| Solution | Marks |
|---|---|
| **ELITE-2425-MOCK-SET 17-MATH-CP 1** | |
| **Suggested solutions** | |
| 1. $\dfrac{h^3k^{-5}}{(h^2k^3)^{-2}} = \dfrac{h^3k^{-5}}{h^{-4}k^{-6}}$ | 1M |
| $= h^{3+4}k^{-5+6}$ | 1M |
| $= h^7k$ | 1A |
| 2. $\dfrac{3+r}{5} = \dfrac{4-s}{3}$ | |
| $9+3r = 20-5s$ | 1M |
| $5s = 11-3r$ | 1M |
| $s = \dfrac{11-3r}{5}$ | 1A |
| 3. (a) $10x^2+3xy-y^2 = (5x-y)(2x+y)$ | 1A |
| (b) $6x+3y-10x^2-3xy+y^2$ | |
| $= 3(2x+y)-(5x-y)(2x+y)$ | 1M |
| $= (2x+y)(3-5x+y)$ | 1A |
| 4. Let $x$ and $y$ be the original number of males and females respectively. | |
| $\dfrac{x+4}{y-8} = \dfrac{1}{2}$ | 1A |
| $2x+8 = y-8$ | |
| $y = 2x+16$ | |
| Suppose 2 males leave the company and 2 females join the company. | |
| $\dfrac{x-2}{y+2} = \dfrac{1}{4}$ | 1A |
| $\dfrac{x-2}{(2x+16)+2} = \dfrac{1}{4}$ | 1M |
| $4x-8 = 2x+18$ | |
| $x = 13$ | |
| Required number $= 13+(2\times 13+16)$ | |
| $= 55$ | 1A |

| Solution | Marks |
|---|---|
| 5. (a) $2(3x-2) \le 11$ | |
| $6x \le 15$ | |
| $x \le \frac{5}{2}$ | 1A |
| $\frac{x-5}{2} < \frac{4x}{3}$ | |
| $-\frac{5x}{6} < \frac{5}{2}$ | |
| $x > -3$ | 1A |
| Thus, we have $-3 < x \le \frac{5}{2}$. | 1A |
| (b) 5 | 1A |
| 6. (a) Percentage profit $= \frac{28}{140-28} \times 100\%$ | 1M |
| $= 25\%$ | 1A |
| (b) Marked price $= \frac{140}{1-20\%}$ | 1M |
| $= \$175$ | 1A |
| 7. (a) Mean $= \frac{3(80°)+2(70°)+1(90°)+0}{360°}$ | 1M |
| $= \frac{47}{36}$ | 1A |
| (b) Required probability $= \frac{360-80}{360}$ | 1M |
| $= \frac{7}{9}$ | 1A |
| 8. $\angle POQ = 150° - 60° = 90°$ | |
| $12^2 + r^2 = 13^2$ | |
| $r = 5$ or $-5$ (rejected) | 1A |
| Area of $\triangle POQ = \frac{1}{2} \times 5 \times 12$ | |
| $= 30$ | |
| Area of $\triangle POR = \frac{1}{2} \times (2 \times 12 \sin 60°) \times (12 \cos 60°)$ | 1M+1M |
| $= 36\sqrt{3}$ | |
| $\approx 62.4$ | |
| $> 2 \times 30$ | |
| The claim is agreed. | 1A |

| Solution | Marks |
|---|---|
| 9. (a) $\angle ABF = \angle DEF$ (alt. $\angle$s, $AB // EC$) | |
| $\angle AFB = \angle DFE$ (vert. opp. $\angle$s) | |
| $\triangle ABF \sim \triangle DEF$ (AA) | |

| **Marking Scheme** | | |
|---|---|---|
| **Case 1** | Any correct proof with correct reasons. | 2 |
| **Case 2** | Any correct proof without reasons. | 1 |

| Solution | Marks |
|---|---|
| (b) $\dfrac{EF}{BF} = \dfrac{DF}{AF}$ | 1M |
| $\dfrac{EF}{13} = \dfrac{15}{5}$ | |
| $EF = 39\text{ cm}$ | |
| $CE = \sqrt{(13+39)^2 - (5+15)^2}$ | 1M |
| $= 48\text{ cm}$ | 1A |
| 10. (a) Let $C = ma^2 + na^3$, where $m$ and $n$ are non-zero constants. | 1A |
| $\begin{cases} 0.28 = 4m + 8n \\ 2.5 = 25m + 125n \end{cases}$ | 1M |
| Solving, we have $m = \dfrac{1}{20}$ and $n = \dfrac{1}{100}$. | 1A |
| Required cost | |
| $= \dfrac{1}{20}(4)^2 + \dfrac{1}{100}(4^3)$ | |
| $= \$1.44$ | 1A |
| (b) $1.5z = \dfrac{1}{20}z^2 + \dfrac{1}{100}z^3$ | 1M |
| $0 = z\left(\dfrac{z^2}{100} + \dfrac{z}{20} - \dfrac{3}{2}\right)$ | |
| $z = 0$ (rejected) or $10$ or $-15$ (rejected) | 1A |

| Solution | Marks |
|---|---|
| 11. (a) $(50 + c) - 24 = 33$ | 1M |
| $c = 7$ | 1A |
| (b) $\dfrac{43 + (40 + b)}{2} - \dfrac{(20 + a) + 26}{2} \geq 20$ | 1M |
| $b - a \geq 3$ | |
| There are only two cases. | |
| Case 1: mode is 24. | |
| We have $a = 4$ and $b \neq 3$. | 1M |
| $b - 4 \geq 3$ | |
| $b \geq 7$ | |
| When $a = 4$ and $b = 7$, standard deviation $\approx 10.3$ | 1M |
| When $a = 4$ and $b = 8$, standard deviation $\approx 10.4$ | |
| Case 2: mode is 43. | |
| We have $a \neq 4$ and $b = 3$. | |
| $3 - a \geq 3$ | |
| $a \leq 0$ (rejected) | |
| Thus, the greatest possible standard deviation is 10.4. | 1A |
| 12. (a) Let $x$ cm$^3$ be the volume of $X$. | |
| $\left(\dfrac{6}{6+3}\right)^3 = \dfrac{x - 114\pi}{x}$ | 1M+1A |
| $8x = 27(x - 114\pi)$ | |
| $x = 162\pi$ | 1A |
| (b) Let $r$ cm be the base radius of $X$. | |
| $\dfrac{1}{3}\pi r^2(9) = 162\pi$ | 1M |
| $r = \sqrt{54}$ | |
| Slant height $= \sqrt{(\sqrt{54})^2 + 9^2}$ | 1M |
| $= \sqrt{135}$ cm | |
| Total surface area | |
| $= \pi\sqrt{54}\sqrt{135} + \pi(\sqrt{54})^2$ | 1A |
| $\approx 438$ cm$^2$ | |
| $\approx 0.0438$ m$^2$ | |
| $> 0.04$ m$^2$ | |
| The total surface area is not less than 0.04 m$^2$. | 1A |

| Solution | Marks |
|---|---|
| 13. (a) Let $f(x) = (2x^2 + 7x + 3)(x^2 + Ax + B) - 160x - 80$, where $A$ and $B$ are constants. | 1M |
| Compare the coefficients of $x^3$. | |
| $-5 = 2A + 7$ | 1M |
| $A = -6$ | |
| Compare the constant terms. | |
| $-5 = 3B - 80$ | |
| $B = 25$ | |
| Required quotient is $x^2 - 6x + 25$. | 1A |
| (b) $f(1) = (2 + 7 + 3)(1 - 6 + 25) - 160 - 80$ | 1M |
| $= 0$ | |
| $x - 1$ is a factor of $f(x)$. | 1A |
| (c) $f(x) = 0$ | |
| $(2x^2 + 7x + 3)(x^2 - 6x + 25) - 160x - 80 = 0$ | |
| $(2x + 1)(x + 3)(x^2 - 6x + 25) - 80(2x + 1) = 0$ | |
| $(2x + 1)[(x + 3)(x^2 - 6x + 25) - 80] = 0$ | |
| $(2x + 1)(x^3 - 3x^2 + 7x - 5) = 0$ | |
| $(2x + 1)(x - 1)(x^2 - 2x + 5) = 0$ | 1M |
| $x = -\frac{1}{2}$ or $1$ or $x^2 - 2x + 5 = 0$ | |
| Consider the equation $x^2 - 2x + 5 = 0$. | |
| $\Delta = 2^2 - 4(1)(5)$ | 1M |
| $= -16$ | |
| $< 0$ | |
| The equation $x^2 - 2x + 5 = 0$ has no real roots and hence no irrational roots. | |
| The equation $f(x) = 0$ has no irrational roots. | |
| The claim is disagreed. | 1A |

| Solution | Marks |
| --- | --- |
| 14. (a) Let the equation of the circle be $x^2 + y^2 + Dx + Ey + F = 0$, where $D$, $E$ and $F$ are constants. | |
| $\begin{cases} 0^2 + 2^2 + 0 + 2E + F = 0 \\ 4^2 + 0^2 + 4D + 0 + F = 0 \\ 9^2 + 5^2 + 9D + 5E + F = 0 \end{cases}$ | 1M |
| Subtract one equation from another. | |
| $\begin{cases} 102 + 9D + 3E = 0 \\ 90 + 5D + 5E = 0 \end{cases}$ | 1M |
| Solving, we have $D = -8$, $E = -10$ and $F = 16$. | |
| Required equation is $x^2 + y^2 - 8x - 10y + 16 = 0$. | 1A |
| (b) The coordinates of the centre of $C$ are $(4, 5)$. | |
| Radius $= \sqrt{4^2 + t^2 - 16} = 5$ | |
| Distance from $M$ to the centre of $C$ | |
| $= \sqrt{(5-4)^2 + (8-5)^2}$ | 1M |
| $= \sqrt{10}$ | |
| $< 5$ | |
| Thus, $M$ lies inside $C$. | 1 |
| (c) (i) $G$, $M$ and $N$ are collinear. | 1A |
| (ii) Slope $= \dfrac{8-5}{5-4} = 3$ | 1M |
| Required equation is | |
| $y - 5 = 3(x - 4)$ | |
| $3x - y - 7 = 0$ | 1A |
| 15. (a) Required probability $= \dfrac{C_2^9 C_2^3 C_2^4}{C_6^{16}}$ | 1M |
| $= \dfrac{81}{1001}$ | 1A |
| (b) Required probability $= \dfrac{C_2^9 (C_4^7 - C_4^4)}{C_6^{16}}$ | 1M |
| $= \dfrac{153}{1001}$ | 1A |

| Solution | Marks |
|---|---|
| 16. (a) Let $r$ be the common ratio. | |
| $8r + 8r^2 = 30$ | 1M |
| $8r^2 + 8r - 30 = 0$ | |
| $r = 1.5$ or $-2.5$ (rejected) | 1A |
| (b) $8 + 8(1.5) + 8(1.5)^2 + \ldots + 8(1.5)^{n-1} > 2.4 \times 10^{12}$ | |
| $\dfrac{8(1.5^n - 1)}{1.5 - 1} > 2.4 \times 10^{12}$ | 1M |
| $1.5^n > 0.5 \times 10^{12} + 1$ | |
| $n \log 1.5 > \log\left(0.5 \times 10^{12} + 1\right)$ | 1M |
| $n > 63.5$ | |
| The least value of $n$ is 64. | 1A |
| 17. (a) Perpendicular distance from $P$ to $AB = \dfrac{20 \times 2}{5} = 8$ | 1M |
| Locus of $P$ is a pair of straight lines parallel to $AB$, maintaining a distance of 8 to $AB$. | |
| Let $\theta$ be the inclination of $AB$. | |
| $\tan\theta = \dfrac{6-3}{4-0} = \dfrac{3}{4}$. So, $\cos\theta = \dfrac{4}{\sqrt{3^2 + 4^2}} = \dfrac{4}{5}$. | |
| Let $c$ be the $y$-intercept of $\Gamma$. | |
| $\dfrac{4}{5} = \dfrac{8}{\pm(c-3)}$ | 1M |
| $c = -7$ or $13$ | |
| The equation of $\Gamma$ is $y = \dfrac{3}{4}x - 7$ or $y = \dfrac{3}{4}x + 13$. | 1A |
| (b) Perpendicular distance from $P$ to $AB = 8 > 5$. | |
| It is not possible to have $AP = AB$ or $BP = AB$. | |
| When $AP = BP$, $P$ lies on perpendicular bisector of $AB$. | |
| Equation of perpendicular bisector of $AB$ is | |
| $y - \dfrac{9}{2} = -\dfrac{4}{3}(x-2)$ | 1M |
| $y = -\dfrac{4}{3}x + \dfrac{43}{6}$ | |
| Solve $\begin{cases} y = -\dfrac{4}{3}x + \dfrac{43}{6} \\ y = \dfrac{3}{4}x - 7 \end{cases}$ or $\begin{cases} y = -\dfrac{4}{3}x + \dfrac{43}{6} \\ y = \dfrac{3}{4}x + 13 \end{cases}$, | |
| the coordinates of $P$ are $\left(\dfrac{34}{5}, -\dfrac{19}{10}\right)$ or $\left(-\dfrac{14}{5}, \dfrac{109}{10}\right)$. | 1A+1A |

| Solution | Marks |
|---|---|
| 18. (a) $AN = AM = a\sin 60° = \dfrac{\sqrt{3}a}{2}$ cm | 1M |
| $MN = \dfrac{BC}{2} = \dfrac{a}{2}$ cm | 1M |
| $MN^2 = AM^2 + AN^2 - 2(AM)(AN)\cos\angle MAN$ | 1M |
| $\dfrac{a^2}{4} = \dfrac{3a^2}{4} + \dfrac{3a^2}{4} - 2\left(\dfrac{3a^2}{4}\right)\cos\angle MAN$ | |
| $\dfrac{1}{4} = \dfrac{3}{2} - \dfrac{3}{2}\cos\angle MAN$ | |
| $\angle MAN \approx 33.6°$ | 1A |
| (b) Since $VX < \dfrac{1}{2}VB$ and $VY < \dfrac{1}{2}VC$, we have $VX < VM$ and $VY < VN$. | 1M |
| Note that $\triangle VXY \sim \triangle VMN$, we have $\dfrac{XY}{MN} = \dfrac{VX}{VM} < 1$ and $XY < MN$. | |
| Note that $AM \perp VB$ and $AN \perp VC$, we have $AX > AM$ and $AY > AN$. | |
| Consider $\triangle XAY$. | |
| $\sin\dfrac{\angle XAY}{2} = \dfrac{XY \div 2}{AX}$ | |
| $< \dfrac{MN \div 2}{AM}$ | 1M |
| $= \sin\dfrac{\angle MAN}{2}$ | |
| We have $\sin\dfrac{\angle XAY}{2} < \sin\dfrac{\angle MAN}{2}$. | 1M |
| Thus, $\dfrac{\angle XAY}{2} < \dfrac{\angle MAN}{2}$ and $\angle XAY < \angle MAN$. | |
| The claim is agreed. | 1A |

| Solution | Marks |
|---|---|
| 19. (a) $f(x) = 2x^2 + (-16k - 8)x + 32k^2 + 31k + 18$ | |
| $= 2[x^2 - 2(4k + 2)x + (4k + 2)^2 - (4k + 2)^2] + 32k^2 + 31k + 18$ | 1M |
| $= 2[x - (4k + 2)]^2 - k + 10$ | |
| The coordinates of $Q$ are $(4k + 2,\ -k + 10)$. | 1A |
| (b) $(2k + 12,\ -k - 4)$ | 1A |
| (c) (i) The coordinates of the mid-point of $QS$ are $(4k,\ -k + 3)$. | |
| Slope of $QS = \dfrac{(-k + 10) - (-k - 4)}{(4k + 2) - (4k - 2)} = \dfrac{7}{2}$ | |
| Required equation is | |
| $y - (-k + 3) = -\dfrac{2}{7}(x - 4k)$ | 1M |
| $2x + 7y - k - 21 = 0$ | 1A |
| (ii) $x$-coordinate of $G = \dfrac{(2k + 12) + (4k - 2)}{2}$ | 1M |
| $= 3k + 5$ | |
| $G$ lies on the perpendicular bisector of $QS$. | |
| $2(3k + 5) + 7y - k - 21 = 0$ | 1M |
| $y = \dfrac{11 - 5k}{7}$ | |
| The coordinates of $G$ are $\left(3k + 5,\ \dfrac{11 - 5k}{7}\right)$. | 1A |
| (iii) Suppose $AQGS$ is a square, we have $GQ \perp GS$. | 1M |
| $\dfrac{\frac{11-5k}{7} - (-k + 10)}{(3k + 5) - (4k + 2)} \times \dfrac{\frac{11-5k}{7} - (-k - 4)}{(3k + 5) - (4k - 2)} = -1$ | |
| $\dfrac{2k - 59}{7} \times \dfrac{2k + 39}{7} = -(3 - k)(7 - k)$ | |
| $\dfrac{53k^2}{49} - \dfrac{530k}{49} - \dfrac{1272}{49} = 0$ | |
| $k = 12$ or $-2$ (rejected) | 1A |
| Note that $\angle AQG = \angle ASG = 90^\circ$. | 1M |
| When $k = 12$, $\angle QAS = 360^\circ - 90^\circ \times 3 = 90^\circ$ and $GQ = GS$. | |
| $AQGS$ is a square when $k = 12$. | |
| It is possible. | 1A |



**END OF PAPER**