm{d}x$	1M
$\displaystyle = -\frac{\pi}{2}\int_1^{-1}(1-u^2)u^2\,\mathrm{d}u$	
$\displaystyle = \frac{\pi}{2}\int_{-1}^{1}(u^2-u^4)\,\mathrm{d}u$	
$\displaystyle = \frac{\pi}{2}\left[\frac{u^3}{3}-\frac{u^5}{5}\right]_{-1}^{1}$	
$\displaystyle = \frac{2\pi}{15}$	
Combine the results.	
$\displaystyle\int_{-\pi}^{\pi}\ln(1+e^x)(\sin^3 x + x)\cos^2 x\,\mathrm{d}x = \frac{2\pi}{15} + \left(\frac{\pi^3}{6}+\frac{\pi}{4}\right)$	
$\displaystyle = \frac{\pi^3}{6} + \frac{23\pi}{60}$	1A

Solution	Marks
11. (a) (i) (1) $\begin{vmatrix} 1 & 2 & 1 \\ 2 & a+3 & -4 \\ a & 7 & 14 \end{vmatrix} = -a^2 + 3a + 28$	1A
$= -(a-7)(a+4)$	
(E) has a unique solution. Then $-(a-7)(a+4) \neq 0$.	1M
Thus, we have $a \neq 7$ and $a \neq -4$.	1A
(2) $x = \dfrac{\begin{vmatrix} 1 & 2 & 1 \\ 5 & a+3 & -4 \\ b & 7 & 14 \end{vmatrix}}{-(a-7)(a+4)}$	1M
$= \dfrac{ab - 14a + 11b + 35}{(a-7)(a+4)}$	1A
$y = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & -4 \\ a & b & 14 \end{vmatrix}}{-(a-7)(a+4)}$	
$= \dfrac{9a - 6b - 42}{(a-7)(a+4)}$	
$z = \dfrac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & a+3 & 5 \\ a & 7 & b \end{vmatrix}}{-(a-7)(a+4)}$	
$= \dfrac{a^2 - ab - 7a + b + 21}{(a-7)(a+4)}$	1A
(ii) The augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & 2 & 1 & 1 \\ 2 & -1 & -4 & 5 \\ -4 & 7 & 14 & b \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & 1 & 1 \\ 0 & -5 & -6 & 3 \\ 0 & 15 & 18 & b+4 \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & 1 & 1 \\ 0 & -5 & -6 & 3 \\ 0 & 0 & 0 & b+13 \end{array}\right)$	1M
(E) is consistent, then $b + 13 = 0$.	
We have $b = -13$.	1A
(iii) The augmented matrix of (E) is	
$\left(\begin{array}{ccc\|c} 1 & 2 & 1 & 1 \\ 0 & -5 & -6 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$	
The solution set is $\left\{ \left(\dfrac{7t+11}{5}, \dfrac{-6t-3}{5}, t \right) : t \in \mathbb{R} \right\}$.	1
(b) The first three equations are equivalent to those in (E) when $a = -4$.	
Since (F) is consistent, $\alpha = 13$.	

Solution	Marks
The solution set to the first three equations of (F) is $\left\{\left(\dfrac{7y+11}{5},\ \dfrac{-6t-3}{t},\ t\right) : t \in \mathbb{R}\right\}$.	1M
$px + qy + rz = \beta$	
$p\left(\dfrac{7t+11}{5}\right) + q\left(\dfrac{-6t-3}{5}\right) + rt = \beta$	
$(7p - 6q + 5r)t + (11p - 3q - 5\beta) = 0$	
(F) has infinitely many solutions. We have $7p - 6q + 5r = 0$ and $11p - 3q - 5\beta = 0$.	1M
$11p - 3\left(\dfrac{7p+5r}{6}\right) - 5\beta = 0$	
$\beta = \dfrac{3p - r}{2}$	1A

Solution	Marks
12. (a) (i) $\overrightarrow{AB} = -\mathbf{i} + (m-7)\mathbf{j} + (21-m)\mathbf{k}$	
$\overrightarrow{AB} \cdot (2\mathbf{i} + 7\mathbf{j} - \mathbf{k}) = 0$	1M
$-2 + 7(m-7) - (21-m) = 0$	
$m = 9$	1A
(ii) $\overrightarrow{AC} = \mathbf{j} + 8\mathbf{k}$	
$\overrightarrow{AB} \times \overrightarrow{AC}$	
$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 12 \\ 0 & 1 & 8 \end{vmatrix}$	1M
$= 4\mathbf{i} + 8\mathbf{j} - \mathbf{k}$	
$\mathbf{v} = \dfrac{1}{\sqrt{4^2+8^2+1^2}}(4\mathbf{i} + 8\mathbf{j} - \mathbf{k})$	1M
$= \dfrac{4}{9}\mathbf{i} + \dfrac{8}{9}\mathbf{j} - \dfrac{1}{9}\mathbf{k}$	1A
(b) (i) $\overrightarrow{AD} = 5\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$	
$\overrightarrow{ED} = \left(\dfrac{\overrightarrow{AD} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}\right)\mathbf{v}$	1M
$= \dfrac{5 \times \frac{4}{9} + 3 \times \frac{8}{9} - 10 \times \left(-\frac{1}{9}\right)}{1}\left(\dfrac{4}{9}\mathbf{i} + \dfrac{8}{9}\mathbf{j} - \dfrac{1}{9}\mathbf{k}\right)$	
$= \dfrac{8}{3}\mathbf{i} + \dfrac{16}{3}\mathbf{j} - \dfrac{2}{3}\mathbf{k}$	1A
(ii) Required distance	
$= ED$	1M
$= \sqrt{\left(\dfrac{8}{3}\right)^2 + \left(\dfrac{16}{3}\right)^2 + \left(-\dfrac{2}{3}\right)^2}$	
$= 6$	1A
(iii) Required vector	
$= \overrightarrow{OD} + \overrightarrow{DE}$	
$= \dfrac{25}{3}\mathbf{i} + \dfrac{17}{3}\mathbf{j} - \dfrac{1}{3}\mathbf{k}$	1A
(iv) $\overrightarrow{CE} = \dfrac{7}{3}\mathbf{i} - \dfrac{10}{3}\mathbf{j} - \dfrac{52}{3}\mathbf{k}$	1M
$\overrightarrow{AB} \cdot \overrightarrow{CE} = -1 \times \dfrac{7}{3} + 2 \times \left(-\dfrac{10}{3}\right) + 12 \times \left(-\dfrac{52}{3}\right)$	1M
$= -217$	
$\neq 0$	
AB is not perpendicular to CE.	
E is not the orthocentre of $\triangle ABC$.	1A

END OF PAPER