Solution	Marks

REG-2425-MOCK-SET 8-MATH-CP 1
Suggested solutions

1. $m = \dfrac{kn + m}{n - 2}$

    $mn - 2m = kn + m$ 1M

    $n(m - k) = 3m$ 1M

           $n = \dfrac{3m}{m - k}$ 1A

2. $\dfrac{(ab^{-3})^4}{a^{-6}b^{-5}} = \dfrac{a^4 b^{-12}}{a^{-6}b^{-5}}$ 1M

          $= \dfrac{a^{4+6}}{b^{-5+12}}$ 1M

          $= \dfrac{a^{10}}{b^7}$ 1A

3.   (a) 308   1A

     (b) 300   1A

     (c) 308.10   1A

4.   (a) $3v^2 - 2uv^2 + v^3 = v^2(3 - 2u + v)$ 1A

     (b) $3 - 2u + v - 3v^2 + 2uv^2 - v^3$

           $= (3 - 2u + v) - v^2(3 - 2u + v)$ 1M

           $= (1 - v^2)(3 - 2u + v)$ 1M

           $= (1 - v)(1 + v)(3 - 2u + v)$ 1A

5. Let \$$x$ be the cost of the heater.

   Marked price $= x(1 + 20\%) = \$1.2x$ 1M

   Selling price $= 1.2x(1 - 15\%) = \$1.02x$ 1M

          $1.02x - x = 5$ 1M

               $x = 250$ 1A

6.   (a) $x - 3(x - 4) > 10$

           $-2x > -2$ 1M

             $x < 1$ 1A

     Thus, x can be any real number.   1M

     (b) 1   1A

Solution	Marks
7. (a) $\angle DOF = 290° - 110° = 180°$	1M
D, O and F are collinear.	1A
(b) $\angle EOF = 290° - 200° = 90°$	
$\dfrac{(3r+r)(2r)}{2} = 1296$	1M
$r = 18$ or -18 (rejected)	1A
8. $x = 180° - \theta$	1A
$\dfrac{\angle BED}{CBE} = \dfrac{7+5}{5+4}$	1M+1A
$\angle BED = \dfrac{4}{3}(180° - \theta)$	
$= \dfrac{720° - 4\theta}{3}$	
$y = \angle BED$	1M
$= \dfrac{720° - 4\theta}{3}$	1A
9. (a) $\dfrac{m}{m+n+6} = \dfrac{5}{8}$	1M
$8m = 5m + 5n + 30$	
$3m - 5n = 30$	
$7\left(\dfrac{m}{m+n+6}\right) + 23\left(\dfrac{n}{m+n+6}\right) + m\left(\dfrac{6}{m+n+6}\right) = 11$	1M
$7m + 23n + 6m = 11m + 11n + 66$	
$m + 6n = 33$	
Solving, we have $m = 15$ and $n = 3$.	1A
(b) Required angle	
$= 360° \times \dfrac{3}{15+3+6}$	1M
$= 45°$	1A
10. (a) $\dfrac{(10+a) + (10+a) + \ldots + 45}{20} = 29$	1M
$2a + 3b = 21$	
Note that a and b are integers with $0 \le a \le 9$ and $4 \le b \le 8$.	
We have $(a,\ b) = (3,\ 5)$ or $(0,\ 7)$.	1A+1A
(b) 35	1A
(c) The inter-quartile range is the least when $b = 5$.	
Least inter-quartile range	
$= 35 - 24.5$	1M
$= 10.5$	1A

Solution	Marks

11. (a) Height of the original cone $= 24$ cm

Curved surface area of the original cone

$= \pi(10)(\sqrt{10^2 + 24^2})$

$= 260\pi \text{ cm}^2$

Total surface area of X

$= 260\pi \times \left(1 - \dfrac{5^2}{10^2}\right) + 2\pi(10)^2 + 5^2\pi$ **1M+1M**

$= 420\pi \text{ cm}^2$ **1A**

(b) Volume of X

$= \dfrac{1}{3}\pi(10)^2(24) \times \left(1 - \dfrac{5^3}{10^3}\right) + \dfrac{2}{3}\pi(10)^3$ **1M**

$= \dfrac{4100\pi}{3} \text{ cm}^3$

Volume of Y

$= \dfrac{4100\pi}{3} \times \left(\sqrt{\dfrac{9}{16}}\right)^3$ **1M**

$\approx 1810 \text{ cm}^3$

$> 1800 \text{ cm}^3$

The claim is disagreed.   **1A**

Solution	Marks

12. (a) $\angle ABE = \angle CDE$ *(alt. $\angle$s, AB//DC)*

      $\angle AEB = \angle CED$ *(vert. opp. $\angle$s)*

      $\triangle ABE \sim \triangle CDE$ *(AA)*

Marking Scheme		
Case 1	Any correct proof with correct reasons.	2
Case 2	Any correct proof without reasons.	1

(b) (i)
$$\frac{AE}{CE} = \frac{AB}{CD}$$
$$\frac{72 - CE}{CE} = \frac{30}{60}$$
$$144 - 2CE = CE$$
$$CE = 48 \text{ cm}$$
$$CE^2 + DE^2 = 48^2 + 36^2 = 3600 \text{ cm}^2$$
$$CD^2 = 60^2 = 3600 \text{ cm}^2 = CE^2 + DE^2$$

    We have $\angle CED = 90°$.

    $\triangle CDE$ is a right-angled triangle.

1M (for $\frac{AE}{CE} = \frac{AB}{CD}$ line)

1M (for CD^2 line)

1A

(ii) Let h cm be the shortest distance from C to AD.
$$AD = \sqrt{AE^2 + DE^2}$$
$$= \sqrt{(72 - 48)^2 + 36^2}$$
$$= \sqrt{1872} \text{ cm}$$

    Consider the area of $\triangle ACD$.
$$\frac{1}{2} \times \sqrt{1872} \times h = \frac{1}{2} \times 72 \times 36$$
$$h \approx 59.9$$
$$h > 58$$

1M

    There is no point F lying on AD such that the distance between C and F is less than 58 cm.

1A

Solution	Marks

13. (a) Let $f(x) = (2x^2 - x - 1)(Ax + B) + (mx + 5)$, where A and B are constants.

1M

$$f\left(-\frac{1}{2}\right) = 0$$

$$\left[2\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 1\right]\left[A\left(-\frac{1}{2}\right) + B\right] + m\left(-\frac{1}{2}\right) + 5 = 0$$

1M

$$m = 10$$

(b) We have $f(0) = -2$.

$$-B + 5 = -2$$

1M

$$B = 7$$

We have $f\left(\frac{1}{2}\right) = 0$.

$$\left(\frac{1}{2} - \frac{1}{2} - 1\right)\left(\frac{A}{2} + 7\right) + \frac{10}{2} + 5 = 0$$

$$A = 6$$

We have $f(x) = (2x^2 - x - 1)(6x + 7) + 10x + 5$.

1A

$$f(x) = 0$$

$$(2x + 1)(x - 1)(6x + 7) + 5(2x + 1) = 0$$

$$(2x + 1)(6x^2 + x - 2) = 0$$

1M

$$(2x + 1)(3x + 2)(2x - 1) = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad -\frac{2}{3} \quad \text{or} \quad \frac{1}{2}$$

1A

All the roots are not integers.

The claim is agreed.

1A

Solution	Marks

14.　(a)　The coordinates of V are $(-3,\ -4)$.　　　　　　　　　　　　　　1A

　　　　　　When $x = 0$, $y = 3^2 - 4 = 5$.

　　　　　　The coordinates of A are $(0,\ 5)$.　　　　　　　　　　　　　　　1A

　　(b)　Slope of $VA = \dfrac{5+4}{0+3} = 3$

　　　　　　Slope of $L = -\dfrac{1}{3}$　　　　　　　　　　　　　　　　　　　1M

　　　　　　Equation of L is

$$y + 4 = -\frac{1}{3}(x + 3)$$　　　　　　　　　　1M

$$x + 3y + 15 = 0$$　　　　　　　　　　　　1A

　　(c)　The coordinates of C and D are $(-15,\ 0)$ and $(0,\ -5)$ respectively.

　　　　　　The coordinates of mid-point of CD are $\left(-\dfrac{15}{2},\ -\dfrac{5}{2}\right)$.　　　1M

　　　　　　Required equation of locus is

$$\left(x + \frac{15}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = 4^2$$　　　　1M

$$\left(x + \frac{15}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = 16$$　　　　1A

15.　(a)　Let $\bar{x}$ be the mean of the scores.

$$\frac{74 - \bar{x}}{12.5} = 0.96$$　　　　　　　　　　　1M

$$\bar{x} = 62$$　　　　　　　　　　　　　　1A

　　(b)　Standard score of Cora

$$= \frac{60 - 62}{12.5}$$

$$= -0.16$$

　　　　　　Note that $-0.16 + 1 = 0.84 < 0.96$.　　　　　　　　　　　　　1M

　　　　　　The claim is not correct.　　　　　　　　　　　　　　　　　1A

Solution	Marks

16. (a) Slope of $L_1 = \dfrac{12 - 6}{9 - 0} = \dfrac{2}{3}$

The equation of L_1 is

$$\frac{y - 6}{x - 0} = \frac{2}{3}$$ 1M

$$2x - 3y + 18 = 0$$ 1A

The equation of L_2 is

$$\frac{y - 12}{x - 9} = -\frac{3}{2}$$

$$3x + 2y - 51 = 0$$

Required system of inequalities is $\begin{cases} 2x - 3y + 18 \geq 0 \\ 3x + 2y - 51 \leq 0 \\ x \geq 0 \\ y \geq 0 \end{cases}$.   1A

(b) The coordinates of the vertices are $(0,\ 0)$, $(0,\ 6)$, $(9,\ 12)$ and $(17,\ 0)$.

$(x,\ y)$	$(0,\ 0)$	$(0,\ 6)$	$(9,\ 12)$	$(17,\ 0)$
$3x - 4y$	0	−24	−21	51

        1M

The greatest value is 51.   1A

17. (a) $\angle PRQ = 180° - 40° - 58° = 82°$

$$\frac{PR}{\sin 58°} = \frac{75}{\sin 82°}$$ 1M

$$PR \approx 64.2 \text{ cm}$$ 1A

(b) Let T and U be the projections of P and M on the horizontal ground respectively.

Required angle is $\angle MQU$.   1M

$$MP = \frac{PR}{2} \approx 32.1 \text{ cm}$$

$$MQ^2 = 75^2 + MP^2 - 2(75)(MP)\cos 40°$$ 1M

$$MQ \approx 54.5 \text{ cm}$$

$$PT = 55\sin 50° \approx 42.1 \text{ cm}$$

$$MU = \frac{PT}{2} \approx 21.1 \text{ cm}$$

$$\sin \angle MQU = \frac{MU}{MQ}$$ 1M

$$\angle MQU \approx 22.8°$$

$$< 25°$$

The claim is not correct.   1A

Solution	Marks

18. (a) $8(\alpha + \beta) - \dfrac{1}{\alpha} = \dfrac{1}{\beta} - 8(\alpha + \beta)$ **1M**

$$16(\alpha + \beta) = \dfrac{\alpha + \beta}{\alpha\beta}$$

$$16 = \dfrac{1}{\alpha\beta}$$

$$\log_4 16 = \log_4 \dfrac{1}{\alpha\beta}$$

$$2 = -\log_4 \alpha - \log_4 \beta \qquad \textbf{1M}$$

$$\log_4 \beta = -2 - \log_4 \alpha \qquad \textbf{1A}$$

(b) $\log_{\alpha\beta} \dfrac{\alpha}{\beta} \div \log_4 \dfrac{4}{\alpha} = \log_4 \dfrac{4}{\beta} \div \log_{\alpha\beta} \dfrac{\alpha}{\beta}$ **1M**

$$\left(\dfrac{\log_4 \alpha - \log_4 \beta}{\log_4 \alpha\beta} \right)^2 = (1 - \log_4 \beta)(1 - \log_4 \alpha) \qquad \textbf{1M}$$

$$\left(\dfrac{\log_4 \alpha - \log_4 \beta}{-2} \right)^2 = (1 - \log_4 \beta)(1 - \log_4 \alpha)$$

Let $u = \log_4 \alpha$.

$$\dfrac{[u - (-2 - u)]^2}{4} = [1 - (-2 - u)](1 - u) \qquad \textbf{1M}$$

$$(u + 1)^2 = (3 + u)(1 - u)$$

$$2u^2 + 4u - 2 = 0$$

$$u = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-2)}}{2(2)}$$

$$u = -1 - \sqrt{2} \quad \text{or} \quad -1 + \sqrt{2} \text{ (rejected)}$$

Common ratio $= \log_{\alpha\beta} \dfrac{\alpha}{\beta} \div \log_4 \dfrac{4}{\alpha}$

$$= \dfrac{\log_4 \alpha - \log_4 \beta}{\log_4 \alpha\beta} \div (1 - \log_4 \alpha)$$

$$= \dfrac{u - (-2 - u)}{-2} \div (1 - u)$$

$$= -(u + 1)(1 - u)$$

$$= -(-\sqrt{2}) \div (2 + \sqrt{2}) \qquad \textbf{1M}$$

$$= \dfrac{\sqrt{2}}{2 + \sqrt{2}} \times \dfrac{2 - \sqrt{2}}{2 - \sqrt{2}}$$

$$= \sqrt{2} - 1$$

$$= \log_4 \beta \qquad \textbf{1}$$

Solution	Marks

19. (a) (i) $r^2 = \left(\dfrac{k-4}{-2}\right)^2 + \left(\dfrac{2+k}{-2}\right)^2 - (-3k-6)$ **1M**

$\qquad = \dfrac{k^2}{2} + 2k + 11$ **1A**

(ii) $r^2 = \dfrac{k^2}{2} + 2k + 11$

$\qquad = \dfrac{1}{2}(k^2 + 4k + 4) + 9$ **1M**

$\qquad = \dfrac{1}{2}(k+2)^2 + 9$

$r = \sqrt{\dfrac{1}{2}(k+2)^2 + 9}$

When $k = -2$, the value of r is the least.

Required equation is $x^2 + y^2 - 6x = 0$.   **1A**

(b) Equation of L is $y = mx + h$.

$\qquad x^2 + (mx+h)^2 - 6x = 0$ **1M**

$\qquad (1+m^2)x^2 + (2mh-6)x + h^2 = 0$

The equation has only one real root.

$\qquad \Delta = (2mh-6)^2 - 4(1+m^2)h^2 = 0$ **1M**

$\qquad\qquad -24mh - 4h^2 + 36 = 0$

$\qquad\qquad\qquad m = \dfrac{9 - h^2}{6h}$ **1**

(c) Coordinates of A are $(3, 0)$.

Let M be the mid-point of OA, then $M\left(\dfrac{3}{2}, 0\right)$.

Coordinates of centroid $= \left(\dfrac{1(0) + 2\left(\frac{3}{2}\right)}{1+2}, \dfrac{1(h) + 2(0)}{1+2}\right)$ **1M**

$\qquad\qquad = \left(1, \dfrac{h}{3}\right)$

Centroid lies on the circle.

$\qquad 1^2 + \left(\dfrac{h}{3}\right)^2 - 6(1) = 0$ **1M**

$\qquad\qquad h^2 = 45$

$\qquad\qquad h = \sqrt{45} \quad \text{or} \quad -\sqrt{45} \text{ (rejected)}$

Slope of $PQ = m$

$\qquad \dfrac{\sqrt{45} - 0}{0 - q} = \dfrac{9 - (\sqrt{45})^2}{6(\sqrt{45})}$ **1M**

$\qquad\qquad q = \dfrac{15}{2}$

Solution	Marks
Area of $\triangle OPQ = \dfrac{(\sqrt{45})\left(\frac{15}{2}\right)}{2}$	1M
≈ 25.2	
> 25	
The claim is disagreed.	1A

END OF PAPER