

REG-EOC-2425-ASM-SET 4-MATH

Suggested solutions

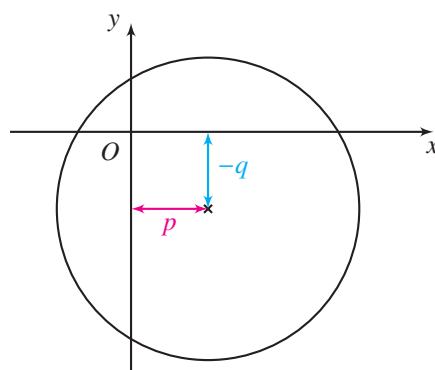
Multiple Choice Questions

- | | | | | |
|-------|-------|-------|-------|-------|
| 1. D | 2. A | 3. D | 4. D | 5. C |
| 6. A | 7. D | 8. A | 9. B | 10. C |
| 11. D | 12. C | 13. C | 14. C | 15. D |
| 16. B | 17. D | 18. A | 19. C | 20. A |
| 21. C | 22. A | 23. D | 24. C | |

1. D

Centre (p, q) lies in quadrant IV. So, $p > 0$ and $q < 0$.

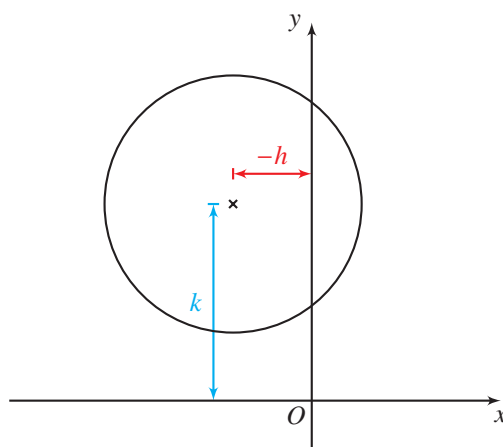
- I. ✓.
- II. ✓. $p - r < 0$
(length p is shorter than the radius)
- III. ✓. $\sqrt{p^2 + q^2} < r$
(distance between origin and centre is smaller than the radius)



2. A

Centre lies in quadrant II. Therefore, $h < 0$ and $k > 0$.

- I. ✓. $k + h = k - (-h) > 0$
(distance from x -axis is longer than that from y -axis)
- II. ✓. $r - h = r + (-h) > 0$
(positive lengths)
- III. ✗. $r - k < 0$
(radius is shorter than distance from x -axis)



3. D

$$\text{I. Radius} = \sqrt{3^2 + 4^2 - 10} = \sqrt{15}$$

$$\text{II. Radius} = \sqrt{4^2 + 3^2 - 10} = \sqrt{15}$$

$$\text{III. Radius} = \sqrt{5^2 - 10} = \sqrt{15}$$

All circles have the same area.

The answer is D.

4. D

$$C: x^2 + y^2 + 3x + 4y - \frac{25}{2} = 0$$

$$\text{Centre} \left(-\frac{3}{2}, -2 \right)$$

$$\text{Radius} = \sqrt{\left(\frac{3}{2}\right)^2 + 2^2 + \frac{25}{2}} = \frac{5\sqrt{3}}{2}$$

5. C

$$\text{Centre} \left(-\frac{k}{2}, -\frac{k+1}{2} \right)$$

$$-\frac{k}{2} - \frac{k+1}{2} + 1 = 0$$

$$k = \frac{1}{2}$$

6. A

$$L \text{ passes through centre of circle } \left(\frac{k}{-2}, -2 \right).$$

$$\frac{3 - (-2)}{16 - \frac{k}{-2}} = \frac{1}{2}$$

$$k = -12$$

7. D

Diameter passes through centre (4, 3).

$$\frac{-5 - 3}{k - 4} = -4$$

$$k = 6$$

8. A

Let the centre of S be G . The coordinates of G are $(1, 2)$. Note that $GM \perp AB$.

$$\text{Slope of } GM = \frac{2+2}{1-3} = -2$$

$$\text{Slope of } AB = \frac{1}{2}$$

Required equation is

$$y + 2 = \frac{1}{2}(x - 3)$$

$$x - 2y - 7 = 0$$

9. B

$$\text{Radius} = \sqrt{1^2 + 2^2 + 4} = 3$$

$$\text{Required area} = 4 \times \frac{(3)(3)}{2}$$

$$= 18$$

10. C

$$\text{Distance between centres} = \sqrt{(3+3)^2 + (7+1)^2}$$

$$= 10$$

$$= 8 + 2$$

The two circles touch each other externally.

11. D

Substitute the coordinates of the points into L.H.S. of the equation.

$$\text{A. } 10^2 + 6^2 - 8(10) + 4(6) - 16 = 64 > 0. \text{ } W \text{ lies outside the circle.}$$

$$\text{B. } 8^2 + 8^2 - 8(8) + 4(8) - 16 = 80 > 0. \text{ } X \text{ lies outside the circle.}$$

$$\text{C. } 6^2 + 6^2 - 8(6) + 4(6) - 16 = 32 > 0. \text{ } Y \text{ lies outside the circle.}$$

$$\text{D. } 9^2 + 0 - 8(9) - 16 = -7 < 0. \text{ } Z \text{ lies inside the circle.}$$

12. C

I. ✓. The coordinates of the centre are $(-1, 0)$.

The centre lies on the x -axis.

II. ✓. Radius $= \sqrt{9} = 3$

III. ✗. When $y = 0$,

$$(x + 1)^2 + 0 = 9$$

$$x = 2 \quad \text{or} \quad -4$$

The circle intersects the x -axis at $(2, 0)$ and $(-4, 0)$.

13. C

$$C_2: x^2 + y^2 + 6x - 8y + \frac{25}{2} = 0$$

I. ✓. $G_1 (8, -6), G_2 (-3, 4)$. $OG_1 = 10 = 2OG_2$

II. ✗. $m_{OG_1} \times m_{OG_2} = \frac{-6}{8} \times \frac{4}{-3} \neq -1$

III. ✓. Radius of $C_1 = \sqrt{8^2 + 6^2 - 75} = 5$, radius of $C_2 = \sqrt{3^2 + 4^2 - \frac{25}{2}} = \frac{5}{\sqrt{2}}$

Ratio of area of C_1 to area of C_2 is $5 : \left(\frac{5}{\sqrt{2}}\right)^2 = 2 : 1$.

14. C

$$C_1: x^2 + y^2 + 2x + 4y - \frac{149}{2} = 0.$$

I. ✓. $(-1)^2 + (2)^2 - 8(-1) - 20(2) - 53 = 0$. It lies on C_2 .

II. ✗. Radius of $C_1 = \sqrt{1^2 + 2^2 + \frac{149}{2}} = \sqrt{79.5}$ and radius of $C_2 = \sqrt{4^2 + 10^2 + 53} = 13$.

III. ✓. Distance between centre = 13, which lies between sum and difference of their radii, i.e., $13 - \sqrt{79.5}$ and $13 + \sqrt{79.5}$.

So, they intersect at two distinct points.

15. D

$$C: x^2 + y^2 - 3x + y - \frac{13}{2} = 0$$

I. ✗.

The coordinates of the centre are $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

II. ✓.

$$AB = \sqrt{(2-1)^2 + (1+2)^2} = \sqrt{10}$$

$$\text{Radius} = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} + \frac{13}{2} = 3 < \sqrt{10}$$

III. ✓.

Denote the centre by G .

$$\text{Slope of } AB = \frac{1+2}{2-1} = 3$$

$$\text{Slope of } BG = \frac{1+\frac{1}{2}}{2-\frac{3}{2}} = 3$$

A, B and G are collinear.

Thus, G lies on the straight line passing through A and B .

16. B

- I. ✓. Radius = $\sqrt{3^2 + 6^2 + 4} = 7$
- II. ✗. Centre (3, -6) lies in the fourth quadrant.
- III. ✓. $0^2 + 0^2 - 6(0) + 12(0) - 4 = -4 < 0$
The origin lies inside the circle.

17. D

- I. ✓. $G_1 (-2, 6), G_2 (2, 4)$. Slope of $OG_2 \times$ slope of $G_1G_2 = \frac{4}{2} \times \frac{6-4}{-2-2} = -1$
- II. ✓. Distance between centres = $\sqrt{(2+2)^2 + (6-4)^2} = \sqrt{20}$
Radius of $C_1 = \sqrt{2^2 + 6^2 + 40} = \sqrt{80} = 2\sqrt{20}$; radius of $C_2 = \sqrt{2^2 + 4^2} = \sqrt{20}$
Since distance between centres = difference in radii, the circles touch each other internally.
- III. ✓. Area ratio = $\left(\frac{\sqrt{80}}{\sqrt{20}}\right)^2 = 4$

18. A

$$x^2 + y^2 - x - \frac{y}{2} + \frac{1}{4} = 0$$

- I. ✗. Centre $\left(\frac{1}{2}, \frac{1}{4}\right)$
- II. ✗. Radius = $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2} - \frac{1}{4} = \frac{1}{4} \neq 2$
- III. ✓. Radius = $\frac{1}{4} = y$ -coordinate of centre.
So, the circle touches the x -axis.

19. C

$$C_1: x^2 + y^2 - 8x - 6y + 20 = 0$$

$$C_2: x^2 + y^2 + 6x - 8y + \frac{33}{2} = 0$$

The coordinates of G_1 and G_2 are (4, 3) and (-3, 4) respectively.

I. ✓.

$$\text{Slope of } G_1O = \frac{3-0}{4-0} = \frac{3}{4}$$

$$\text{Slope of } G_2O = \frac{4-0}{-3-0} = -\frac{4}{3} = -1 \div \frac{3}{4}$$

Thus, G_1O is perpendicular to G_2O .

II. ✗.

$$\text{Area of } C_1 = \pi \left(\sqrt{4^2 + 3^2 - 20} \right)^2 = 5\pi$$

$$\text{Area of } C_2 = \pi \left(\sqrt{3^2 + 4^2 - \frac{33}{2}} \right)^2 = \frac{17\pi}{2} > 5\pi$$

III. ✓.

$$OG_1 = \sqrt{4^2 + 3^2} = 5$$

$$OG_2 = \sqrt{3^2 + 4^2} = 5$$

20. A

$$x^2 + y^2 - 9x + 8y - \frac{1}{2} = 0$$

I. ✓. $0 + y^2 - 0 + 8y - \frac{1}{2} = 0$

$$y \approx 0.0620 \quad \text{or} \quad -8.06$$

The circle intersect y-axis at two points.

II. ✗. Coordinates of centre are $\left(\frac{9}{2}, -4\right)$.

III. ✗. Sub (0, 0), L.H.S. = $-\frac{1}{2} < 0$. Origin lies inside the circle.

21. C

I. ✗. The coordinates of the centres of C_1 and C_2 are (-4, 3) and (4, -3) respectively. They are not concentric circle.

II. ✓. Radius of both circles = $\sqrt{4^2 + 3^2 + 25} = \sqrt{50}$

The lengths of diameters are the same.

III. ✓. Distance between the centre and the y-axis is 4, which is smaller than the radius.

Both C_1 and C_2 cut the y-axis at two distinct points.

22. A

- I. ✓. Radius of $C_1 = \sqrt{3^2 + 4^2} = 5$; radius of $C_2 = \sqrt{25} = 5$
- II. ✓. Distance between centres $= \sqrt{(3-0)^2 + (0+4)^2} = 5$
- III. ✗. $(0, 0)$ does not satisfy the equation of C_2 . C_2 does not pass through the origin.

23. D

- I. ✓. $(0, 0)$ satisfies the equation.
- II. ✗. The coordinates of the centre are $(0, -4)$.
It does not lie on the x -axis.
- III. ✓. Radius $= \sqrt{0^2 + 4^2} = 4$
Distance between centre and x -axis is equal to the radius.
 C touches the x -axis.

24. C

Denote the centre by G .

Let M be a point on the x -axis such that GM is perpendicular to the x -axis.

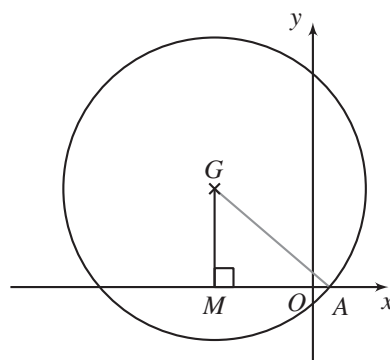
Let A be the intersection of the circle and the positive x -axis.

The coordinates of M are $(-3, 0)$.

$AM = \frac{8}{2} = 4$ and the coordinates of A are $(1, 0)$.

Radius of circle $= \sqrt{(1+3)^2 + (0-3)^2} = 5$

Required equation is $(x+3)^2 + (y-3)^2 = 25$.



Conventional Questions

25. (a) $(x - 6)^2 + (y + 5)^2 = 6^2 + 5^2$ 1M
 $(x - 6)^2 + (y + 5)^2 = 61$ 1A
- (b) (i) $H = (12, 0)$ and $K = (0, -10)$ 1A+1A
(ii) O, P and Q are collinear. 1A
(iii) Required area $= 12 \times 10$ 1M
 $= 120$ 1A
26. (a) Since $FB = FE$, $\angle FBE = \angle FEB$.
 $\angle FCA = \frac{1}{2}\angle FEA$ 1M
In $\triangle ABC$,
 $\angle ABC + \angle BCA = \angle CAE$
 $\angle ABC + \frac{1}{2}\angle ABC = \theta$ 1M
 $\angle ABC = \frac{2\theta}{3}$ 1A
- (b) (i) $\angle ABC = \frac{2}{3}(45^\circ) = 30^\circ$ 1M
 $BE = \frac{CE}{\tan 30^\circ} = \sqrt{3}CE$ and $AE = \frac{CE}{\tan 45^\circ} = CE$
 $AB = BE - AE$
 $0 - (1 - \sqrt{3}) = CE(\sqrt{3} - 1)$ 1M
 $CE = 1$ 1A
Coordinates of $C = (0 + 1, 0 + 1) = (1, 1)$ 1A
Coordinates of $D = (0 + 1 + 1, 0) = (2, 0)$ 1A
(ii) Equation of circle $ADCF$ is $(x - 1)^2 + y^2 = 1$ 1A
27. (a) $y^2 - 12y + 32 = 0$
 $y = 4$ or 8
Coordinates of A are $(0, 4)$. 1A
- (b) $c = 4$ 1A
Coordinates of P are $(6, 6)$. 1M
Slope of $AP = \frac{6 - 4}{6 - 0} = \frac{1}{3}$
Slope of $L = m = -3$ 1A
- (c) Let the x -coordinate of B be b .
Since B lies on $y = -3x + 4$, the coordinates of B are $(b, -3b + 4)$.
 $\sqrt{b^2 + (-3b + 4 - 4)^2} = \sqrt{(0 - 6)^2 + (4 - 6)^2}$ 1M
 $b^2 + 9b^2 = 40$
 $b = 2$ or -2 (rejected)

The coordinates of B are $(2, -2)$. 1A

The equation of C_2 is

$$(x + 10)^2 + (y + 6)^2 = (2 + 10)^2 + (-2 + 6)^2 \quad 1M$$

$$(x + 10)^2 + (y + 6)^2 = 160 \quad 1A$$

28. (a) $OQ = OP = r$ 1A

$$AP = AQ = 4 - r \text{ and } BP = BR = 3 - r \quad 1M+1A$$

$$(b) (3 - r) + (4 - r) = \sqrt{3^2 + 4^2} \quad 1M$$

$$r = 1$$

The coordinates of C are $(1, 1)$. 1A

(c) The equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = 1^2$$

$$x^2 + y^2 - 2x - 2y + 1 = 0 \quad 1A$$

29. (a) Let the equation of circle be $x^2 + y^2 + Dx + Ey + F = 0$, where D, E and F are constants. 1A

$$\begin{cases} 1 + 4 + D + 2E + F = 0 & (1) \end{cases}$$

$$\begin{cases} 9 + 3E + F = 0 & (2) \quad 1M \end{cases}$$

$$\begin{cases} 16 + 4D + F = 0 & (3) \end{cases}$$

Consider $(2) - (1)$ and $(3) - (2)$.

$$\begin{cases} -D + E = -4 \\ 4D - 3E = -7 \end{cases} \quad 1M$$

Solving, we have $D = -19, E = -23$. 1A

When $D = -19, E = -23, F = -9 - 3(-23) = 60$.

The equation of the circle is $x^2 + y^2 - 19x - 23y + 60 = 0$. 1A

(b) Centre of the circle = $\left(\frac{19}{2}, \frac{23}{2}\right)$ 1A

$$\text{Radius of the circle} = \sqrt{\left(\frac{19}{2}\right)^2 + \left(\frac{23}{2}\right)^2 - 60} = \frac{5\sqrt{26}}{2} \quad 1A$$

(c) If two points on the circle form a diameter, then the mid-point of them must be at the centre of circle.

$$\text{mid-point of } AB = \left(\frac{1}{2}, \frac{5}{2}\right) \quad 1M$$

$$\text{mid-point of } BC = \left(2, \frac{3}{2}\right)$$

$$\text{mid-point of } CA = \left(\frac{5}{2}, 1\right)$$

None of the above is at the centre of the circle.

Thus, the claim is incorrect. 1A