REG-2425-MOCK-SET 8-MATH-CP 2

Answers:

1. B	2. C	3. A	4. C	5. C	6. B	7. D	8. B	9. D	10. A
11. B	12. B	13. B	14. D	15. D	16. A	17. C	18. C	19. A	20. D
21. C	22. D	23. D	24. A	25. A	26. B	27. A	28. C	29. B	30. A
31. D	32. B	33. D	34. C	35. D	36. C	37. B	38. A	39. A	40. D
41. B	42. C	43. C	44. A	45. A					

Suggested Solutions:

1. B

$$\begin{aligned}\left(\frac{1}{4^{3001}}\right)8^{2001} &= \frac{1}{2^{2\times 3001}} \times 2^{3\times 2001} \\ &= 2^{6003-6002} \\ &= 2\end{aligned}$$

2. C

$$\begin{aligned}2h(2-a) &= a(a-2h) \\ 4h-2ah &= a^2-2ah \\ h &= \frac{a^2}{4}\end{aligned}$$

3. A

$$\begin{aligned}g(2x)-g(x-1) &= (9-9)-10[(2x)^2-(x-1)^2] \\ &= -10(3x^2+2x-1) \\ &= 10-20x-30x^2\end{aligned}$$

4. C

Put $x = 1$.

$$\begin{aligned}(3+a)(1-2) &= 3 \\ a &= -6\end{aligned}$$

Put $x = 2$.

$$0 = 12 + b(2-1)$$

$$b = -12$$

5. C

A. ✗. It should be 0.019 instead.

B. ✗. It should be 0.0187 instead.

C. ✓.

D. ✗. The number has 5 significant figures only.

6. B

$$-2 = 2\left(-\frac{1}{2}\right)^3 - 5\left(-\frac{1}{2}\right)^2 - a^2\left(-\frac{1}{2}\right) + a$$

$$0 = \frac{a^2}{2} + a + \frac{1}{2}$$

$$a = -1$$

$$\text{Required remainder} = 2\left(\frac{3}{2}\right)^3 - 5\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) - 1$$

$$= -7$$

7. D

$2x + 3 > 7$ and $3 - \dfrac{x+3}{4} < x + 1$

$2x > 4$ $\qquad -\dfrac{5x}{4} < -\dfrac{5}{4}$

$x > 2$ $\qquad x > 1$

Thus, $x > 2$.

8. B

Assign suitable values to the intercepts.

L_1:

$(1, 0) \rightarrow b = 1$

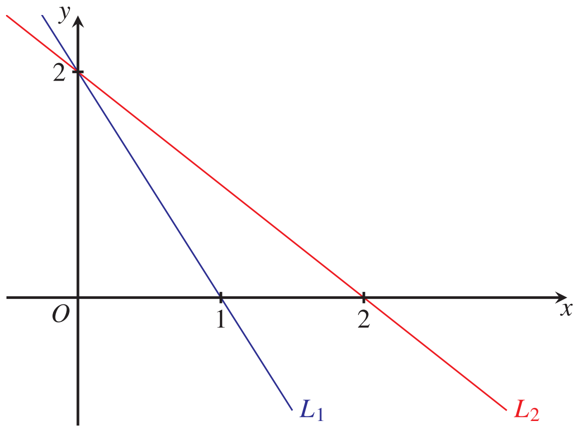
$(0, 2) \rightarrow a = \dfrac{1}{2}$

L_2:

$(2, 0) \rightarrow d = 2$

$(0, 2) \rightarrow c = 1$

Only statement III is true.



9. D

A. ✗.

y-intercept $= -(-5)(1) = 5$

B. ✗.

The coefficient of x^2 is -1, which is negative.

The graph opens downward.

C. ✗.

When $y = 0$, $x = -1$ or 5.

The x-intercepts are -1 and 5.

D. ✓.

The equation of axis of symmetry is

$$x = \frac{-1+5}{2}$$

$$x = 2$$

10. A

$$\text{Interest} = 76\,000\left(1+\frac{4\%}{2}\right)^{6\times 2} - 76\,000$$

$$\approx \$20\,386$$

11. B

$$\frac{3a+b}{a+3b} = \frac{2}{5}$$

$$15a + 5b = 2a + 6b$$

$$b = 13a$$

$$\frac{2a+b}{3a+4b} = \frac{2a+(13a)}{3a+4(13a)}$$

$$= \frac{15a}{55a}$$

$$= \frac{3}{11}$$

12. B

Let $y = \frac{kx^2}{\sqrt{w}}$, where k is a non-zero constant.

Then $k = \frac{y\sqrt{w}}{x^2}$.

Thus, $\frac{x^4}{wy^2} = \frac{1}{k^2}$ is a constant.

13. B

Let $a_4 = x$.
Put $n = 4$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_6 = a_4 + a_5$$
$$29 = x + a_5$$
$$a_5 = 29 - x$$

Put $n = 5$, $n = 6$ and $n = 7$ into $a_{n+2} = a_n + a_{n+1}$.

$$a_7 = a_5 + a_6 \qquad a_8 = a_6 + a_7 \qquad a_9 = a_7 + a_8$$
$$a_7 = (29 - x) + 29 \qquad a_8 = 29 + (58 - x) \qquad 123 = (58 - x) + (87 - x)$$
$$a_7 = 58 - x \qquad a_8 = 87 - x \qquad x = 11$$

14. D

Lower limit of P

$$= 3.5 \times 1.5 + \frac{(3.5 + 5.5) \times (4.5 - 1.5)}{2}$$
$$= 18.75$$

Upper limit of P

$$= 4.5 \times 2.5 + \frac{(4.5 + 6.5) \times (5.5 - 2.5)}{2}$$
$$= 27.75$$

15. D

Let A km^2 be the actual area of the farmland.

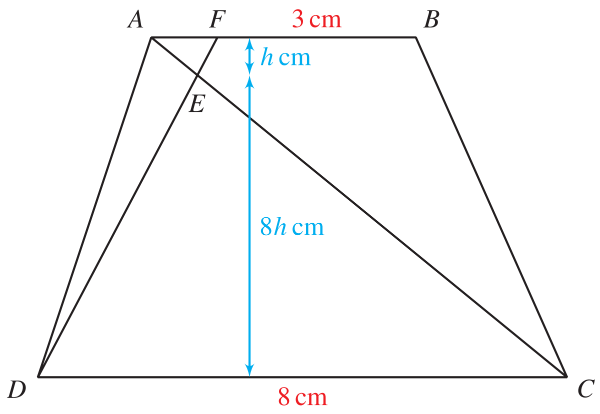
$$\frac{A \times (1000 \times 100)^2}{15} = \left(\frac{60\,000}{1}\right)^2$$
$$A = 5.4$$

16. A

$$u : v : w = \frac{1}{3}\pi r^2(2r) : \pi r^2(r) : \frac{2}{3}\pi r^3$$
$$= \frac{2}{3} : 1 : \frac{2}{3}$$
$$= 2 : 3 : 2$$

17. C

Let $AF = 1$ cm. Then we have the lengths as shown in the figure.



Point E Note that $\triangle AFE \sim \triangle CDE$ (ratio 1 : 8).
Consider the area of $\triangle CED$.

$$64 = \frac{(8)(8h)}{2}$$
$$h = 2$$

$$\text{Required area} = \frac{(4+8)(9h)}{2} = 108\text{ cm}^2$$

18. C

Let θ be the exterior angle of the polygon.

$$\theta + 7\theta = 180°$$
$$\theta = 22.5°$$

I. ✓.
$n = \dfrac{360°}{22.5°} = 16$

II. ✗.

III. ✓.
Number of diagonals $= C_2^{16} - 16 = 104$

19. A

Note that $\triangle ABD \sim \triangle CAD$.

$$\frac{AD}{CD} = \frac{BD}{AD}$$
$$\frac{12}{25 - BD} = \frac{BD}{12}$$
$$0 = -(BD)^2 + 25BD - 144$$
$$BD = 9\,\text{cm} \quad \text{or} \quad 16\,\text{cm}$$

When $BD = 9\,\text{cm}$, then $CD = 25 - 9 = 16\,\text{cm}$.
Perimeter $= \sqrt{9^2 + 12^2} + \sqrt{12^2 + 16^2} + 25 = 60\,\text{cm}$
When $BD = 16\,\text{cm}$, then $CD = 25 - 16 = 9\,\text{cm}$.
Perimeter $= \sqrt{16^2 + 12^2} + \sqrt{12^2 + 9^2} + 25 = 60\,\text{cm}$
Thus, perimeter is $60\,\text{cm}$.

20. D

In $\triangle QRU$, we have $\angle RQU = \angle QUR = 30°$.
Since $PR // RU$, we have $\angle PQU = \angle QUR = 30°$.
Consider $\triangle PQS$.

$$\angle SQP + \angle PSQ + \angle QPS = 180°$$
$$30° + 80° + \angle QPS = 180°$$
$$\angle QPS = 70°$$

Consider $\triangle PQR$.

$$\angle RQP + \angle QPR + \angle PRQ = 180°$$
$$(30° + 30°) + 70° + \angle PRQ = 180°$$
$$\angle PRQ = 50°$$

Since RT is the angle bisector of $\angle PRQ$, we have $\angle PRT = \dfrac{50°}{2} = 25°$.
Consider $\triangle PTR$.

$$\angle TPR + \angle PTR + \angle TRP = 180°$$
$$70° + \angle PTR + 25° = 180°$$
$$\angle PTR = 85°$$

21. C

Let $\angle BAD = \theta$.

$$\begin{aligned}\angle CDF &= \angle DAE + \angle AED \\ &= \theta + 32^\circ\end{aligned}$$

$$\begin{aligned}\angle BCD &= \angle CFD + \angle CDF \\ &= 62^\circ + (\theta + 32^\circ) \\ &= \theta + 94^\circ\end{aligned}$$

$$\begin{aligned}\angle BAD + \angle BCD &= 180^\circ \\ \theta + (\theta + 94^\circ) &= 180^\circ \\ \theta &= 43^\circ\end{aligned}$$

22. D

$\angle CBD = \angle CDB$

$$\begin{aligned}\angle CBD + \angle CDB + \angle BCD &= 180^\circ \\ \angle CDB + 114^\circ &= 180^\circ \\ \angle CDB &= 33^\circ\end{aligned}$$

$$\begin{aligned}\angle BAE + \angle BDE &= 180^\circ \\ 126^\circ + \angle BDE &= 180^\circ \\ \angle BDE &= 54^\circ\end{aligned}$$

$$\begin{aligned}\angle CDE &= \angle BDE + \angle CDB \\ &= 54^\circ + 33^\circ \\ &= 87^\circ\end{aligned}$$

23. D

$$\begin{aligned}\frac{\tan 45^\circ}{1-\cos\theta} + \frac{2\sin 30^\circ}{1+\sin(90^\circ+\theta)} &= \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \\ &= \frac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta} \\ &= \frac{2}{\sin^2\theta}\end{aligned}$$

24. A

The coordinates of B and C are $(3,\ 8)$ and $(-8,\ 3)$ respectively.

Reflect point C with respect to the line $x = -2$, the image is at $(4,\ 3)$.

25. A

The coordinates of A and B are $(2,\ 0)$ and $(0,\ -8)$ respectively.
Let $P\,(x,\ y)$.

$$\sqrt{(x-2)^2+y^2}=\sqrt{x^2+(y+8)^2}$$
$$x^2+y^2-4x+4=x^2+y^2+16y+64$$
$$-4x-16y=60$$
$$x+4y=-15$$

26. B

The coordinates of the mid-point of BC are $(9,\ 5)$.
Slope of required straight line $=\dfrac{5-11}{9-1}=-\dfrac{3}{4}$
Required equation is

$$y-11=-\frac{3}{4}(x-1)$$
$$3x+4y-47=0$$

27. A

$$x^2+y^2+\frac{kx}{2}+6y=0$$

The coordinates of the centre are $\left(-\dfrac{k}{4},\ -3\right)$.

$$3\left(-\frac{k}{4}\right)-2(-3)+6=0$$
$$k=16$$

The coordinates of the centre are $(-4,\ -3)$.
Radius $=\sqrt{4^2+3^2}=5$

28. C

$$\text{Required probability}=\frac{3\times 2}{6^2}=\frac{1}{6}$$

29. B

$$\text{Mean} = \frac{32(68) + 30(58) + 30(62) + 28(53)}{32 + 30 + 30 + 28}$$
$$= 60.5$$

30. A

I. ✓.

The mean is maximum when $m = n = 20$.

$$\text{Maximum mean} = \frac{5 + 7 + 8 + \ldots + 20}{12} \approx 12.7 < 13$$

II. ✗.

When $m = n = 20$, median is 14.

III. ✗.

When $m = 9$ and $n = 20$, the mode has two values 9 and 15.

31. D

$$\log_9 y - 4 = \frac{4 - 0}{0 - (-2)}(x - 0)$$
$$\log_9 y = 2x + 4$$
$$y = 9^{2x+4}$$
$$y = 6561 \cdot 81^x$$

Thus, $b = 81$.

32. B

$$\log_a 2016 = \frac{\log 2016}{\log a}$$

Note that $\log 0.5 < 0 < \log 5 < \log 6$.

We have $\log_{0.5} 2016 < 0 < \log_6 2016 < \log_5 2016$.

33. D

$$11 \times 16^{11} + 17 \times 16^9 + 82$$
$$= 11 \times 16^{11} + 1 \times 16^{10} + 1 \times 16^9 + 82$$
$$= \text{B}11000000052_{16}$$

34. C

$$\frac{i^{2024}+2i^{2025}}{i^{2026}+i^{2027}}=\frac{1+2i}{-1-i}$$
$$=-\frac{3}{2}-\frac{1}{2}i$$

The imaginary part is $-\frac{1}{2}$.

35. D

Maximum value of $3x-2y+15$ occurs at the bottom right corners, $B\,(3,\ 3)$ or $C\,(2,\ 0)$.

$(x,\ y)$	$B\,(3,\ 3)$	$C\,(2,\ 0)$
$3x-2y+15$	18	21

Maximum value = 21

36. C

$$\frac{5a}{1-a}=4+2a$$
$$5a=(1-a)(4+2a)$$
$$0=-2a^2-7a+4$$
$$a=-4\text{ (rejected)}\quad\text{or}\quad\frac{1}{2}$$

37. B

Let $(h,\ k)$ be the coordinates of the centre.
The centre is the mid-point of AB.
The coordinates of B are $(2h-4,\ 2k+1)$.
B lies on $4x-3y+31=0$.

$$4(2h-4)-3(2k+1)+31=0$$
$$8h-6k=-12$$

AB is perpendicular to the line $4x-3y+31=0$.

$$\frac{k+1}{h-4}\times\frac{4}{3}=-1$$
$$3h+4k=8$$

Solving, we have $h=0$ and $k=2$.

38. A

$\angle BCA = 90°$ and $\angle CBA = 180° - 90° - 42° = 48°$
$\angle OBC = \dfrac{48°}{2} = 24°$
$\angle BOC = 180° - 90° - 24° = 66°$

39. A

I. ✓.

II. ✓.

III. ✗.

Note that θ, α and β lie between 0° and 90°.
Since $AB > AX > AC$, we have $\dfrac{1}{AB} < \dfrac{1}{AX} < \dfrac{1}{AC}$.
We have $\tan\theta = \dfrac{VA}{AB}$, $\tan\beta = \dfrac{VA}{AX}$ and $\tan\alpha = \dfrac{VA}{AC}$.
Thus, $\tan\theta < \tan\beta < \tan\alpha$ and $\theta < \beta < \alpha$.

40. D

I. ✗.

Put $f(x) = 0$.

$$0 = -2x^2 + 4x + 16$$
$$x = -2 \quad \text{or} \quad 4$$

Difference between x-intercept of the graph of $y = g(x)$

$= \dfrac{1}{2}(4 - (-2))$

$= 3$

II. ✓.

$$\begin{aligned} f(x) &= -2x^2 + 4x + 16 \\ &= -2(x-1)^2 + 18 \end{aligned}$$
$$\begin{aligned} g(x) &= -\left[-2[(-2x+4) - 1]^2 + 18\right] \\ &= 2(-2x+3)^2 - 18 \\ &= 8\left(x - \frac{3}{2}\right)^2 - 18 \end{aligned}$$

The y-coordinate of the vertex is -18.

III. ✓.

When $x = 0$, $y = -f(0+4) = -f(4) = -[-2(4)^2 + 4(4) + 16] = 0$.
The y-intercept is 0.

41. B

I. ✓.

The coordinates of the circumcentre are $\left(6,\ \frac{7}{2}\right)$.

The coordinates of the mid-point of QR are $\left(6,\ \frac{7}{2}\right)$.

Thus, the circumcentre of $\triangle PQR$ is the mid-point of QR.

II. ✗.

Note that $\angle QPR = 90°$.

The orthocentre of $\triangle PQR$ is at P and it does not lie outside $\triangle PQR$.

III. ✓.

The incentre of a triangle always lie inside the triangle.

42. C

Required number $= (C_4^8 + C_3^8 C_1^6) \times 5!$

$= 48\,720$

43. C

Required probability $= 1 - 0.7^5 - C_4^5 (0.7)^4 (0.3)$

$= 0.471\,78$

44. A

Let the mean of the scores be $\bar{x}$ marks.

$$\frac{96 - \bar{x}}{3} = 4$$

$$\bar{x} = 84$$

Required standard score $= \frac{81 - 84}{3}$

$= -1$

45. A

Required variance is equal to the variance of the numbers $-7, -1, 0, 4, 4, 6, 9$ and 17.

Required variance is 45.