REG-POLY-2425-ASM-SET 4-MATH
Suggested solutions

Conventional Questions

1. (a) Let $f(x) = (x^2 + x + 2)(2x - 1) + Ax + B$, where A and B are constants. 1M

We have $f(1) = 2$.

$$(1 + 1 + 2)(2 - 1) + A + B = 2 \quad \text{1M}$$
$$A + B = -4$$

We have $f(-3) = -42$.

$$(9 - 3 + 2)(-6 - 1) - 3A + B = -42$$
$$-3A + B = 14$$

Solving, we have $A = -4$ and $B = 2$.

Required remainder is $-4x + 2$. 1A

(b)
$$f(x) = 0$$
$$(x^2 + x + 2)(2x - 1) - 4x + 2 = 0$$
$$(x^2 + x + 2)(2x - 1) - 2(2x - 1) = 0$$
$$(2x - 1)(x^2 + x) = 0 \quad \text{1M}$$
$$x(2x - 1)(x + 1) = 0$$
$$x = 0 \quad \text{or} \quad -1 \quad \text{or} \quad \frac{1}{2} \quad \text{1M}$$

The equation has 3 rational roots. 1A

2. (a) Let $f(x) = (x^2 + 7x + 16)(Ax + B)$, where A and B are constants. 1M

We have $f(1) = 72$.

$$(1 + 7 + 16)(A + B) = 72 \quad \text{1M}$$
$$A + B = 3$$

We have $f(-2) = -18$.

$$(4 - 17 + 16)(-2A + B) = -18$$
$$-2A + B = -3$$

Solving, we have $A = 2$ and $B = 1$.

Required quotient is $2x + 1$. 1A

(b)
$$f(x) = 0$$
$$(x^2 + 7x + 16)(2x + 1) = 0$$
$$x = -\frac{1}{2} \quad \text{or} \quad x^2 + 7x + 16 = 0$$

Consider the equation $x^2 + 7x + 16 = 0$.

$$\Delta = 7^2 - 4(1)(16) \quad \text{1M}$$
$$= -15$$
$$< 0$$

The equation $x^2 + 7x + 16 = 0$ has no real roots and hence no rational roots. 1M

The equation $f(x) = 0$ has 1 rational root. 1A

3. (a) Let $f(x) = (2x^2 + 7x + 3)(x^2 + Ax + B) - 160x - 80$, where A and B are constants. 1M

Compare the coefficients of x^3.

$$-5 = 2A + 7 \quad \text{1M}$$
$$A = -6$$

Compare the constant terms.

$$-5 = 3B - 80$$
$$B = 25$$

Required quotient is $x^2 - 6x + 25$. 1A

(b) $f(1) = (2 + 7 + 3)(1 - 6 + 25) - 160 - 80$ 1M

$= 0$

$x - 1$ is a factor of $f(x)$. 1A

(c)
$$f(x) = 0$$
$$(2x^2 + 7x + 3)(x^2 - 6x + 25) - 160x - 80 = 0$$
$$(2x + 1)(x + 3)(x^2 - 6x + 25) - 80(2x + 1) = 0$$
$$(2x + 1)[(x + 3)(x^2 - 6x + 25) - 80] = 0$$
$$(2x + 1)(x^3 - 3x^2 + 7x - 5) = 0$$
$$(2x + 1)(x - 1)(x^2 - 2x + 5) = 0 \quad \text{1M}$$
$$x = -\frac{1}{2} \quad \text{or} \quad 1 \quad \text{or} \quad x^2 - 2x + 5 = 0$$

Consider the equation $x^2 - 2x + 5 = 0$.

$$\Delta = 2^2 - 4(1)(5) \quad \text{1M}$$
$$= -16$$
$$< 0$$

The equation $x^2 - 2x + 5 = 0$ has no real roots and hence no irrational roots.

The equation $f(x) = 0$ has no irrational roots.

The claim is disagreed. 1A

4. (a) Let $f(x) = (x^2 - 4)(Ax + B) + kx - 12$, where A and B are constants. 1M

$$f(2) = 0$$

$$(4 - 4)(2A + B) + 2k - 12 = 0 \quad \text{1M}$$

$$k = 6 \quad \text{1A}$$

(b)

$$f(0) = 20$$

$$(0 - 4)(0 + B) + 0 - 12 = 20 \quad \text{1M}$$

$$B = -8 \quad \text{1A}$$

$$f(-3) = -40$$

$$(9 - 4)(-3A - 8) + 6(-3) - 12 = -40$$

$$A = -2 \quad \text{1A}$$

$$f(x) = 0$$

$$(x^2 - 4)(-2x - 8) + 6x - 12 = 0$$

$$(x + 2)(x - 2)(-2x - 8) + 6(x - 2) = 0$$

$$(x - 2)[(x + 2)(-2x - 8) + 6] = 0$$

$$2(x - 2)(-x^2 - 6x - 5) = 0$$

$$-2(x - 2)(x + 1)(x + 5) = 0 \quad \text{1M}$$

$$x = 2 \quad \text{or} \quad -1 \quad \text{or} \quad -5$$

All the roots are integers.
The claim is correct. 1A

5. (a) Let $f(x) = (x^2 - 4)(Ax + B) + kx + 24$, where A and B are constants. 1M

$$f(2) = 0$$

$$(4 - 4)(2A + B) + 2k + 24 = 0 \quad \text{1M}$$

$$k = -12 \quad \text{1A}$$

(b)

$$f(0) = 8$$

$$(0 - 4)(0 + B) + 0 + 24 = 8 \quad \text{1M}$$

$$B = 4 \quad \text{1A}$$

$$f(-1) = 0$$

$$(1 - 4)(-A + 4) + 12 + 24 = 0$$

$$A = -8 \quad \text{1A}$$

$$f(x) = 0$$
$$(x^2 - 4)(-8x + 4) - 12x + 24 = 0$$
$$(x + 2)(x - 2)(-8x + 4) - 12(x - 2) = 0$$
$$(x - 2)[(x + 2)(-8x + 4) - 12] = 0$$
$$(x - 2)(-8x^2 - 12x - 4) = 0$$
$$-4(x - 2)(x + 1)(2x + 1) = 0 \quad \text{1M}$$
$$x = 2 \quad \text{or} \quad -1 \quad \text{or} \quad -\frac{1}{2}$$

$-\frac{1}{2}$ is not an integer.

The claim is not correct. 1A

6. (a) $f(x) = (x^2 + 1)(2x^2 + ax - 6) + bx + 9$. 1A

We have $f(-1) = 0$.

$$(1 + 1)(2 - a - 6) - b + 9 = 0 \quad \text{1M}$$
$$-2a - b = -1$$

We have $f(-2) = 35$.

$$(4 + 1)(8 - 2a - 6) - 2b + 9 = 35$$
$$-10a - 2b = 16$$

Solving, we have $a = -3$ and $b = 7$. 1A

(b) $f\left(\frac{3}{2}\right) = \left(\frac{9}{4} + 1\right)\left(\frac{9}{2} - 3 \times \frac{3}{2} - 6\right) + 7 \times \frac{3}{2} + 9$

$= 0$

Thus, $2x - 3$ is a factor of $f(x)$. 1

(c)
$$f(x) = 0$$
$$(x^2 + 1)(2x^2 - 3x - 6) + 7x + 9 = 0$$
$$2x^4 - 3x^3 - 4x^2 + 4x + 3 = 0$$
$$(x + 1)(2x^3 - 5x^2 + x + 3) = 0 \quad \text{1M}$$
$$(2x - 3)(x + 1)(x^2 - x - 1) = 0$$
$$x = \frac{3}{2} \quad \text{or} \quad -1 \quad \text{or} \quad x^2 - x - 1 = 0$$

Consider the equation $x^2 - x - 1 = 0$, we have $x \approx 1.62$ or $x \approx -0.618$. 1A

The equation $f(x) = 0$ has 4 real roots.

The claim is not correct. 1A

7. (a) Let $f(x) = (x^2 - 3x + 3)(Ax + B) + x - 39$, where A and B are constants. 1M

Compare the constant terms.

$-15 = 3B - 39$ 1M

$B = 8$

Compare the coefficients of x^2.

$2 = 8 - 3A$

$A = 2$

$f(x) = (x^2 - 3x + 3)(2x + 8) + x - 39$

$= 2x^3 + 2x^2 - 17x - 15$

Thus, $a = 2$ and $b = -17$. 1A

(b) $f(-3) = 2(-3)^3 + 2(-3)^2 - 17(-3) - 15$ 1M

$= 0$

Thus, $x + 3$ is a factor of $f(x)$. 1A

(c) $f(x) = 0$

$2x^3 + 2x^2 - 17x - 15 = 0$

$(x + 3)(2x^2 - 4x - 5) = 0$ 1M

$x = -3$ or $2x^2 - 4x - 5 = 0$

Consider the equation $2x^2 - 4x - 5 = 0$, we have $x \approx 2.87$ or $x \approx -0.871$. 1M

The equation $f(x) = 0$ has three real roots.

The claim is not correct. 1A

8. (a) $p(x) = (x^2 + 3x)(2x^2 - 19) + cx - c - 64$ 1M

$p(3) = 0$

$(9 + 9)(18 - 19) + 3c - c - 64 = 0$ 1M

$c = 41$ 1A

(b) $p(-5) = (25 - 15)(50 - 19) + 41(-5) - 105$

$= 0$

Thus, $x + 5$ is a factor of $p(x)$. 1

(c) $p(x) = 0$

$(x^2 + 3x)(2x^2 - 19) + 41x - 105 = 0$

$2x^4 + 6x^3 - 19x^2 - 16x - 105 = 0$

$(x - 3)(2x^3 + 12x^2 + 17x + 35) = 0$

$(x - 3)(x + 5)(2x^2 + 2x + 7) = 0$ 1M

$x = 3$ or -5 or $2x^2 + 2x + 7 = 0$

Consider the equation $2x^2 + 2x + 7 = 0$.

$$\begin{aligned}\Delta &= 2^2 - 4(2)(7) \\ &= -52 \\ &< 0\end{aligned}$$ 1M

The roots of the equation are not real numbers.
The claim is not correct. 1A

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