

REG-2425-MOCK-SET 9-MATH-CP 1**Suggested solutions**

$$1. \frac{ab^7}{(a^3b^{-4})^{-2}} = \frac{ab^7}{a^{-6}b^8}$$

$$= \frac{a^{1+6}}{b^{8-7}}$$

$$= \frac{a^7}{b}$$

1M

$$= \frac{a^{1+6}}{b^{8-7}}$$

1M

$$= \frac{a^7}{b}$$

1A

$$2. \text{ (a) } x^2 - 10xy + 25y^2 = (x - 5y)^2$$

1A

$$\text{ (b) } x^2 - 10xy + 25y^2 - 2x^2y + 10xy^2$$

$$= (x - 5y)^2 - 2xy(x - 5y)$$

1M

$$= (x - 5y)(x - 5y - 2xy)$$

1A

3. We have $y + z = 5 : 1$ and $x + y + z = 15 : 20 : 4$.

1M

Let $x = 15k$, $y = 20k$ and $z = 4k$, where k is a non-zero number.

$$\frac{2x + 3y}{2y - z} = \frac{2(15k) + 3(20k)}{2(20k) - 4k}$$

1M

$$= \frac{5}{2}$$

1A

$$4. \frac{9}{n + 5 + 9} = \frac{3}{8}$$

1M+1A

$$72 = 3n + 42$$

$$n = 10$$

1A

5. (a) Let $\$x$ be the cost.

$$x(1 + 30\%) = 4940$$

1M

$$x = 3800$$

1A

(b) Selling price

$$= 4940 \times (1 - 20\%)$$

1M

$$= \$3952$$

$$> \$3800$$

The claim is agreed.

1A

6. (a) $\frac{4x-5}{2} > 3x+2$

$$-x > \frac{9}{2}$$

$$x < -\frac{9}{2}$$

$$1-2x \geq 13$$

$$x \leq -6$$

Thus, we have $x < -\frac{9}{2}$.

1M

1A

1A

(b) -5

1A

7. (a) $\angle POQ = 215^\circ - 125^\circ = 90^\circ$

1A

(b) $r^2 + 24^2 = 26^2$

1M

$$r = 10 \quad \text{or} \quad -10 \text{ (rejected)}$$

1A

(c) Required area = $\frac{(24)(10)}{2}$

1M

$$= 120$$

1A

8. (a) Let $z = ax^2 + \frac{b}{\sqrt{y}}$, where a and b are non-zero constants.

1A

$$\begin{cases} 5 = a + b \\ 4 = a + \frac{b}{\sqrt{4}} \end{cases}$$

1M

Solving, we have $a = 3$ and $b = 2$.

1A

Thus, $z = 3x^2 + \frac{2}{\sqrt{y}}$.

(b) New value of z

$$= 3(2)^2 + \frac{2}{\sqrt{1}}$$

1M

$$= 14$$

Increase in the value of z

$$= 14 - 4$$

$$= 10$$

1A

9. (a) $\angle ADC + \angle ABC = 180^\circ$

$$\angle ADC = 180^\circ - \theta$$

$$\angle ACD = 90^\circ$$

$$\angle CAD + \angle ADC + \angle ACD = 180^\circ$$

$$\angle CAD + (180^\circ - \theta) + 90^\circ = 180^\circ$$

$$\angle CAD = \theta - 90^\circ$$

1M

1M

1A

(b) $\angle AEO = 90^\circ$

$$\angle BOD = \angle CAD + \angle AEO$$

$$= (\theta - 90^\circ) + 90^\circ$$

$$= \theta$$

1M

1A

10. (a) $\angle BAC = \angle DAE$ (common \angle)

$$BD = BE \quad (\text{given})$$

$$\angle ADE = \angle BED \quad (\text{base } \angle\text{s, isos. } \triangle)$$

$$\angle ABC = \angle BED \quad (\text{given})$$

$$= \angle ADE$$

$$\triangle ABC \sim \triangle ADE \quad (\text{AA})$$

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) $\frac{DE}{BC} = \frac{AE}{AC}$
 $\frac{DE}{4} = \frac{2AC}{AC}$

$$DE = 8 \text{ cm}$$

$$\angle AED = \angle ACB = 90^\circ$$

Consider the area of $\triangle ADE$.

$$\frac{1}{2} \times AE \times 8 = 24$$

$$AE = 6 \text{ cm}$$

$$AD = \sqrt{6^2 + 8^2}$$

$$= 10 \text{ cm}$$

AD is a diameter of the circle ADE .

$$\text{Required area} = \pi \left(\frac{10}{2} \right)^2$$

$$= 25\pi \text{ cm}^2$$

1M

1M

1M

1A

$$11. \quad (a) \quad 2 \times \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{1}{3} \pi r^2 h$$

$$h = 4r$$

1M

1A

(b) Volume of the liquid

$$= \frac{1}{2} \times \frac{4}{3} \pi (2)^3$$

$$= \frac{16\pi}{3} \text{ cm}^3$$

1M

Volume of the cone

$$= \frac{1}{3} \pi (2)^2 (4 \times 2)$$

$$= \frac{32\pi}{3} \text{ cm}^3$$

1M

Let h cm be the depth of liquid in the cone.

$$\frac{16\pi}{3} \div \frac{32\pi}{3} = \left(\frac{h}{8}\right)^3$$

$$h^3 = 256$$

$$h = \sqrt[3]{256}$$

$$\approx 6.35$$

1M

The depth of the liquid in the cone is 6.35 cm.

1A

12. (a) Let $f(x) = (x^2 - 4)Q(x) + kx + 64$, where $Q(x)$ is a polynomial.

1M

$$f(2) = 0$$

$$(4 - 4)Q(2) + 2k + 64 = 0$$

$$k = -32$$

1A

We have $f(-2) = 0 + 64 + 64 = 128$.

$$(-2)(-6 + 5)^2 - 2p + q = 128$$

$$-2p + q = 130$$

1M

We have $f(2) = 0$.

$$2(6 + 5)^2 + 2p + q = 0$$

$$2p + q = -242$$

Solving, we have $p = -93$ and $q = -56$.

1A

(b) $f(x) = 0$

$$x(3x + 5)^2 - 93x - 56 = 0$$

$$9x^3 + 30x^2 - 68x - 56 = 0$$

$$(x - 2)(9x^2 + 48x + 28) = 0$$

$$(x - 2)(3x + 2)(3x + 14) = 0$$

1M

1M

$$x = 2 \quad \text{or} \quad -\frac{2}{3} \quad \text{or} \quad -\frac{14}{3}$$

The equation $f(x) = 0$ has 3 rational roots.

1A

Solution	Marks
<p>13. (a) The mode is 9. We have $x > 10$. The median is 9.5.</p>	
$7 + 9 + x = 10 + y + 8$ $y = x - 2$	1M
<p>We have $(x, y) = (11, 9)$ or $(12, 10)$.</p>	1A+1A
<p>(b) When $x = 11$ and $y = 9$, standard deviation ≈ 1.61.</p>	1M
<p>When $x = 12$ and $y = 10$, standard deviation ≈ 1.59.</p>	
<p>The least possible standard deviation is 1.59.</p>	1A
<p>(c) The ages of the four leaving members are 7, 7, 7 and 9.</p>	1M
<p>When $x = 11$ and $y = 9$, mean = $\frac{7(4) + 8(9) + \dots + 12(8)}{4 + 9 + \dots + 8} = 9.7$.</p>	
<p>When $x = 12$ and $y = 10$, mean = $\frac{7(4) + 8(9) + \dots + 12(8)}{4 + 9 + \dots + 8} \approx 9.71$.</p>	
<p>The greatest possible mean is 9.71.</p>	1A

14. (a) Let (h, k) be the coordinates of A .

$$\begin{cases} \frac{k-0}{h-0} = \frac{3}{4} \\ 4h + 3k - 50 = 0 \end{cases}$$

Solving, we have $h = 8$ and $k = 6$.

Required equation is

$$(x-8)^2 + (y-6)^2 = (0-8)^2 + (0-6)^2$$

$$(x-8)^2 + (y-6)^2 = 100$$

(b) (i) $(x-8)^2 + (0-6)^2 = 100$

$$x^2 - 16x = 0$$

$$x = 0 \quad \text{or} \quad 16$$

The coordinates of B are $(16, 0)$.

(ii) Let θ be the inclination of BD .

$$\tan \theta = \frac{3}{4}$$

$$\theta \approx 36.9^\circ$$

$$\angle OAD = 2\angle OBD$$

$$= 2\theta$$

$$\approx 73.7^\circ$$

(iii) $\angle ADM = \angle OAD \approx 73.7^\circ$

Required area

$$= \frac{1}{2}(DM)(AM)$$

$$= \frac{1}{2}(AD \cos \angle ADM)(AD \sin \angle ADM)$$

$$\approx 13.4$$

$$< 14$$

The claim is not correct.

15. (a) Required probability = $\frac{C_4^5 C_1^7 + C_5^5}{C_5^{12}}$

$$= \frac{1}{22}$$

(b) Required probability = $1 - \frac{C_3^5 C_2^7}{C_5^{12}} - \frac{1}{22}$

$$= \frac{91}{132}$$

1M

1A

1A

1M

1A

1M

1A

1M

1A

1M

1A

1M

1A

16. (a) Let the mean and standard deviation of the distribution be μ and σ respectively.

$$\begin{cases} \frac{60 - \mu}{\sigma} = 1.25 \\ \frac{44 - \mu}{\sigma} = 0.25 \end{cases}$$

1M

Solving, we have $\mu = 40$ and $\sigma = 16$.

1A+1A

(b) New standard score of Carol = $\frac{44(1 + 10\%) - 40(1 + 10\%)}{16(1 + 10\%)}$
 $= 0.25$

1M

The claim is not correct.

1A

17. (a) Let a and r be the first term and the common ratio respectively.

$$\begin{cases} \frac{a(1 - r^3)}{1 - r} = 7320 \\ \frac{a}{1 - r} = 15\,000 \end{cases}$$

1M

$$15\,000(1 - r^3) = 7320$$

1M

$$r^3 = 0.512$$

$$r = 0.8$$

$$\text{First term} = 15\,000(1 - r) = 3000$$

1A

(b) $3000(0.8)^n - 3000(0.8)^{2n} > 200$

$$15(0.8^n)^2 - 15(0.8^n) + 1 < 0$$

$$0.0718 < 0.8^n < 0.928$$

1M

$$\log 0.0718 < n \log 0.8 < \log 0.928$$

1M

$$11.8 > n > 0.334$$

The greatest value of n is 11.

1A

18. (a) $f(x) = ax^2 + 8a^2x + 16a^3 + a$

$$= a(x^2 + 8ax + 16a^2) + a$$

1M

$$= a(x + 4a)^2 + a$$

1M

The coordinates of vertex are $(-4a, a)$.

1A

- (b) (i) The graph of $y = f(x)$ is first translated rightwards by $5a$ units,
then is enlarged along the y -axis to 4 times the original.

1A

1A

(ii) $(a, 4a)$

1A

(iii) Slope of $OP \times$ slope of $OQ = \frac{a}{-4a} \times \frac{4a}{a} = -1$

So, $OP \perp OQ$.

1M

Thus, the orthocentre of $\triangle OPQ$ is at O .

The coordinates of orthocentre are $(0, 0)$.

1A

$$19. \quad (a) \quad AC^2 = 11^2 + 24^2 - 2(11)(24) \cos 120^\circ$$

$$AC = 31 \text{ cm}$$

$$\frac{\sin \angle CAB}{24} = \frac{\sin 120^\circ}{31}$$

$$\angle CAB \approx 42.1^\circ$$

1M

1A

1M

1A

(b) (i) Let E be a point on AB produced such that $CE \perp AB$.

$$CE = AC \sin \angle CAB \approx 20.8 \text{ cm}$$

$$\text{Required distance} = CE \sin 40^\circ$$

$$\approx 13.4 \text{ cm}$$

1M

1M

1A

(ii) Let F be the projection of C on the horizontal ground.

$$AF = \sqrt{31^2 - CF^2} \approx 28.0 \text{ cm}$$

$$AE = 31 \cos \angle CAB = 23 \text{ cm}$$

$$\cos \angle EAF = \frac{AE}{AF}$$

$$\angle EAF \approx 34.7^\circ$$

$$\angle BAD = 2\angle EAF \approx 69.4^\circ$$

Required volume

$$= \frac{1}{3} \left[\frac{1}{2} \times AB \times AD \times \sin \angle BAD \right] CF$$

$$\approx 252 \text{ cm}^3$$

1M

1A

(iii) Area of $\triangle ABC$

$$= \frac{1}{2} \times 11 \times 24 \times \sin 120^\circ$$

$$= 66\sqrt{3} \text{ cm}^2$$

Let G be the projection of D on the plane ABC .

Consider the volume of the tetrahedron $ABDC$.

$$\text{volume} = \frac{1}{3} (66\sqrt{3})(DG)$$

$$DG \approx 6.62 \text{ cm}$$

Let θ be the required angle.

$$\sin \theta = \frac{DG}{CD}$$

$$\theta \approx 16.0^\circ$$

$$> 15^\circ$$

1M

The claim is disagreed.

1A