| Solution | Marks |
|---|---|

**REG-2425-MOCK-SET 10-MATH-EP(M2)**
**Suggested solutions**

1. $f'(0) = \lim_{h\to 0} \dfrac{\frac{e^h}{h+2} - \frac{e^0}{0+2}}{h}$ — 1M

$= \lim_{h\to 0} \dfrac{2e^h - (h+2)}{2h(h+2)}$ — 1M

$= \lim_{h\to 0} \left[\dfrac{e^h - 1}{h} \cdot \dfrac{1}{h+2} - \dfrac{1}{2(h+2)}\right]$ — 1M

$= 1 \cdot \dfrac{1}{2} - \dfrac{1}{2(2)}$

$= \dfrac{1}{4}$ — 1A

2. (a) $(1+3x)^n = 1 + C_1^n(3x) + C_2^n(3x)^2 + C_3^n(3x)^3 + C_4^n(3x)^4 + \ldots$ — 1M

$= 1 + 3C_1^n x + 9C_2^n x^2 + 27C_3^n x^3 + 81C_4^n x^4 + \ldots$

$\left(x - \dfrac{4}{x}\right)^2 = x^2 - 8 + \dfrac{16}{x^2}$

$(1)(-8) + (9C_2^n)(16) = 5176$ — 1M

$-8 + \dfrac{9n(n-1)}{2}(16) = 5176$

$72n^2 - 72n - 5184 = 0$

$n = 9 \quad \text{or} \quad -8 \text{ (rejected)}$ — 1A

(b) Required coefficient $= (1)(1) + (9C_2^9)(-8) + (81C_4^9)(16)$ — 1M

$= 160\,705$ — 1A

| Solution | Marks |
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| 3. (a) (i) $\cos\left(3\times\frac{3\pi}{10}\right)+\sin\left(2\times\frac{3\pi}{10}\right)$ | |
| $=\cos\left(\pi-\frac{\pi}{10}\right)+\sin\left(\frac{\pi}{2}+\frac{\pi}{10}\right)$ | |
| $=-\cos\frac{\pi}{10}+\cos\frac{\pi}{10}$ | |
| $=0$ | |
| Thus, $\frac{3\pi}{10}$ is a root of the equation $\cos 3\theta+\sin 2\theta=0$. | 1 |
| (ii) $\cos 3\theta$ | |
| $=\cos(2\theta+\theta)$ | |
| $=\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$ | 1M |
| $=(2\cos^2\theta-1)\cos\theta-(2\sin\theta\cos\theta)\sin\theta$ | |
| $=2\cos^3\theta-\cos\theta-2\cos\theta(1-\cos^2\theta)$ | |
| $=4\cos^3\theta-3\cos\theta$ | 1 |
| (b) $\cos 3\theta+\sin 2\theta=0$ | |
| $(4\cos^3\theta-3\cos\theta)+2\sin\theta\cos\theta=0$ | 1M |
| $\cos\theta(4\cos^2\theta-3+2\sin\theta)=0$ | |
| $\cos\theta[4(1-\sin^2\theta)+2\sin\theta-3]=0$ | |
| $\cos\theta(-4\sin^2\theta+2\sin\theta+1)=0$ | |
| $\cos\theta=0$ or $-4\sin^2\theta+2\sin\theta+1=0$ | |
| Consider the equation $-4\sin^2\theta+2\sin\theta+1=0$. | |
| $\sin\theta=\frac{-2\pm\sqrt{2^2-4(-4)(1)}}{2(-4)}$ | |
| $=\frac{1\pm\sqrt{5}}{4}$ | |
| By (a), $\frac{3\pi}{10}$ is a root of the equation. | |
| Since $0<\frac{3\pi}{10}<\frac{\pi}{2}$, $\cos\frac{3\pi}{10}\neq 0$ and $\sin\frac{3\pi}{10}>0$. | |
| Thus, we have $\sin\frac{3\pi}{10}=\frac{1+\sqrt{5}}{4}$. | 1A |
| 4. (a) $\int u(21^u)\,\mathrm{d}u=\frac{1}{\ln 21}\left[u(21^u)-\int 21^u\,\mathrm{d}u\right]$ | 1M+1M |
| $=\frac{u(21^u)}{\ln 21}-\frac{21^u}{(\ln 21)^2}+\text{constant}$ | 1A |
| (b) Required area | |
| $=\int_0^1 x(21^x)\,\mathrm{d}x$ | 1M |
| $=\left[\frac{x(21^x)}{\ln 21}-\frac{21^x}{(\ln 21)^2}\right]_0^1$ | 1M |
| $=\frac{21}{\ln 21}-\frac{20}{(\ln 21)^2}$ | 1A |

| Solution | Marks |
|---|---|
| 5. (a) Let $x - 1 = 6\sin\theta$. Then $dx = 6\cos\theta\, d\theta$. | 1M |
| $\displaystyle\int \frac{1}{\sqrt{35+2x-x^2}}\,dx = \int \frac{1}{\sqrt{36-(x-1)^2}}\,dx$ | |
| $\displaystyle = \int \frac{6\cos\theta}{\sqrt{36-36\sin^2\theta}}\,d\theta$ | |
| $\displaystyle = \int d\theta$ | 1M |
| $= \theta + \text{constant}$ | |
| $\displaystyle = \sin^{-1}\frac{x-1}{6} + \text{constant}$ | 1A |
| (b) Let $u = 35 + 2x - x^2$. Then $du = (2-2x)\,dx$. | 1M |
| $\displaystyle y = \int \frac{-2x+3}{\sqrt{35+2x-x^2}}\,dx$ | |
| $\displaystyle = \int \frac{-2x+2}{\sqrt{35+2x-x^2}}\,dx + \int \frac{1}{\sqrt{35+2x-x^2}}\,dx$ | |
| $\displaystyle = \int u^{-\frac{1}{2}}\,du + \sin^{-1}\frac{x-1}{6} + C_1$ | 1M |
| $\displaystyle = 2u^{\frac{1}{2}} + \sin^{-1}\frac{x-1}{6} + C$ | |
| $\displaystyle = 2\sqrt{35+2x-x^2} + \sin^{-1}\frac{x-1}{6} + C$ | |
| $G$ passes through the point $\left(4,\ 6\sqrt{3}\right)$. | |
| $\displaystyle 6\sqrt{3} = 2\sqrt{35+8-16} + \sin^{-1}\frac{4-1}{6} + C$ | 1M |
| $\displaystyle C = -\frac{\pi}{6}$ | |
| Put $x = 1$. | |
| $\displaystyle y = 2\sqrt{35+2-1} + \sin^{-1}\frac{1-1}{6} - \frac{\pi}{6}$ | |
| $\displaystyle = 12 - \frac{\pi}{6}$ | |
| Thus, $G$ passes through $\left(1,\ 12 - \dfrac{\pi}{6}\right)$. | 1A |

| Solution | Marks |
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| 6. (a) When $n = 1$, | |
| L.H.S. $= 1(1+1)(1+2) = 6$ | |
| R.H.S. $= \dfrac{1(1+1)(1+2)(1+3)}{4} = 6 =$ L.H.S. | |
| It is true for $n = 1$. | 1 |
| Assume $\displaystyle\sum_{k=1}^{m} k(k+1)(k+2) = \frac{m(m+1)(m+2)(m+3)}{4}$, where $m \in \mathbb{Z}^+$. | 1M |
| $\displaystyle\sum_{k=1}^{m+1} k(k+1)(k+2)$ | |
| $\displaystyle = \sum_{k=1}^{m} k(k+1)(k+2) + (m+1)(m+2)(m+3)$ | |
| $\displaystyle = \frac{m(m+1)(m+2)(m+3)}{4} + (m+1)(m+2)(m+3)$ | 1M |
| $\displaystyle = \frac{(m+1)(m+2)(m+3)}{4}(m+4)$ | |
| $\displaystyle = \frac{(m+1)(m+2)(m+3)(m+4)}{4}$ | |
| It is true for $n = m+1$. | |
| By M.I., it is true $\forall n \in \mathbb{Z}^+$. | 1 |
| (b) $\displaystyle\sum_{k=44}^{66} k\left(\frac{k+1}{8}\right)\left(\frac{k+2}{9}\right)$ | |
| $\displaystyle = \frac{1}{72}\left[\sum_{k=1}^{66} k(k+1)(k+2) - \sum_{k=1}^{43} k(k+1)(k+2)\right]$ | 1M |
| $\displaystyle = \frac{1}{72}\left[\frac{66\times 67\times 68\times 69}{4} - \frac{43\times 44\times 45\times 46}{4}\right]$ | 1M |
| $= 58\,443$ | 1A |

| Solution | Marks |
|---|---|
| 7. (a) $\det M = \cos^2\theta + \sin^2\theta = 1$ | |
| $M^{-1} = \frac{1}{1}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$ | 1M |
| $= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ | |
| (b) $M^2 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ | |
| $= \begin{pmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$ | 1M |
| $= \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$ | |
| $M^3 = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ | |
| $= \begin{pmatrix} \cos 2\theta\cos\theta & -\cos 2\theta\sin\theta - \sin 2\theta\cos\theta \\ \sin 2\theta\cos\theta + \cos 2\theta\sin\theta & -\sin 2\theta\sin\theta + \cos 2\theta\cos\theta \end{pmatrix}$ | |
| $= \begin{pmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{pmatrix}$ | 1A |
| (c) $(M^2)^{-1}M(M^4+I) = (M^{-1})^2 M(M^4+I)$ | |
| $= M^3 + M^{-1}$ | 1M |
| $= \begin{pmatrix} \cos 3\theta + \cos\theta & -\sin 3\theta + \sin\theta \\ \sin 3\theta - \sin\theta & \cos 3\theta + \cos\theta \end{pmatrix}$ | |
| Compare the corresponding elements. | |
| $\cos 3\theta + \cos\theta = 0$ | 1M |
| $2\cos 2\theta\cos\theta = 0$ | |
| $\cos 2\theta = 0$ or $\cos\theta = 0$ | |
| $2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ ; $\theta = \frac{\pi}{2}$ | 1A |
| $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ | |
| When $\theta = \frac{\pi}{4}$, $a = b = 0$. Rejected. | |
| When $\theta = \frac{3\pi}{4}$, $a = b = 0$. Rejected. | |
| When $\theta = \frac{\pi}{2}$, $a = 2$ and $b = -2$. | |
| Thus, $a = 2$ and $b = -2$. | 1A |

| Solution | Marks |
|---|---|
| 8. (a) $f'(x) = \dfrac{(x-4)(2x) - x^2(1)}{(x-4)^2}$ | 1M |
| $= \dfrac{x^2 - 8x}{(x-4)^2}$ | 1A |
| (b) $\dfrac{x^2 - 8x}{(x-4)^2} = 0$ | |
| $x = 0$ or $8$ | 1A |
| (see table below) | 1M |
| $f(0) = 3$ and $f(8) = 19$ | |
| The local maximum is 3 and the local minimum is 19. | 1A |
| (c) Vertical asymptote is $x = 4$. | 1A |
| $f(x) = 3 + \dfrac{x^2}{x-4}$ | |
| $= x + 7 + \dfrac{16}{x-4}$ | 1M |
| Oblique asymptote is $y = x + 7$. | 1A |

| $x$ | $x < 0$ | $0 < x < 4$ | $4 < x < 8$ | $x > 8$ |
|---|---|---|---|---|
| $f'(x)$ | $+$ | $-$ | $-$ | $+$ |

| Solution | Marks |
|---|---|
| 9. (a) $A(\theta) = \dfrac{[(1-\sin\theta)+1](2-\cos\theta)}{2}$ | |
| $= \dfrac{(2-\sin\theta)(2-\cos\theta)}{2}$ | 1A |
| $\dfrac{\mathrm{d}A(\theta)}{\mathrm{d}\theta} = \dfrac{1}{2}[(-\cos\theta)(2-\cos\theta)+(2-\sin\theta)(\sin\theta)]$ | 1M |
| $= \dfrac{1}{2}[2(\sin\theta-\cos\theta)-(\sin^2\theta-\cos^2\theta)]$ | |
| $= \dfrac{1}{2}(\sin\theta-\cos\theta)(2-\sin\theta-\cos\theta)$ | 1 |
| (b) (i) For $0<\theta<\dfrac{\pi}{2}$, we have $\sin\theta<1$ and $\cos\theta<1$. | 1M |
| $2-\sin\theta-\cos\theta > 2-1-1$ | |
| $2-\sin\theta-\cos\theta > 0$ | 1 |
| (ii) When $A(\theta)=0$, | |
| $\sin\theta-\cos\theta=0$ | 1M |
| $\tan\theta = 1$ | |
| $\theta = \dfrac{\pi}{4}$ | |

| $\theta$ | $0<\theta<\dfrac{\pi}{4}$ | $\dfrac{\pi}{4}<\theta<\dfrac{\pi}{2}$ |
|---|---|---|
| $\dfrac{\mathrm{d}A(\theta)}{\mathrm{d}\theta}$ | $-$ | $+$ |

(Marks: 1M)

| Solution | Marks |
|---|---|
| $A(\theta)$ attains its minimum value at $\theta=\dfrac{\pi}{4}$. | 1A |
| Required value | |
| $= \dfrac{1}{2}\left(2-\sin\dfrac{\pi}{4}\right)\left(2-\cos\dfrac{\pi}{4}\right)$ | |
| $= \dfrac{9-4\sqrt{2}}{4}$ | 1A |
| (c) Let $t$ s be the time. | |
| $\dfrac{\mathrm{d}A(\theta)}{\mathrm{d}t} = \dfrac{\mathrm{d}A(\theta)}{\mathrm{d}\theta}\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$ | 1M |
| When $\theta=\dfrac{\pi}{3}$ and $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=-\dfrac{1}{10}$, | |
| $\dfrac{\mathrm{d}A(\theta)}{\mathrm{d}t} = \dfrac{1}{2}\left(\sin\dfrac{\pi}{3}-\cos\dfrac{\pi}{3}\right)\left(2-\sin\dfrac{\pi}{3}-\cos\dfrac{\pi}{3}\right)\left(-\dfrac{1}{10}\right)$ | 1M |
| $= \dfrac{1}{2}\cdot\dfrac{\sqrt{3}-1}{2}\cdot\dfrac{4-\sqrt{3}-1}{2}\cdot\left(-\dfrac{1}{10}\right)$ | |
| $= \dfrac{3-2\sqrt{3}}{40}$ | |
| Required rate is $\dfrac{3-2\sqrt{3}}{40}$ $\mathrm{m}^2/\mathrm{s}$. | 1A |

| Solution | Marks |
|---|---|
| 10. (a) (i) $\cos^4 x \sin^2 x = \cos^2 x \left(\dfrac{2\sin x \cos x}{2}\right)^2$ | |
| $= \left(\dfrac{1+\cos 2x}{2}\right)\left(\dfrac{\sin^2 2x}{4}\right)$ | 1M |
| $= \dfrac{\sin^2 2x + \cos 2x \sin^2 2x}{8}$ | 1 |
| (ii) Let $x = \dfrac{\pi}{2} - u$. Then $\mathrm{d}x = -\mathrm{d}u$. | |
| $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \,\mathrm{d}x = -\int_{\frac{\pi}{2}}^{0} \sin^4\left(\dfrac{\pi}{2} - u\right)\cos^2\left(\dfrac{\pi}{2} - u\right)\mathrm{d}u$ | 1M |
| $= \int_0^{\frac{\pi}{2}} \cos^4 u \sin^2 u \,\mathrm{d}u$ | |
| $= \int_0^{\frac{\pi}{2}} \dfrac{\sin^2 2u + \cos 2u \sin^2 2u}{8}\,\mathrm{d}u$ | |
| $= \dfrac{1}{16}\int_0^{\frac{\pi}{2}} (1 - \cos 4u)\,\mathrm{d}u + \dfrac{1}{16}\int_0^{\frac{\pi}{2}} \sin^2 2u \,\mathrm{d}(\sin 2u)$ | 1M+1M |
| $= \dfrac{1}{16}\left[u - \dfrac{\sin 4u}{4}\right]_0^{\frac{\pi}{2}} + \dfrac{1}{16}\left[\dfrac{\sin^3 2u}{3}\right]_0^{\frac{\pi}{2}}$ | |
| $= \dfrac{\pi}{32}$ | 1A |
| (b) (i) Let $x = k - u$. Then $\mathrm{d}x = -\mathrm{d}u$. | |
| $\int_0^k f(x)\,\mathrm{d}x = \int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x + \int_{\frac{k}{2}}^{k} f(x)\,\mathrm{d}x$ | |
| $= \int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x - \int_{\frac{k}{2}}^{0} f(k-u)\,\mathrm{d}u$ | 1M |
| $= \int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x + \int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x$ | |
| $= 2\int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x$ | 1 |
| (ii) Let $x = k - w$. Then $\mathrm{d}x = -\mathrm{d}w$. | |
| $\int_0^k x f(x)\,\mathrm{d}x = -\int_k^0 (k-w) f(k-w)\,\mathrm{d}w$ | 1M |
| $= k\int_0^k f(w)\,\mathrm{d}w - \int_0^k w f(w)\,\mathrm{d}w$ | |
| $= k\int_0^k f(x)\,\mathrm{d}x - \int_0^k x f(x)\,\mathrm{d}x$ | |
| $2\int_0^k x f(x)\,\mathrm{d}x = k\int_0^k f(x)\,\mathrm{d}x$ | |
| $\int_0^k x f(x)\,\mathrm{d}x = \dfrac{k}{2}\int_0^k f(x)\,\mathrm{d}x$ | |
| $= k\int_0^{\frac{k}{2}} f(x)\,\mathrm{d}x$ | 1 |
| (iii) Let $f(x) = \cos^4 x \sin^2 x$. | |
| Then $f(\pi - x) = \cos^4(\pi - x)\sin^2(\pi - x) = \cos^4 x \sin^2 x = f(x)$. | 1M |

| Solution | Marks |
|---|---|
| $\text{Volume} = \pi \int_0^{\pi} x \sin^4 x \cos^2 x \, dx$ | 1M |
| $= \pi^2 \int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ | |
| $= \dfrac{\pi^3}{32}$ | 1A |

| Solution | Marks |
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| 11. (a) (i) (1) $\begin{vmatrix} 1 & 2 & -1 \\ 3 & a & -4 \\ 3a+1 & 2-a & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 3 & a-6 & -1 \\ 3a+1 & -7a & 3a+4 \end{vmatrix}$ | |
| $= (a-6)(3a+4) - 7a$ | |
| $= 3(a-8)(a+1)$ | 1A |
| $(E)$ has unique solution if and only if $3(a-8)(a+1) \neq 0$. | 1M |
| Thus, $a \neq 8$ and $a \neq -1$. | 1 |
| (2) $x = \dfrac{\begin{vmatrix} 12 & 2 & -1 \\ b & a & -4 \\ 2-6b & 2-a & 3 \end{vmatrix}}{3(a-8)(a+1)}$ | 1M |
| $= -\dfrac{5(b+2)}{3(a+1)}$ | 1A |
| $y = \dfrac{\begin{vmatrix} 1 & 12 & -1 \\ 3 & b & -4 \\ 3a+1 & 2-6b & 3 \end{vmatrix}}{3(a-8)(a+1)} = \dfrac{3ab - 144a - 2b - 154}{3(a-8)(a+1)}$ | |
| $z = \dfrac{\begin{vmatrix} 1 & 2 & 12 \\ 3 & a & b \\ 3a+1 & 2-a & 2-6b \end{vmatrix}}{3(a-8)(a+1)} = \dfrac{-36a^2 + ab - 46a + 36b + 60}{3(a-8)(a+1)}$ | 1A |
| (ii) (1) When $a = -1$, the augmented matrix of $(E)$ is | |
| $\left(\begin{array}{ccc\|c} 1 & 2 & -1 & 12 \\ 3 & -1 & -4 & b \\ -2 & 3 & 3 & 2-6b \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & -1 & 12 \\ 0 & -7 & -1 & b-36 \\ 0 & 7 & 1 & 26-6b \end{array}\right) \sim \left(\begin{array}{ccc\|c} 1 & 2 & -1 & 12 \\ 0 & 7 & 1 & 36-b \\ 0 & 0 & 0 & -10-5b \end{array}\right)$ | 1M |
| Therefore, $-10-5b = 0$, i.e., $b = -2$. | 1A |
| (2) When $a = -1$ and $b = -2$, the augmented matrix of $(E)$ is | |
| $\left(\begin{array}{ccc\|c} 1 & 2 & -1 & 12 \\ 0 & 7 & 1 & 38 \\ 0 & 0 & 0 & 0 \end{array}\right)$ | |
| Let $z = t$, where $t \in \mathbb{R}$. Then $y = \dfrac{38-t}{7}$ and $x = \dfrac{9t+8}{7}$. | |
| The solution set is $\left\{\left(\dfrac{9t+8}{7}, \dfrac{38-t}{7}, t\right) : t \in \mathbb{R}\right\}$. | 1A |
| (b) By (a), the solution is $\left\{\left(\dfrac{9t+8}{7}, \dfrac{38-t}{7}, t\right) : t \in \mathbb{R}\right\}$. | |
| $x^2 - 9y^2 + 36z = \left(\dfrac{9t+8}{7}\right)^2 - 9\left(\dfrac{38-t}{7}\right)^2 + 36t$ | 1M |
| $= \dfrac{72t^2}{49} + \dfrac{2592t}{49} - \dfrac{12\,932}{49}$ | |
| $= \dfrac{72}{49}(t+18)^2 - 740$ | 1M |
| The least value is $-740$. | 1A |

| Solution | Marks |
|---|---|
| 12. (a) (i) $\mathbf{s}=\dfrac{k}{h+k}\mathbf{q}+\dfrac{h}{h+k}\mathbf{r}$ | 1A |
| (ii) $\dfrac{\mathbf{q}\cdot\mathbf{s}}{\mathbf{r}\cdot\mathbf{s}}=\dfrac{\lvert\mathbf{q}\rvert\lvert\mathbf{s}\rvert\cos\angle QPS}{\lvert\mathbf{r}\rvert\lvert\mathbf{s}\rvert\cos\angle RPS}$ | 1M |
| $\dfrac{\mathbf{q}\cdot\left(\frac{k}{h+k}\mathbf{q}+\frac{h}{h+k}\mathbf{r}\right)}{\mathbf{r}\cdot\left(\frac{k}{h+k}\mathbf{q}+\frac{h}{h+k}\mathbf{r}\right)}=\dfrac{\lvert\mathbf{q}\rvert}{\lvert\mathbf{r}\rvert}$ | 1M |
| $\dfrac{k\lvert\mathbf{q}\rvert^2+h(\mathbf{q}\cdot\mathbf{r})}{k(\mathbf{q}\cdot\mathbf{r})+h\lvert\mathbf{r}\rvert^2}=\dfrac{\lvert\mathbf{q}\rvert}{\lvert\mathbf{r}\rvert}$ | |
| $k(\lvert\mathbf{q}\rvert^2\lvert\mathbf{r}\rvert-\lvert\mathbf{q}\rvert(\mathbf{q}\cdot\mathbf{r}))=h(\lvert\mathbf{q}\rvert\lvert\mathbf{r}\rvert^2-\lvert\mathbf{r}\rvert(\mathbf{q}\cdot\mathbf{r}))$ | |
| $\dfrac{h}{k}=\dfrac{\lvert\mathbf{q}\rvert(\lvert\mathbf{q}\rvert\lvert\mathbf{r}\rvert-\mathbf{q}\cdot\mathbf{r})}{\lvert\mathbf{r}\rvert(\lvert\mathbf{q}\rvert\lvert\mathbf{r}\rvert-\mathbf{q}\cdot\mathbf{r})}$ | |
| $=\dfrac{\lvert\mathbf{q}\rvert}{\lvert\mathbf{r}\rvert}$ | 1 |
| (b) (i) $\overrightarrow{OE}\times\overrightarrow{OF}=\begin{pmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\6&-8&0\\3&-4&3\end{pmatrix}$ | 1M |
| $=-24\mathbf{i}-18\mathbf{j}$ | |
| Let $\overrightarrow{OB}=t(-24\mathbf{i}-18\mathbf{j})$, where $t$ is a real number. | |
| $OA=25$ | |
| $OB=\sqrt{(24t)^2+(18t)^2}=30\sqrt{t^2}$ | |
| $\dfrac{AD}{DB}=\dfrac{OA}{OB}$ | |
| $\dfrac{5}{3}=\dfrac{25}{30\sqrt{t^2}}$ | 1M |
| $t^2=\dfrac{1}{4}$ | |
| $t=-\dfrac{1}{2}$ or $\dfrac{1}{2}$ | |
| Since $\angle AOB$ is an acute angle, $\overrightarrow{OA}\cdot\overrightarrow{OB}$ is positive. | |
| We have $t=-\dfrac{1}{2}$. | |
| $\overrightarrow{OB}=12\mathbf{i}+9\mathbf{j}$ | |
| $\overrightarrow{OD}=\dfrac{3}{5+3}\overrightarrow{OA}+\dfrac{5}{5+3}\overrightarrow{OB}=\dfrac{15}{2}\mathbf{i}+15\mathbf{j}$ | 1A |
| (ii) Let $G$ be a point on $OB$ such that $AG$ is the angle bisector of $\angle BAO$. | |
| $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=12\mathbf{i}-16\mathbf{j}$ | |
| $AB=\sqrt{12^2+16^2}=20$ | |
| From (a), $BG:GO=AB:AO=20:25=4:5$. | 1M |
| $\overrightarrow{AG}=\dfrac{5}{4+5}\overrightarrow{AB}+\dfrac{4}{45}\overrightarrow{AO}=\dfrac{20}{3}\mathbf{i}-20\mathbf{j}$ | |

| Solution | Marks |
| --- | --- |
| Let $\overrightarrow{OI} = m\overrightarrow{OD}$ and $\overrightarrow{IA} = n\overrightarrow{AG}$, where $m$ and $n$ are real numbers. | |
| $\overrightarrow{OA} = \overrightarrow{OI} + \overrightarrow{IA}$ | |
| $25\mathbf{j} = m\left(\frac{15}{2}\mathbf{i} + 15\mathbf{j}\right) + n\left(\frac{20}{3}\mathbf{i} - 20\mathbf{j}\right)$ | 1M |
| $25\mathbf{j} = \left(\frac{15m}{2} + \frac{20}{3}\right)\mathbf{i} + (15m - 20n)\mathbf{j}$ | |
| We have $\frac{15m}{2} + \frac{20n}{3} = 0$ and $15m - 20n = 25$. | 1M |
| Solving, we have $m = \frac{2}{3}$ and $n = -\frac{3}{4}$. | |
| Thus, $\overrightarrow{OI} = \frac{2}{3}\overrightarrow{OD}$ and $OI = \frac{2}{3}OD$. | 1 |
| (iii) $\overrightarrow{OI} = \frac{2}{3}\overrightarrow{OD} = 5\mathbf{i} + 10\mathbf{j}$ | |
| $HI = \frac{\left\lvert \overrightarrow{OI} \cdot \overrightarrow{OB} \right\rvert}{OB}$ | |
| $= \frac{\lvert(5\mathbf{i} + 10\mathbf{j}) \cdot (12\mathbf{i} + 9\mathbf{j})\rvert}{\sqrt{12^2 + 9^2}}$ | 1M |
| $= 10$ | |
| $OI = \sqrt{5^2 + 10^2} = \sqrt{125}$ | |
| $OH = \sqrt{OI^2 - HI^2}$ | |
| $= \sqrt{125 - 10^2}$ | |
| $= 5$ | 1A |



**END OF PAPER**