REG-NSCN-2425-ASM-SET 1-MATH
Suggested solutions

Multiple Choice Questions

1. A	2. A	3. B	4. A	5. C
6. B	7. C	8. D	9. B	10. C
11. B	12. C	13. D	14. D	15. C
16. C	17. D	18. B	19. A	20. D
21. A	22. C	23. C	24. D	25. B
26. B	27. B	28. C	29. D	30. B

1. A

   I. ✗. $\frac{1}{i^3} = \frac{i^4}{i^3} = i \neq -i$.

   II. ✗. $\sqrt{-8} = \sqrt{8}i = 2\sqrt{2}i \neq 8i$.

   III. ✓. $5 = 5 + 0i$ is a complex number.

2. A

   $i^{10} - \frac{1}{i^3} = i^2 - \frac{i^4}{i^3} = -1 - i$

3. B

   $i^{2012} = (i^4)^{503} = 1^{503} = 1$

4. A

   $i + i^2 + i^3 + \ldots + i^{100}$
   $= (i + i^2 + i^3 + i^4) + \ldots + (i^{97} + i^{98} + i^{99} + i^{100})$
   $= 0 + \ldots + 0$
   $= 0$

5. C

   I. ✗.

   II. ✓. $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{-1} = -i$

   III. ✓. $i(2 + 5i) = 2i + 5i^2 = 2i - 5$

6. B

   $\sqrt{7} - \sqrt{-9} = \sqrt{7} - \sqrt{9}i$
   $= \sqrt{7} - 3i$

7. C

$$\sqrt{a^2 - bc} = \sqrt{(6k)^2 - (8k)(5k)}$$
$$= \sqrt{-4k^2}$$
$$= \sqrt{4k^2}i$$
$$= 2ki$$

8. D

$$\sqrt{-36k^2} + k\sqrt{-49} - \sqrt{25}k = \sqrt{36k^2}i + k\sqrt{49}i - \sqrt{25}k$$
$$= 6ki + 7ki - 5k$$
$$= -5k + 13ki$$

9. B

$6i(1 - 3i) = 18 + 6i$

$$\begin{cases} x + y = 18 \\ x - 2y = 6 \end{cases}$$

Solving, we have $x = 14$ and $y = 4$.

10. C

$(a - 8i) - 4 = 6 + (b + 3)i$

$(a - 4) - 8i = 6 + (b + 3)i$

We have $a - 4 = 6$ and $-8 = b + 3$.

Solving, we have $a = 10$ and $b = -11$.

11. B

$$(a + bi)^3 = (a + bi)^2(a + bi)$$
$$= [(a^2 - b^2) + 2abi](a + bi)$$
$$= [a(a^2 - b^2) - 2ab^2] + [b(a^2 - b^2) + 2a^2b]i$$
$$= a(a^2 - 3b^2) + b(3a^2 - b^2)i$$

Thus, we have $c = a(a^2 - 3b^2)$.

12. C

$a(2 - i) = -6 + 3bi$

$2a - ai = -6 + 3bi$

We have $2a = -6$ and $-a = 3b$.

Solving, we have $a = -3$ and $b = 1$.

13. D

$z = (x - 4) + 3i$ is purely imaginary.

$$x - 4 = 0$$
$$x = 4$$

14. D

A. $(5+2i)(2+5i) = (10-10)+(4+25)i = 29i$

B. $(5-2i)(2-5i) = (10-10)+(-25-4)i = -29i$

C. $(5+2i)(2-5i) = (10+10)+(-25+4)i = 20-21i$

D. $(5+2i)(5-2i) = 25+4 = 29$

Thus, $(5+2i)(5-2i)$ is a real number.

15. C

$(x+i)(1-i) = (x-i^2)+(-x+1)i$

$= (x+1)+(-x+1)i$

We have $-x+1=0$.

Solving, we have $x=1$.

16. C

$(2-i)z = (2-i)(1+i)(2x-yi)$

$= (3+i)(2x-yi)$

$= (6x+y)+(2x-3y)i$

Since x and y are positive integers such that $6x+y=25$ and $x>y$,

we have $x=4$ and $y=1$.

Thus, $x-y=4-1=3$.

17. D

$2+i+4i^2-5i^4 = 2+i-4-5$

$= -7+i$

The real part is -7.

18. B

$i-2i^2+3i^3-4i^4 = i+2-3i-4 = -2-2i$

Imaginary part $= -2$

19. A

$i^3(ai-4) = -i(ai-4) = a+4i$

Real part $= a$

20. D

Let $x=1$. Using calculator CMPLX mode, $(x-5i)(4+i) = (1-5i)(4+i) = 9-19i$.

Only option D gives a value of 9 when $x=1$.

21. A

$i(1+2i+i^2+3i^3) = i+2i^2+i^3+3i^4$

$= i-2-i+3$

$= 1$

The imaginary part is 0.

22. C

$(5-2i)(3+4i) = (15-8i^2)+(20-6)i$

$= 23+14i$

23. C

$(3-4i)(7+i) = (21-4i^2)+(3-28)i$

$= 25-25i$

24. D

$(a+bi)^2-(a-bi)^2 = [(a+bi)+(a-bi)][(a+bi)-(a-bi)]$

$= (2a)(2bi)$

$= 4abi$

25. B

$(2+ki)\times(5i-4) = 22+34i$

$(-8+5ki^2)+(10-4k)i = 22+34i$

$(-8-5k)+(10-4k)i = 22+34i$

We have $-8-5k = 22$ and $10-4k = 34$.

Solving, we have $k = -6$.

26. B

Let $\alpha = 1$.

Using calculator CMPLX mode, $-i(\alpha-8i^3) = 8-i$.

The answer is B.

27. B

$(3-5i)\times(i-4)-(7-2i) = [(-12-5i^2)+(20+3)i]-(7-2i)$

$= (-7+23i)-(7-2i)$

$= -14+25i$

28. C

$(2+3i)(5-3i) = (10-9i^2)+(15-6)i$

$= 19+9i$

29. D

$$\begin{aligned} i^5(\beta - 3i) &= \beta i^5 - 3i^6 \\ &= 3 + \beta i \end{aligned}$$

30. B

I. ✓. Calculator CMPLX mode.

II. ✓. $1 + z + z^2 = 0$ by calculator. So, $z^3 + z^4 + z^5 = z^3(1 + z + z^2) = 0$.

III. ✗. $(z^{3n} + z^{3n+1} + z^{3n+2}) + \ldots + (z^{12n-3} + z^{12n-2} + z^{12n-1}) + z^{12n} = 0 + 1 = 1 \neq 0$.

Conventional Questions

31. (a) $z^2 = (a + bi)^2$

$= (a^2 + b^2i^2) + 2abi$ 1M

$= (a^2 - b^2) + 2abi$ 1A

(b) $z^2 = bi$

$(a^2 - b^2) + 2abi = bi$

We have $a^2 - b^2 = 0$ and $2ab = b$. 1M

Solving, we have $a = \frac{1}{2}$ and $b = \pm\frac{1}{2}$.

Thus, $z = \frac{1}{2} + \frac{1}{2}i$ or $\frac{1}{2} - \frac{1}{2}i$. 1A+1A

32. (a) $(2 - 3i)(3 - 2i) = (6 + 6i^2) + (-4 - 9)i$ 1M

$= -13i$ 1A

(b) $(2 - 3i)^2(3 - 2i)^3 = [(2 - 3i)(3 - 2i)]^2(3 - 2i)$

$= (-13i)^2(3 - 2i)$ 1M

$= -169(3 - 2i)$

$= -507 + 338i$ 1A

33. If z is a rational number, then z is real.

$2x - 6 = 0$ 1M

$x = 3$

$z = \sqrt{4(3)} + 0i = 2\sqrt{3}$, which is not a rational number. 1A

There is no real number x such that z is a rational number. 1A

34. (a) $6x + 24 = 0$ 1M

$x = -4$ 1A

(b) $z(9 - 5i) = 18 + ki$

$[y - 4(-4)](9 - 5i) = 18 + ki$ 1M

$9(y + 16) - 5(y + 16)i = 18 + ki$

We have $9(y + 16) = 18$ and $-5(y + 16) = k$. 1M

Solving, we have $y = -14$ and $k = -10$. 1A