**REG-POLY-2425-ASM-SET 3-MATH**
**Suggested solutions**

**Conventional Questions**

1. (a)

$$f(-2) = 0$$

$$4(-2)^3 + b(-2)^2 + c(-2) + 18 = 0 \quad \text{1M}$$

$$2b - c = 7$$

$$f(1) = f\left(-\frac{3}{2}\right)$$

$$4 + b + c + 18 = 4\left(-\frac{3}{2}\right)^3 + b\left(-\frac{3}{2}\right)^2 + c\left(-\frac{3}{2}\right) + 18 \quad \text{1M}$$

$$b - 2c = 14$$

Solving the simultaneous equations $\begin{cases} 2b - c = 7 \\ b - 2c = 14 \end{cases}$, 1M

we have $b = 0$ and $c = -7$. 1A

(b) $f(x) = 4x^3 - 7x + 18$

$= (x + 2)(4x^2 - 8x + 9)$ 1M+1A

(c) Remainder $= f(-1 - 1)$

$= f(-2)$

$= 4(-2)^3 - 7(-2) + 18$ 1M

$= 0$ 1A

2. (a) $f(-1) = h = -h - 1 + 9 + k$ 1M

$2h - k = 8$

$0 = f\left(\frac{1}{2}\right) = \frac{h}{8} - \frac{1}{4} - \frac{9}{2} + k$ 1M

$h + 8k = 38$

Solving, we have $h = 6$ and $k = 4$. 1A+1A

(b)

$$f(x) = 6$$

$$6x^3 - x^2 - 9x - 2 = 0$$

$(x + 1)(6x^2 - 7x - 2) = 0$ 1M

$x = -1 \quad \text{or} \quad \frac{7 \pm \sqrt{7^2 - 4(6)(-2)}}{2(6)}$ 1M

$= -1 \quad \text{or} \quad \frac{7 \pm \sqrt{97}}{12}$

There are some irrational roots in $f(x) = 6$.

The claim is disagreed. 1A

3. (a) $f(-1) = g(-1)$

$a(-1)^3 - b(-1)^2 - (-1) + 2 = (-1)^3 + a(-1)^2 - b$ 1M

$-a - b + 3 = -1 + a - b$

$a = 2$ 1A

$f(1) = 0$

$2(1)^3 - b(1)^2 - 1 + 2 = 0$ 1M

$b = 3$ 1A

(b) $f(x) - 2g(x) = -1$

$7x^2 + x - 9 = 0$ 1M

$$x = \frac{-1 \pm \sqrt{1^2 - 4(7)(-9)}}{2(7)}$$

$$= \frac{-1 \pm \sqrt{253}}{14}$$

The roots are irrational.

The claim is agreed. 1A

4. (a) $3(-3)(-3+1)(-3+2) + k = 0$ 1M

$k = 18$ 1A

(b) $3x(x+1)(x+2) + 18 = 0$

$3x^3 + 9x^2 + 6x + 18 = 0$

$3(x+3)(x^2+2) = 0$ 1M+1A

$x = -3$ or $x^2 = -2$

Since the equation $x^2 = -2$ has no real roots, the claim is disagreed. 1A

5. (a) $f(-1) = 0 = -4(-1)^3 + (a+2)(-1)^2 + 2(-1) - 3b$ 1M

$a - 3b = -4$

$f(2) = 9 = -4(2)^3 + (a+2)(2)^2 + 2(2) - 3b$ 1M

$4a - 3b = 29$

Solving, we have $a = 11$ and $b = 5$. 1A

(b) $f(x) = -4x^3 + 13x^2 + 2x - 15$

$= (-x^2 + 2x + 3)(4x - 5)$ 1M

So, $g(x) = 4x - 5$.

$kx(4x - 5) = (-x^2 + 2x + 3)(4x - 5)$ 1M

$(4x - 5)(x^2 + (k-2)x - 3) = 0$ 1M

$x = \frac{5}{4}$ or $x^2 + (k-2)x - 3 = 0$

$\Delta = (k-2)^2 - 4(1)(-3) = (k-2)^2 + 12 > 0$ for all real values of $k$.

$x^2 + (k-2)x - 3 = 0$ has two distinct real roots.

The claim is agreed. 1A

6. (a) $f(x) = (x+2)(ax^2 + bx + c)$.

Compare like terms.

$a = 4$ 1A

$b + 2a = 10$ 1M

$b = 2$ 1A

$2c = 10$

$c = 5$ 1A

(b) $f(x) = 0$

$(x+2)(4x^2 + 2x + 5) = 0$

$x = -2 \quad \text{or} \quad 4x^2 + 2x + 5 = 0$

Consider the equation $4x^2 + 2x + 5 = 0$.

$\Delta = 2^2 - 4(4)(5)$ 1M

$= -76$

$< 0$

The equation $4x^2 + 2x + 5 = 0$ has non-real roots. 1M

The claim is agreed. 1A

7. (a) $f(1) = f(-1) - 22$

$2(1)^3 + a(1)^2 + b(1) - 6 = 2(-1)^3 + a(-1)^2 + b(-1) - 6 - 22$ 1M

$b = -13$

$f(-2) = 0$

$2(-2)^3 + a(-2)^2 - 13(-2) - 6 = 0$ 1M

$a = -1$ 1A

(b) $f(x) = 2x^3 - x^2 - 13x - 6$

$= (x+2)(2x^2 - 5x - 3)$ 1M

$= (x+2)(x-3)(2x+1)$ 1A

(c) Let $u = x + 999$.

We have $g(u) = 1000f(u - 999)$.

$g(x) = 0$

$1000f(x - 999) = 0$

$1000(x - 997)(x - 1002)(2x - 1997) = 0$ 1M

$x = 997 \quad \text{or} \quad 1002 \quad \text{or} \quad \frac{1997}{2}$

Note that $\frac{1997}{2}$ is not an integer.

The claim is disagreed. 1A

8. (a) $2(4)^3 - 7(4)^2 + m(4) + n = 0$ 1M

$$n = -4m - 16$$ 1A

(b) $2x^3 - 7x^2 + mx - 4m - 16 = 0$

$(x - 4)(2x^2 + x + m + 4) = 0$ 1M+1A

$$x = 4 \quad \text{or} \quad 2x^2 + x + m + 4 = 0$$

Consider the equation $2x^2 + x + m + 4 = 0$.

Note that that sum of roots is $-\frac{1}{2}$, which is not an integer. 1M

It is not possible that both roots are integers.

The claim is disagreed. 1A

 **END OF PAPER**