REG-NSCN-2425-ASM-SET 2-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. A	3. A	4. C	5. B
6. D	7. D	8. A	9. A	10. A
11. B	12. B	13. C	14. B	15. D
16. D	17. B	18. B	19. A	20. C
21. A	22. A	23. A	24. C	25. A
26. C	27. A	28. B	29. C	30. A

1. $\boxed{\text{D}}$

   $(i^3)^5 = i^{15} = (i^4)^3 i^3 = i^3 = -i$

2. $\boxed{\text{A}}$

   $i + 2i^2 + 3i^3 + 4i^4 = i - 2 - 3i + 4 = 2 - 2i$.
   Real part is 2.

3. $\boxed{\text{A}}$

   Put $\beta = 1$, using calculator CMPLX mode, $i^3(\beta i - 3) = i^3(i - 3) = 1 + 3i$.
   Only option A satisfies this result.

4. $\boxed{\text{C}}$

   $\dfrac{a + bi}{a - bi} = i$

   $a + bi = i(a - bi)$

   $a + bi = b + ai$
   We have $b = a$.

5. $\boxed{\text{B}}$

   $$\begin{aligned}(3 + 5i) + (-2 + 6i) - (7 - 2i) &= (3 - 2 - 7) + (5 + 6 + 2)i \\ &= -6 + 13i\end{aligned}$$

6. $\boxed{\text{D}}$

   $$\begin{aligned}i^3(\alpha + 4i) &= (-i)(\alpha + 4i) \\ &= -\alpha i - 4i^2 \\ &= 4 - \alpha i\end{aligned}$$

7. $\boxed{\text{D}}$

   Using calculator CMPLX mode, $\dfrac{1}{i - 2} = -\dfrac{2}{5} - \dfrac{1}{5}i$.

8. $\boxed{\text{A}}$

By calculator, $\dfrac{5}{2-i} = 2 + i$.

9. $\boxed{\text{A}}$

$$\begin{aligned}(3+2i)z &= 22-7i \\ z &= \frac{22-7i}{3+2i} \\ &= 4-5i\end{aligned}$$

10. $\boxed{\text{A}}$

Using calculator CMPLX mode,

$$(4+i) + \frac{5-3i}{3+4i} = \frac{103}{25} - \frac{4}{25}i$$

11. $\boxed{\text{B}}$

Using calculator CMPLX mode, $\dfrac{4-i}{2+i} = \dfrac{7}{5} - \dfrac{6}{5}i$.

12. $\boxed{\text{B}}$

Using calculator CMPLX mode, $\dfrac{1+3i}{3-i} = i$.

13. $\boxed{\text{C}}$

$z = (a+5)i^6 + (a-3)i^7 = -(a+5) - (a-3)i$ is a real number.

$$\begin{aligned}-(a-3) &= 0 \\ a &= 3\end{aligned}$$

14. $\boxed{\text{B}}$

Using calculator CMPLX mode, $\dfrac{5}{2-i} = (2+i)$.

$\dfrac{5}{2-i} + ki = 2 + (k+1)i$ is a real number.

$$\begin{aligned}k+1 &= 0 \\ k &= -1\end{aligned}$$

15. $\boxed{\text{D}}$

$$\begin{aligned}&\frac{4i^{2020} + 5i^{2019} + 6i^{2018} + 7i^{2017} + 8i^{2016}}{1+i} \\ &= \frac{4 + 5i^3 + 6i^2 + 7i + 8}{1+i} \\ &= \frac{6+2i}{1+i} \\ &= 4-2i\end{aligned}$$

Imaginary part $= -2$

16. $\boxed{\text{D}}$

Using calculator CMPLX mode, $\dfrac{2i^{12}+3i^{13}+4i^{14}+5i^{15}+6i^{16}}{1-i} = \dfrac{2+3i+(-4)+(-5i)+6}{1-i} = 3+i$.
Real part is 3.

17. $\boxed{\text{B}}$

$$\begin{aligned}\frac{6i^6+7i^7+8i^8+9i^9+10i^{10}}{1+i} &= \frac{-6-7i+8+9i-10}{1+i}\\ &= -3+5i\end{aligned}$$

The real part is -3.

18. $\boxed{\text{B}}$

When $k=1$,

$$\begin{aligned}\frac{16k-4i}{-2i} - \frac{9i+3k}{3i} &= \frac{16-4i}{-2i} - \frac{9i+3}{3i}\\ &= -1+9i\end{aligned}$$

Only option B gives $-1+9i$ when $k=1$.

19. $\boxed{\text{A}}$

$$\begin{aligned}5k - \frac{3+9ki}{i} &= 5k - \frac{3}{i} - 9k\\ &= -4k+3i\end{aligned}$$

20. $\boxed{\text{C}}$

Put $k=1$.

$$\frac{5k+10i}{1-2i} = \frac{5+10i}{1-2i} = -3+4i$$

Imaginary part is 4.
Check the value of each option when $k=1$.

A. 5

B. -3

C. 4

D. 0

21. $\boxed{\text{A}}$

Put $\beta = 1$, using calculator CMPLX mode, $\dfrac{\beta^2+4}{\beta+2i} = \dfrac{1+4}{1+2i} = 1-2i$.
Only option A satisfies this result.

22. $\boxed{\text{A}}$

Let $\alpha = 1$. Using calculator CMPLX mode, $\dfrac{\alpha^2+9}{\alpha-3i} = 1+3i$.
Only option A satisfies this.

23. $\boxed{\text{A}}$

$$\frac{bi}{2+3i} = a - 2i$$

$$\begin{aligned} bi &= (2+3i)(a-2i) \\ &= (2a+6) + (3a-4)i \end{aligned}$$

$$\begin{cases} 2a+6=0 \\ 3a-4=b \end{cases}$$

Solving, $a = -3$ and $b = -13$.

24. $\boxed{\text{C}}$

$$\begin{aligned} \frac{3+5ki}{i} - 2i &= \frac{3}{i} + 5k - 2i \\ &= -3i + 5k - 2i \\ &= 5k - 5i \end{aligned}$$

25. $\boxed{\text{A}}$

$$\begin{aligned} 2k - \frac{5+ki}{i} &= 2k - \frac{(5+ki)(i)}{i^2} \\ &= 2k + (5i + ki^2) \\ &= k + 5i \end{aligned}$$

26. $\boxed{\text{C}}$

$$\frac{3+ai}{2-i} = 2 - bi$$

$$\begin{aligned} 3+ai &= (2-bi)(2-i) \\ &= (4-b) + (-2b-2)i \end{aligned}$$

Compare real parts,

$$3 = 4 - b$$

$$b = 1$$

27. $\boxed{\text{A}}$

When $k = 1$,

$$\begin{aligned} \frac{3k}{2i} - i\left(\frac{1}{2}k - 3i\right) &= \frac{3}{2i} - i\left(\frac{1}{2} - 3i\right) \\ &= -3 - 2i \end{aligned}$$

Only option A gives $-3 - 2i$ when $k = 1$.

28. $\boxed{\text{B}}$

I. ✗. $uv = \dfrac{49}{a^2+1}$ is irrational if $a = \pi$.

II. ✓. $u = \dfrac{7(a-i)}{a^2+1} = \dfrac{7a}{a^2+1} - \dfrac{7}{a^2+1}i$ and $v = \dfrac{7(a+i)}{a^2+1} = \dfrac{7a}{a^2+1} + \dfrac{7}{a^2+1}i$ have equal real parts.

III. ✗. Take $a = 1$, using calculator CMPLX mode, $\dfrac{1}{u} = \dfrac{1}{7} + \dfrac{1}{7}i$ and $\dfrac{1}{v} = \dfrac{1}{7} - \dfrac{1}{7}i$.
The imaginary parts are different.

29. $\boxed{\text{C}}$

Note that $z_1 - z_2$ is a real number. Let $z_1 - z_2 = r$, where r is a real number.

$$\frac{2+ki}{1+i} - \frac{k+5i}{2-i} = r$$
$$(2+ki)(2-i) - (k+5i)(1+i) = r(1+i)(2-i)$$
$$[(4+k)+(2k-2)i] - [(k-5)+(5+k)i] = r(3+i)$$
$$9 + (k-7)i = 3r + ri$$

Compare real parts,

$$3r = 9$$
$$r = 3$$

30. $\boxed{\text{A}}$

$$\frac{k+10i}{3+4i} + \frac{k-i}{2+3i} = \frac{(k+10i)(3-4i)}{3^2+4^2} + \frac{(k-i)(2-3i)}{2^2+3^2}$$
$$= \frac{(3k+40)+(30-4k)i}{25} + \frac{(2k-3)+(-3k-2)i}{13}$$

$z_1 + z_2$ is a purely imaginary number.

$$\frac{3k+40}{25} + \frac{2k-3}{13} = 0$$
$$k = -5$$

$z_1 + z_2 = \dfrac{50i}{25} + \dfrac{13i}{13} = 3i$
The imaginary part is 3.

Conventional Questions

31. (a) Let $z = a + bi$.

$$z + i = a + bi + i = a + (b + 1)i$$

Since $z + i$ is real, $b + 1 = 0$, i.e., $b = -1$. 1M+1A

$$(1 - 2i)z = (1 - 2i)(a - i) = (a - 2) - (2a + 1)i$$

Since $(1 - 2i)z$ is purely imaginary, $a = 2$.

Thus, $z = 2 - i$. 1A

(b) $\left[\frac{(1 - 2i)z}{5}\right]^{2014} = \left(\frac{-5i}{5}\right)^{2014}$

$= (-i)^{2014}$

$= i^{2012}i^2$ 1M

$= -1$ 1A

32. (a) $\frac{1 + 5i}{1 - 5i} = \frac{1 + 5i}{1 - 5i} \times \frac{1 + 5i}{1 + 5i}$ 1M

$= \frac{1 + 10i - 25}{1^2 + 5^2}$

$= -\frac{12}{13} + \frac{5}{13}i$ 1A

(b) $2m + \frac{1 + 5i}{1 - 5i} = n - mi$

$2m - \frac{12}{13} + \frac{5}{13}i = n - mi$

So, $2m - \frac{12}{13} = n$ and $\frac{5}{13} = -m$. 1M

Solving, we have $m = -\frac{5}{13}$ and $n = -\frac{22}{13}$. 1A

33. $(1 + i)^2 + a(1 + i) + b = 0$ 1M

$(1 + 2i - 1) + a + ai + b = 0$

$(a + b) + (a + 2)i = 0$

Therefore, $\begin{cases} a + b = 0 \\ a + 2 = 0 \end{cases}$. 1M

Solving, we have $a = -2$ and $b = 2$. 1A

34. (a) $\frac{3 + 6i}{2 - i} = \frac{3 + 6i}{2 - i} \times \frac{2 + i}{2 + i}$ 1M

$= \frac{6 + 12i + 3i + 6i^2}{2^2 - i^2}$

$= \frac{6 - 6 + 15i}{4 + 1}$

$= 0 + 3i$ 1A

(b) $(3i)^4 - 2(3i)^3 + 3(3i)^2 - p(3i) + 9q = 0$ 1M

$$81 + 54i - 27 - 3pi + 9q = 0$$

$$(54 + 9q) + (54 - 3p)i = 0$$

So, $\begin{cases} 54 + 9q = 0 \\ 54 - 3p = 0 \end{cases}$. 1M

Thus, $p = 18$ and $q = -6$. 1A

35. (a) $\dfrac{3+9i}{3-i} = \dfrac{3+9i}{3-i} \times \dfrac{3+i}{3+i}$ 1M

$$= \frac{9 + 3i + 27i + 9i^2}{3^2 - i^2}$$

$$= \frac{9 - 9 + 30i}{9+1}$$

$$= \frac{30i}{10}$$

$= 0 + 3i$ 1A

(b) Substitute $x = 3i$,

$(3i)^4 + (3i)^3 + (3i)^2 + a(3i) + 6b = 0$ 1M

$$81i^4 + 27i^3 + 9i^2 + 3ai + 6b = 0$$

$$81 - 27i - 9 + 3ai + 6b = 0$$

$$72 + 6b + (3a - 27)i = 0$$

Therefore, $\begin{cases} 72 + 6b = 0 \\ 3a - 27 = 0 \end{cases}$. 1M

Thus, $b = -12$ and $a = 9$. 1A

 END OF PAPER