REG-CP2B-2425-ASM-SET 4-MATH
Suggested solutions

Multiple Choice Questions

1. D	2. B	3. C	4. B	5. B
6. D	7. C	8. C	9. A	10. A
11. B	12. C	13. D	14. B	15. A
16. C	17. B	18. A	19. A	20. D
21. A	22. D	23. C	24. B	25. D
26. A	27. A	28. B	29. A	30. C
31. B	32. D	33. C	34. D	35. B
36. C	37. A	38. B	39. A	

1. $\boxed{\text{D}}$

Volume of tetrahedron $ABCD = \frac{1}{3}\left[\frac{(3)(2)}{2}\right](4)$

$= 4\text{ cm}^3$

$BC = \sqrt{3^2+2^2} = \sqrt{13}\text{ cm}$
$AC = \sqrt{2^2+4^2} = \sqrt{20}\text{ cm}$
$AB = \sqrt{3^2+4^2} = 5\text{ cm}$
Let $s = \dfrac{AB+AC+BC}{2} \approx 6.54\text{ cm}$.
Area of $\triangle ABC = \sqrt{s(s-AB)(s-AC)(s-BC)}$

$\approx 7.81\text{ cm}^2$

Let the required angle be θ. Consider the volume of the tetrahedron.

$$4 = \frac{1}{3}(\text{area of }\triangle ABC)(CD\sin\theta)$$
$$\theta \approx 50^\circ$$

2. $\boxed{\text{B}}$

Let M be a point on BC such that $AM \perp BC$.
Note that AMD is a plane of reflection of the tetrahedron and it is perpendicular to the plane BCD.
We have $\angle AMD = 80^\circ$ and the required angle is $\angle ADM$.
Consider $\triangle ABC$.
$AM = 56\sin 60^\circ = 28\sqrt{3}$ cm
Consider $\triangle BCD$.
$DM = \sqrt{60^2 - 28^2} = \sqrt{2816}$ cm
Consider $\triangle AMD$.
$AD^2 = AM^2 + DM^2 - 2(AM)(DM)\cos 80^\circ$
$AD \approx 65.4$ cm
$AM^2 = AD^2 + DM^2 - 2(AD)(DM)\cos\angle ADM$
$\angle ADM \approx 47^\circ$

3. $\boxed{\text{C}}$

Let K be the mid-point of EF.
Note that $M_5K \perp M_4M_5$.
We have $M_1M_5 \perp M_4M_5$ by theorem of three perpendiculars.
Thus, $\theta = \angle KM_5M_1$.
Let $AB = 2x$ cm.

$$\tan\theta = \frac{KM_1}{M_1M_5} = \frac{2x}{\sqrt{x^2 + x^2}} = \sqrt{2}$$

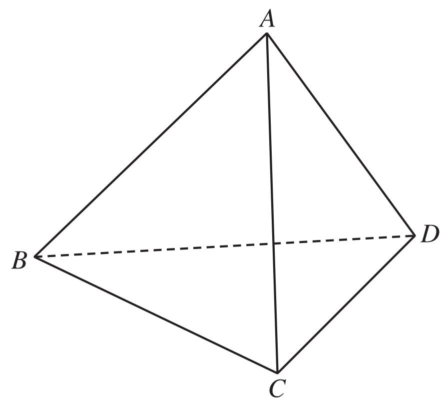
4. $\boxed{\text{B}}$

Let Q be a point on BG such that $PQ \perp BG$.
Then $\theta = \angle PCQ$.
$CQ = \sqrt{2^2 + 5^2} = \sqrt{29}$ cm

$$\cos\theta = \frac{CQ}{PC} = \frac{\sqrt{29}}{\sqrt{14^2 + (\sqrt{29})^2}} = \frac{\sqrt{29}}{15}$$

5. B

Refer to the figure. Let E be the mid-point of BC and length of each side be x cm.



In $\triangle AED$, $AE = DE = x \sin 60° = \frac{\sqrt{3}x}{2}$ cm.

$$x^2 = AE^2 + DE^2 - 2(AE)(DE)\cos\angle AED$$

$$\angle AED = \cos^{-1}\frac{1}{3}$$

Consider the volume of the tetrahedron.

$$\frac{1}{3}\left(\frac{x^2}{2}\sin 60°\right)(AE\sin\angle AED) = 576$$

$$x \approx 17.0$$

Let X be the projection of A on the plane BCD.

Note that D, X, E are collinear.

Consider $\triangle AXE$.

Required height $= AE\sin\angle AED$

≈ 13.9 cm

6. D

Note that $\angle CAB = 50°$, $\angle PBA = \angle PBC = 90°$.

We have $BC = \frac{PB}{\tan 35°}$ and $AB = \frac{PB}{\tan 40°}$.

$$\frac{AB}{\sin\angle ACB} = \frac{BC}{\sin\angle BAC}$$

$$\frac{PB}{\tan 40°\sin\angle ACB} = \frac{PB}{\tan 35°\sin 50°}$$

$$\angle ACB \approx 39.7°$$

Required bearing is N39.7°W.

7. $\boxed{\text{C}}$

Let K be a point on AC such that $DK \perp AC$.
We have $x = \angle DKE$.
Consider the area of $\triangle ACD$.

$$\frac{(AD)(CD)}{2} = \frac{(AC)(DK)}{2}$$
$$\frac{(3)(4)}{2} = \frac{\sqrt{3^2+4^2}(DK)}{2}$$
$$DK = 2.4\,\text{cm}$$

Consider $\triangle DKE$.

$$\tan x = \frac{DE}{DK} = \frac{2}{2.4} = \frac{5}{6}$$

8. $\boxed{\text{C}}$

Let $PQ = 2$. Then $RS = 3$.
$RQ = \dfrac{PQ}{\tan 45^\circ} = 2$
$QS = \dfrac{PQ}{\tan 30^\circ} = 2\sqrt{3}$
Consider $\triangle QRS$.

$$RS^2 = RQ^2 + QS^2 - 2(RQ)(QS)\cos\angle RQS$$
$$\angle RQS \approx 60^\circ$$

9. $\boxed{\text{A}}$

Let K be a point on ME such that $FK \perp ME$. [In fact, K is at the position of point M.]
Since $AF \perp EM$, we also have $AK \perp ME$. The angle required is therefore $\angle AKF$.
Since $MH = EH = 12\,\text{cm}$, $\angle EMH = 45^\circ$ and so $\angle FEM = 45^\circ$
$FK = FE \sin\angle FEM = 12\sqrt{2}\,\text{cm}$
Required angle = $\tan^{-1}\dfrac{AF}{FK} = \dfrac{10}{12\sqrt{2}} \approx 31^\circ$

10. $\boxed{\text{A}}$

Note that $\theta = \angle BKA$.
$AK = \sqrt{12^2+9^2} = 15\,\text{cm}$
$BK = \sqrt{15^2+8^2} = 17\,\text{cm}$

$$\cos\theta = \frac{AK}{BK} = \frac{15}{17}$$

11. B

PR (equation $x = 3$) is vertical and is the tangent to the circle.
Consider the centre and radius of the circles.

A. ✗. Centre $\left(9,\ \frac{9}{2}\right)$, radius $= \sqrt{9^2 + \left(\frac{81}{4}\right)^2 - 59} = 6.5$.
Distance between centre and $x = 3$ is $6 \neq 6.5$.

B. ✓. Centre $(5,\ 4)$, radius $= \sqrt{5^2 + 4^2 - 37} = 2$.
Distance between centre and $x = 3$ is 2.

C. ✗. Centre $\left(\frac{5}{2},\ 2\right)$, radius $= \sqrt{\left(\frac{5}{2}\right)^2 + 2^2 - 37}$, which is not a real number.

D. ✗. Centre $\left(\frac{9}{2},\ \frac{9}{4}\right)$, radius $= \sqrt{\left(\frac{9}{2}\right)^2 + \left(\frac{9}{4}\right)^2 - 59}$, which is not a real number.

12. C

Solve the simultaneous equations $\begin{cases} mx - y - 5 = 0 \\ x^2 + y^2 - 11x + 7y + 20 = 0 \end{cases}$ using the calculator program.

Value of m	Number of intersections	Sign of Δ
-3	0	$-$

Required range contains -3 and -3 is not a boundary value.
The answer is C.

13. D

Solve the system $\begin{cases} mx - y - 1 = 0 \\ x^2 + y^2 - 16x - 2y + 31 = 0 \end{cases}$ using the calculator program.

The system has repeated solutions when $m = \frac{5}{3}$ and when $m = -\frac{3}{5}$.
Thus, $m = \frac{5}{3}$ or $-\frac{3}{5}$.

14. B

Solve the system $\begin{cases} 2x - y + b = 0 \\ x^2 + y^2 - 2x - y + \frac{5}{4} = 0 \end{cases}$ using the calculator program.

A. ✗. No intersections

B. ✓. 1 intersection: $\left(1,\ \frac{1}{2}\right)$

C. ✗. No intersections

D. ✗. No intersections

15. A

Solve the system $\begin{cases} 3x + 4y + k = 0 \\ x^2 + y^4 - \frac{9}{4} = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
$-\frac{15}{2}$	1	0
0	2	+

Required range has $-\frac{15}{2}$ as a boundary value (not equal to) and includes 0.
The answer is A.

16. C

When the straight line passes through centre $(-1,\ 2)$, $k = -1 - 2 = -3$.
mid-point of AB is then centre of the circle.
When $k = -3$, y-coordinate of mid-point = 2.
Only option C satisfies this.

17. B

Solve the system $\begin{cases} 2x - y + k = 0 \\ x^2 + y^2 - 8x - 10y - 39 = 0 \end{cases}$ using the calculator program.

A. ✗. 2 distinct intersections.

B. ✓. 1 intersection: $(4,\ 9)$.

C. ✗. 2 distinct intersections.

D. ✗. 2 distinct intersections.

18. A

Solve the system $\begin{cases} mx - y = 0 \\ x^2 + y^2 - 6x + 4 = 0 \end{cases}$ using the calculator program.

A. 1 intersection → tangent

B. No intersections

C. No intersections

D. No intersections

19. A

Solve the system $\begin{cases} kx - y + 2 = 0 \\ x^2 + y^2 - 5x - 9y + 24 = 0 \end{cases}$ using calculator program.

Value of k	Number of intersection	Sign of Δ
$\frac{1}{3}$	1	0
-3	0	$-$

Required range has $\frac{1}{3}$ as one of the boundary value, and it contains $k = -3$.
The answer is A.

20. D

Solve the system $\begin{cases} x - y + k = 0 \\ x^2 + y^2 - 2x + 4y - 3 = 0 \end{cases}$ using the calculator program.

Value of k	Number of intersections	Sign of Δ
-7	1	0
0	2	+

Required range has -7 as a boundary value and excludes 0.
The answer is D.

21. A

Solve the system $\begin{cases} x + 2y + k = 0 \\ x^2 + y^2 + 2y - 4 = 0 \end{cases}$ using the calculator program.

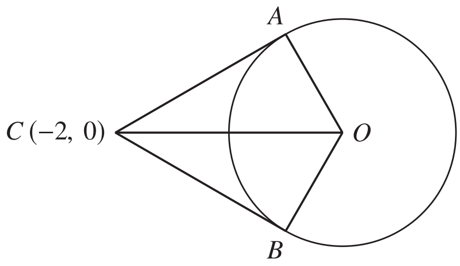
Value of k	Number of intersections	Sign of Δ
7	1	0
0	2	+

Required range contains "7" as a boundary value, and does not include 0.
The answer is A.

22. D

Radius of circle = 1; $\angle AOC = \cos^{-1}\dfrac{1}{2} = 60^\circ$

$\angle AOB = 120^\circ$ and $AB = \sqrt{1^2 + 1^2 - 2(1)(1)\cos 120^\circ} = \sqrt{3}$



23. C

Let the standard deviation of the scores and the score of Billy be σ and x respectively.

$$\begin{cases} 2a = \dfrac{60 - 74}{\sigma} \\ -a = \dfrac{x - 74}{\sigma} \end{cases}$$

$$\frac{2a}{-a} = \frac{60 - 74}{\sigma} \div \frac{x - 74}{\sigma}$$

$$-2 = \frac{-14}{x - 74}$$

$$x = 81$$

24. B

Let z be the standard score of Alice.

$$\frac{196 - 136}{62} < z < \frac{200 - 119}{62}$$

$$0.968 < z < 1.31$$

The answer is B.

25. D

I. ✗. New median = $(m + 7) \times 4 = 4m + 28$

II. ✗. New variance = $4^2 v = 16v$

26. A

I. ✓. m_1 lies between the max and min data of the original dataset.

II. ✓. The new dataset is less dispersed.

III. ✗. The mean remains unchanged.

27. A

Let the mean of the scores be $\overline{x}$ marks.

$$\frac{96-\overline{x}}{3}=4$$
$$\overline{x}=84$$

Required standard score $=\frac{81-84}{3}$
$=-1$

28. B

Let μ and σ be the mean and the standard deviation of the scores in the test respectively.

$$\frac{96.5-\mu}{\sigma}-\frac{58-\mu}{\sigma}=4-(-1.5)$$
$$\frac{96.5-58}{\sigma}=5.5$$
$$\sigma=7$$

29. A

Let the score of the new student be y, then $x=\frac{y-m}{s}$.

New standard score $=\frac{(1.5y+10)-(1.5m+10)}{1.5s}$
$=\frac{1.5(y-m)}{1.5s}$
$=\frac{y-m}{s}$
$=x$

30. C

Let the score of Mary be M.

$$M-M(1-12.5\%)=10$$
$$M=80$$

Peter scores 70 marks in the test.

$$\begin{cases}-1=\dfrac{70-\mu}{\sigma}\\ 1.5=\dfrac{80-\mu}{\sigma}\end{cases}$$

Solving, we have $\mu=74$ and $\sigma=4$.
The mean score of the test is 74 marks.

31. B

The marks are multiplied by a constant h and then added by a constant k.
Then $32h+k=52$ and $4h=8$. Solving, we have $h=2$ and $k=-12$.
Simon's new mark $=23(2)-12=34$

32. D

The new group of numbers is formed by the following procedures:

- Multiply each number by 3.
- Add 2 to each resulting number.

	Mean	Inter-quartile range	Variance
Original group	m_1	r_1	v_1
$\times 3$	$3m_1$	$3r_1$	$9v_1$
$+2$	$2+3m_1$	$3r_1$	$9v_1$

33. C

$\times 2 \quad \longrightarrow \quad +15$

New variance $= k^2(2^2) = 4k^2$

34. D

Variance of the group of numbers $\{b, 2b, 3b\}$ is also 4.

Variance of the group of numbers $\{1, 2, 3\}$ is $\frac{2}{3}$.

$$\frac{2}{3} \times b^2 = 4$$
$$b^2 = 6$$

Variance of the group of numbers $\{1, 3, 5\}$ is $\frac{8}{3}$.

Required variance = variance of the group of numbers $\{b, 3b, 5b\}$

$$= \frac{8}{3} \times b^2$$
$$= 16$$

35. B

$\mu_2 = \mu_1$ and the data set is less dispersed $\Rightarrow$ standard deviation is decreased.

However, when all data are equal, the new and original standard deviations are both 0.

36. C

There are two steps for the transformation of data set: $\times 2$, $+23$.

New standard deviation $= 14 \times 2 = 28$

37. A

The new group of numbers is formed by inserting a mean x_1 and then add 1.

I. ✓. $x_2 = x_1 + 1$

II. ✗. We cannot find the median of the new group of numbers.

III. ✗. $z_2 = z_1$

38. B

Let the mean and standard deviation be μ marks and σ marks respectively.

$$\begin{cases} \dfrac{16-\mu}{\sigma} = 2 \\ \dfrac{4-\mu}{\sigma} = -1 \end{cases}$$

Solving, we have $\mu = 8$ and $\sigma = 4$.

39. A

Each datum is multiplied by 3 and then add 10.

I. ✓.

II. ✗. $q_2 = 3q_1$.

III. ✗. $v_2 = 3^2 v_1$

END OF PAPER